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Original Paper

Unified Lattice Framework G4: Primitive Geometry, Derived Constants, and a Discriminating Test of Equilibrium Radiation

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Abstract: The preceding G–series (G1–G3) established that equilibrium black–body radiation admits a finite, well–posed description based on discrete cavity geometry, finite boundary response, and standard equilibrium statistics, without invoking a continuum density of states. These results were obtained strictly within static, finite cavities and do not modify the dynamical structure or empirical predictions of quantum electrodynamics. In this capstone work, that reconstruction is elevated to an origin–framework statement. We formulate the Unified Lattice Framework (ULF) at the origin level by: (i) stating its primitive ontology, bounds, and observables explicitly; (ii) demonstrating that within the equilibrium radiation sector, the constant appearing in the Planck–Einstein relation may be interpreted as an operational calibration of bounded geometric exchange, without altering photon counting or field quantization; and (iii) proposing a concrete, falsifiable experimental discriminator based on geometry– dependent equilibrium linewidth scaling in high–Q cavities. The ontological and epistemological commitments of ULF are clarified in Appendix A, formal primitive closure across matter, radiation, and vacuum boundary exchange is established in Appendix B, and the Casimir effect is shown in Appendix C to arise as a curvature–bounded boundary–exchange phenomenon requiring no additional primitives. Taken together, these results complete the equilibrium radiation arc of the G–series and place ULF in a testable origin–level form in which physical constants and classical limits emerge from finite geometric structure while remaining fully compatible with standard quantum and classical predictions.

Keywords: Bounded geometric exchange; primitive ontology; Planck constant calibration; equilibrium cavity linewidth; origin-level radiation framework

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Global Introduction

The black–body radiation problem historically marked a decisive turning point in the foundations of physics. The Rayleigh–Jeans ultraviolet catastrophe exposed the failure of classical continuum assumptions at thermal equilibrium and motivated Planck's introduction of energy quantization. While Planck's law successfully reproduces observed spectra, its standard derivations elevate Planck's constant to the status of a fundamental axiom and retain continuum densities of states and idealized boundary conditions as foundational assumptions.

The Unified Lattice Framework (ULF) approaches this problem from a different direction. Across quantum matter stability, Yang–Mills confinement, gravitation, cosmology, and magnetism, ULF has demonstrated that apparent divergences and singularities arise not from intrinsic physical inconsistencies, but from unbounded geometric idealizations [1–4]. Finite curvature, discrete geometry, and bounded exchange suffice to regularize these theories without modifying their underlying dynamical equations.

Within this geometric program, equilibrium radiation represents the simplest and most controlled setting in which origin–level questions may be addressed explicitly. The G–series was therefore constructed as a staged reconstruction of black–body radiation from first principles.

In G1, equilibrium radiation was reformulated as a population of discrete cavity modes regulated by finite boundary quality, resolving the ultraviolet catastrophe at its geometric origin [5]. In G2, the Planck spectral envelope was shown to emerge as a macroscopic consequence of dense mode overlap under finite spectral resolution, without invoking a continuum density of states [6]. In G3, Wien displacement and Stefan–Boltzmann scaling were recovered through explicit mode counting and energy accumulation, demonstrating that all classical equilibrium radiation laws arise as emergent geometric limits [7].

Together, G1–G3 establish that equilibrium radiation is fully determined by discrete geometry, finite boundary response, and equilibrium statistics, up to a single global normalization constant. What remains unresolved at that stage is not the structure of equilibrium radiation, but the origin–level status of that constant itself. In conventional treatments, this role is played by Planck's constant $\hbar$, introduced as a fundamental quantum postulate.

The purpose of the present work is to show that within ULF, $\hbar$ is not an ontological primitive, but an operational calibration of bounded geometric exchange. We demonstrate that $\hbar$ may be reduced to a function of curvature bounds, boundary correlation lengths, and measurement conventions, placing it on the same conceptual footing as constants such as k_B and c.

More broadly, this paper clarifies when a unifying physical framework attains explanatory closure at the level of origins. An origin framework is understood here not by definition but by construction: it states its primitive ontology explicitly, eliminates pathologies at their geometric source, derives universal laws as emergent limits, reduces normalization constants to operational calibrations, and yields experimentally discriminating consequences.

By deriving equilibrium radiation from bounded geometry, interpreting Planck's constant as a measure of finite exchange rather than a quantization axiom, and proposing a falsifiable test based on geometry–dependent equilibrium linewidth scaling, the present work places the ULF radiation sector in a closed and testable form.

ULF in Context: Relation to QFT, Photon Counting, and Discrete Geometry

Before addressing the origin-level status of Planck's constant within the G–series, it is essential to specify the *scope* of the present framework and clarify its relationship to well-established theories. ULF is not proposed as an alternative to quantum electrodynamics (QED), quantum field theory (QFT), or any dynamical theory of matter. All conventional predictions of QED—including photon counting, field quantization, occupation statistics, and discrete detection events—remain unchanged. ULF operates only in the restricted domain treated in G1–G4: static equilibrium radiation in finite cavities with physically realized, finite-response boundaries.

In this work, the term *origin-level* refers to explanations that specify the primitive ontology, admissibility constraints, and operational structure prior to the introduction of continuum equations or quantization rules. Origin-level analysis concerns the explanatory order, not the empirical predictions, and therefore does not modify established dynamical laws.

Compatibility with QFT, photon counting, and quantum optics

ULF is fully compatible with the standard machinery of QED:

- Photon-number operators, discrete detection events, and Poissonian or Bose–Einstein counting statistics remain unchanged.
- The Planck–Einstein relation $E = \hbar\omega$ retains its full quantum mechanical status.
- All quantum-optical processes (photoelectric effect, spontaneous emission, stimulated emission, coherent states) remain as in standard theory.

Bounded exchange at the geometric level is compatible with discrete detection because photodetectors have finite thresholds and effectively implement quantized response to exchange events. ULF does not alter these thresholds or the quantum description that governs them. Its contribution is to explain why a *single* global action scale appears universally when equilibrium radiation is modeled from geometric primitives.

Distinction from lattice QFT, causal sets, and loop quantum gravity

The discrete geometric substrate invoked in ULF is not:

- a Planck-scale spacetime discretization,
- a regularization of continuum QFT,
- a causal-set model,
- a loop-quantized spatial complex,
- nor a proposal for quantum gravity or modified relativistic kinematics.

Discreteness in ULF refers only to *the admissibility of exchange channels*: a finite geometric complex whose adjacency relations determine which boundary-coupled modes may participate in equilibrium exchange. This structure is non-dynamical, non-relativistic, and does not encode spacetime geometry. No violation or modification of Lorentz invariance is implied or required.

Relation to renormalization and continuum limits

Because ULF treats equilibrium radiation directly in discrete cavities with finite boundary response, no continuum limit is ever taken and no divergences arise. Renormalization does not appear at the primitive level because the derivation does not require a continuum field vacuum or infinite density of states. This should not be interpreted as a replacement for renormalization in QFT; it simply reflects that the G–series operates entirely within finite, static geometries where renormalization is not invoked.

Bounded geometry as the unifying structure of the G–series

The unifying element of G1–G4 is the same geometric principle: exchange is physically finite because geometry and boundary response are finite. This boundedness plays distinct, but coherent, roles across the series:

1. **G1:** Finite boundary quality smooths the discrete cavity spectrum and removes the ultraviolet catastrophe without continuum densities of states.
2. **G2:** The Planck envelope arises from dense-but-finite mode overlap under finite spectral resolution.

3. **G3:** Wien displacement and Stefan–Boltzmann scaling follow from temperature-driven accumulation of discrete modes constrained by geometry.
4. **G4:** The remaining global normalization constant is shown to be an operational calibration of bounded exchange rather than a postulated quantum axiom.

The same geometric boundedness that regularizes equilibrium radiation in G1–G3 also determines the action-scale calibration in Section 3 and produces a falsifiable prediction (geometry-dependent equilibrium linewidth scaling) in Section 4.

Domain of validity

ULF makes no claims about:

- non-equilibrium quantum dynamics,
- interacting relativistic quantum fields,
- gravitational physics or quantum gravity,
- renormalization flow or ultraviolet completion in QFT,
- or any domain outside static equilibrium and finite cavity geometries.

Within these limits, ULF provides an origin-level account of equilibrium radiation that is fully consistent with standard quantum and classical predictions while offering a geometric interpretation of the normalization constant and a concrete, reproducible experimental test.

1. Scope and Structure of This Work

This paper is organized as follows. In Section 2, we state the primitive ontology of ULF, distinguishing what exists physically, what is fundamentally bounded, and what is operationally measured. Section 3 demonstrates how Planck's constant emerges as a calibration of bounded geometric exchange rather than as a postulated quantum axiom. Section 4 presents a concrete experimental discriminator based on equilibrium cavity radiation, identifying a measurable deviation absent from conventional continuum and quantum treatments. We conclude with a capstone statement outlining the implications of these results for future experimental and theoretical work.

The present work does not introduce new physical postulates beyond those already established in the G–series and the broader Unified Lattice Framework [1–4]. Rather, it consolidates and sharpens their implications.

Specifically, G4 makes explicit what is already latent in the preceding analysis: that equilibrium radiation within ULF is governed by a well-defined primitive ontology, that its classical pathologies are eliminated at their geometric source, that its universal laws arise as emergent limits, and that its remaining normalization constant admits an operational interpretation tied to bounded exchange.

The sections that follow therefore proceed constructively rather than axiomatically. We state the primitive ontology required for equilibrium radiation, derive the status of Planck's constant within that ontology, and identify a concrete experimental test. The framework is evaluated by the coherence, closure, and falsifiability of the resulting physical description.

1.1. The role of the G–series in the equilibrium radiation program

The G–series was designed as a minimal test-bed for foundational closure because equilibrium radiation is simultaneously historically decisive, highly constrained, and dominated by geometric idealizations in standard derivations. In particular, the ultraviolet catastrophe arises only after replacing the discrete cavity spectrum by an unbounded continuum.

G1 established that equilibrium radiation in a finite cavity can be treated as a population of discrete modes with finite boundary response, yielding convergent energy and a smooth Planck–like envelope without invoking a continuum density of states or ad hoc ultraviolet regulators [5]. The pathology is traced to a geometric idealization (unbounded continuum replacement) and removed by restoring physical discreteness and finite boundary coupling.

G2 and G3 then established that the remaining black–body laws arise as successive emergent limits of the same discrete, bounded exchange picture: the Planck spectral envelope as a dense mode overlap phenomenon (G2), and Wien displacement plus Stefan–Boltzmann scaling as geometric consequences of mode accumulation under temperature variation (G3) [6,7]. In particular, G3 emphasizes that the temperature scaling exponent should be measured from the discrete model rather than imposed *a priori*, and that deviations from the classical T^4 exponent reflect intrinsic geometric effects rather than a breakdown of equilibrium physics.

Taken together, G1–G3 fix the *relative* structure of equilibrium radiation within ULF: spectral shape, peak displacement, and temperature scaling are determined once discrete geometry and finite boundary response are specified. What remains for the present capstone is to state the primitive ontology explicitly, interpret the remaining global normalization constant as an operational calibration rather than a postulate, and identify a discriminating experimental consequence of bounded exchange.

2. Primitive Ontology of the Unified Lattice Framework

This section states the minimal primitive content required for the G–series capstone. The goal is not to re-derive the full Unified Lattice Framework synthesis, but to specify

(i) what exists, (ii) what is fundamentally bounded, and (iii) what is measured, in a form sufficient to make the $\hbar$–reduction and the discriminating experimental test logically well posed. The primitives and bounds stated here are chosen to match the equilibrium radiation construction developed in G1–G3.

2.1. Primitives and state descriptors

Definition 1 (Discrete substrate)**.** *The* ULF substrate *is a discrete geometric complex $\mathcal{L}$ (cells with adjacency) whose adjacency relations define admissible exchange channels. Smooth fields, when used, are by definition coarse–grained descriptions of collective excitations of $\mathcal{L}$ rather than ontological primitives.*

Definition 2 (Boundaries as exchange agents)**.** *A* boundary *is a physically realized interface $\partial\mathcal{L}$ coupling substrate modes to one another and to measurement channels. It is characterized by a finite response (or correlation) length $\ell_b > 0$ and a finite relaxation time $\tau_b > 0$ (equivalently, a finite bandwidth of exchange). Boundaries are not ideal constraints; they are dynamical exchange agents.*

Definition 3 (Exchange process)**.** *An* exchange process *is a bounded transfer of energy or information between discrete modes of $\mathcal{L}$ and/or between modes and $\partial\mathcal{L}$, mediated exclusively through admissible adjacency channels and boundary response.*

Remark 1 (Non-primitives)**.** *Neither a continuum density of states nor an* a priori *quantization rule is primitive in this sector. Both appear, if at all, as derived coarse–grained descriptions or operational calibrations. This conceptual inversion is already enacted in G1–G3 for the construction of equilibrium radiation.*

2.2. Fundamental bounds

The origin-remedial force of ULF is expressed through explicit boundedness assumptions at the primitive level. For the present capstone, only three such bounds are required.

Axiom 1 (Curvature bound)**.** *There exists a universal finite curvature bound $\kappa_{\max}$ such that every physically admissible exchange configuration satisfies*

$$\kappa \leq \kappa_{\max}. \tag{1}$$

This bound is a physical property of the substrate rather than a mathematical regulator.

Axiom 2 (Boundary response bound)**.** *Boundaries possess finite correlation length ℓ_b and finite relaxation time τ_b. Consequently, there exists a maximal boundary–mediated exchange rate.*

Axiom 3 (Observability bound)**.** *A mode is* physically observable *only insofar as it is coupled to a measurement channel through the boundary response. Modes that are uncoupled, or coupled below the resolution set by ℓ_b, τ_b, and instrumental limits, are not operational observables.*

Remark 2 (What is being asserted)**.** *Axioms 1–3 are not fixes appended to an otherwise continuum model. They define the domain of physically admissible idealizations. In the radiation setting, this is precisely what removes the classical ultraviolet pathology at its geometric origin: unbounded continuum replacement is excluded as nonphysical rather than regulated post hoc.*

2.3. Measured quantities and the operational map

ULF distinguishes sharply between primitive structure and measured observables. In the radiation sector, the operational map is minimal and concrete.

Definition 4 (Radiation-sector observables)**.** *Given a fixed cavity geometry (a bounded region with boundary $\partial\mathcal{L}$), the measured observables are:*

1. *Mode frequencies ω_n determined by geometry through standing–wave constraints.*
2. *Boundary–induced linewidths Γ_n and corresponding quality factors*

$$Q_n := \frac{\omega_n}{\Gamma_n}. \tag{2}$$

3. *Spectral outputs obtained from finite–resolution aggregation of modal energy (e.g. binned spectra) and integrated equilibrium quantities such as the total cavity energy $U(T)$.*

These are precisely the quantities employed throughout G1–G3.

Proposition 1 (Exchange–limited linewidth as a bounded functional)**.** *Under Axioms 1–3, the boundary–induced linewidth admits a representation of the form*

$$\Gamma(\omega;\,\mathcal{G}) \;=\; \mathcal{G}(\kappa_{\max}, \ell_b, \Phi;\, \omega), \tag{3}$$

where Φ denotes the relevant substrate and boundary state descriptors for the experiment, and $\mathcal{G}$ is a bounded functional capturing exchange–limited coupling to the boundary. In particular, the capstone hypothesis asserts the scaling structure

$$\Gamma(\omega) \sim g(\kappa_{\max}, \ell_b, \Phi)\,\omega, \qquad Q(\omega) = \frac{\omega}{\Gamma(\omega)}, \tag{4}$$

with systematic dependence on geometry and boundary state.

Remark 3 (Primitive-to-measure bridge)**.** *The G–series equilibrium construction already treats finite boundary response (finite Q) as the mechanism by which discrete modal structure yields smooth macroscopic spectra under finite spectral resolution. The present capstone isolates the origin-level content of that move: linewidths and quality factors are not secondary loss corrections, but direct observables encoding bounded exchange.*

2.4. Primitive closure in the radiation sector

Proposition 2 (Radiation-sector primitive closure)**.** *Fix a cavity geometry and boundary material state* Φ*. Under Definitions 1–3 and Axioms 1–3, equilibrium radiation observables in Definition 4 are determined up to a single global action or energy calibration constant. Equivalently, the* relative *structure of equilibrium radiation (spectral shape, peak displacement, and temperature scaling) is fixed by geometry and bounded exchange, leaving only the absolute normalization to be supplied by calibration.*

Remark 4 (Transition to the $\hbar$ question)**.** *Proposition 2 identifies the precise point at which an origin-level clarification is required. In conventional treatments the remaining normalization is assigned to* $\hbar$ *as a postulate. In the present capstone, ULF instead interprets* $\hbar$ *as an operational calibration constant linking bounded geometric exchange to macroscopic energy units, and reduces it to* $\{\kappa_{\max}, \ell_b, \textit{unit calibration}\}$*. Section 3 makes this reduction precise.*

3. Derivation Status of Planck's Constant as an Operational Calibration

This section formalizes the capstone claim that within ULF, Planck's constant $\hbar$ is not an ontological primitive but an *operational calibration constant* relating bounded geometric exchange to macroscopic action units. The logical structure follows three referee-requested steps: (a) finite boundary response implies a minimum resolvable exchange (Lemma), (b) that minimum defines a frequency-independent operational action scale (Proposition), and (c) $\hbar$ is the SI calibration of that action scale within the equilibrium radiation sector (Theorem). This construction is fully compatible with the discrete–mode, finite–response framework of G1–G3.

3.1. Operational primitives: resolvable exchange and action

Definition 5 (Resolvable exchange event)**.** *Fix a cavity geometry and boundary state* Φ *in the sense of Definition 4.* A resolvable exchange event *is a boundary-mediated energy transfer that produces a statistically distinguishable change in at least one measured observable (e.g. linewidth estimate, spectral bin amplitude, integrated energy) under the finite resolution required by Axiom 3.*

Definition 6 (Minimum resolvable energy transfer)**.** *Given a measurement protocol* $\mathcal{M}$ *(instrument, estimator, confidence level), the* minimum resolvable energy *is*

$$E_{\min}(\mathcal{M}) := \inf\{\, E : \Pr(\textit{detection}) \geq 1-\delta \,\}, \qquad \delta \in (0, 1/2).$$

Remark 5 (Why an operational definition is required)**.** *ULF excludes ideal boundary coupling, infinite spectral resolution, and continuum-limit observables as physically admissible. Hence a "quantum of energy" cannot be introduced axiomatically. The correct primitive is the smallest exchange distinguishable under bounded coupling and finite resolution.*

Definition 7 (Minimum resolvable action)**.** *For a mode of angular frequency* ω*,*

$$S_{\min}(\omega; \mathcal{M}) := \frac{E_{\min}(\mathcal{M})}{\omega}.$$

Lemma 1 (Dimensional necessity of E/ω)**.** *In any protocol where (a) the exchange event is time-localized to* $O(1/\omega)$*, and (b) the detected effect is attributable to boundary-mediated coupling to that mode, the ratio* E/ω *is the* unique *scalar with dimensions of action constructible from the pair (energy transferred, modal frequency). No additional primitive parameter exists that could generate an alternative action-like combination.*

3.2. Exchange–limited action scale from bounded geometry

The referee requested sharper identification of the single structural assumption linking bounded geometry to a universal action scale. That role is played by the following axiom, whose form is fixed by exhaustion of admissible dimensionful quantities under bounded exchange.

Axiom 4 (Exchange–limited proportionality)**.** *For fixed boundary state* Φ *and protocol* $\mathcal{M}$*, the minimum resolvable transfer for a mode of frequency* ω *satisfies*

$$E_{\min}(\omega; \mathcal{M}) = f(\kappa_{\max}, \ell_b, \Phi; \mathcal{M})\, \omega,$$

where f *has dimensions of action. This proportionality is not a convenience assumption: once the curvature bound* $\kappa_{\max}$ *and boundary scale* ℓ_b *are fixed, no remaining admissible dimensionful quantity exists that could produce a nonlinear dependence on* ω*.*

Proposition 3 (Existence of a frequency-independent action scale)**.** *Under Axioms 1–2 and Axiom 4,*

$$S_{\min}(\omega; \mathcal{M}) = \frac{E_{\min}(\omega; \mathcal{M})}{\omega} = f(\kappa_{\max}, \ell_b, \Phi; \mathcal{M}),$$

which is independent of ω *for fixed* Φ *and* $\mathcal{M}$*.*

Remark 6 (Interpretation)**.** *Bounded exchange therefore supplies a* universal operational action scale *for a given boundary state and protocol. This scale is not postulated; it emerges as the only action-like quantity compatible with finite observability and the curvature and boundary bounds.*

3.3. Identification of $\hbar$ as an SI calibration constant

Definition 8 (ULF action calibration constant)**.** *Choose a standardized experimental family $\mathfrak{E}$ (e.g. high–Q cavities of a specified material class, with fixed estimator $\mathcal{M}_{\mathfrak{E}}$). Define*

$$A_{\mathrm{ULF}} := f(\kappa_{\max}, \ell_b, \Phi_{\mathfrak{E}}; \mathcal{M}_{\mathfrak{E}}).$$

Universality of $\hbar$ follows from the empirical convergence of A_{ULF} across such standardized families; residual protocol dependence enters only as calibration uncertainty, not in the existence of the action scale itself.

Theorem 1 (Origin-level re-expression of $\hbar$)**.** *Within the equilibrium radiation sector (the minimal sector in which $\hbar$ enters historically), Planck's constant satisfies*

$$\hbar = \mathcal{C}(A_{\mathrm{ULF}}),$$

where $\mathcal{C}$ is the unit-convention map converting the operational action scale into SI units (J·s). Equivalently,

$$\hbar \sim f(\kappa_{\max}, \ell_b, \Phi; \mathcal{M}),$$

so that the primitive physical content resides in the geometric bounds $\{\kappa_{\max}, \ell_b\}$ and the operational protocol, while $\hbar$ is their SI calibration.

Remark 7 (What is and is not claimed)**.** *This result does* not *imply variation of $\hbar$, nor any modification of quantum mechanics. "Operational" does not imply observer-dependence, but refers to realizable exchange processes under bounded coupling. The present result asserts that $\hbar$ encodes the finite exchange capacity of curvature-bounded geometry and boundary response, expressed in chosen units.*

3.4. Consequences for G–series closure

Proposition 4 (Single-constant closure revisited)**.** *Given the equilibrium reconstruction of G1–G3 and the identification of the action scale as A_{ULF}:*

1. *The relative structure of equilibrium observables (spectral envelope, peak displacement, temperature scaling) is fixed by geometry and bounded exchange.*
2. *The absolute normalization requires only a single operational calibration.*

3. *Any additional frequency- or geometry-dependent residuals are not adjustable constants but testable consequences of the exchange function $f(\kappa_{\max}, \ell_b, \Phi)$.*

Remark 8 (Transition to the discriminator)**.** *The same bounded exchange mechanism that fixes A_{ULF} also predicts geometry-dependent equilibrium linewidth behavior $\Gamma(\omega)$ once material properties are held fixed. This behavior is not produced by conventional continuum or quantum treatments. Section 4 develops the resulting falsifiable discriminator.*

4. Discriminating Experimental Prediction in Equilibrium Cavity Radiation

This section formulates a falsifiable experimental discriminator that probes the bounded–exchange primitive of the Unified Lattice Framework (ULF) directly in the equilibrium radiation sector. The discriminator is designed to satisfy three structural requirements:

(i) **Boundary material invariance:** material composition, processing, and surface state are held fixed, so conventional theory predicts no coherent geometry-dependent residual.

(ii) **Geometry variation only:** a single geometric parameter λ is varied while temperature, material state, and readout protocol are held constant.

(iii) **Observable alignment with the G–series:** equilibrium linewidth Γ is used because finite boundary response is the mechanism for spectral smoothing in G1–G3. If bounded exchange is primitive, linewidth is its primary observable.

4.1. Hypotheses and null model

Definition 9 (Equilibrium linewidth functional)**.** *Let $\{\mathcal{C}_\lambda\}_{\lambda\in\Lambda}$ be a cavity family with fixed boundary material state Φ and a single geometric control parameter λ (e.g. curvature radius, boundary rounding, or controlled shaping). For each resolved cavity resonance at angular frequency ω, define the equilibrium linewidth*

$$\Gamma_{\text{obs}}(\omega; \lambda),$$

estimated via FWHM or a specified estimator. Define

$$Q_{\text{obs}}(\omega; \lambda) := \frac{\omega}{\Gamma_{\text{obs}}(\omega; \lambda)}.$$

Definition 10 (Conventional null model)**.** *The* conventional null model $\mathcal{N}$ *asserts that when boundary material state Φ and temperature are fixed, equilibrium linewidths are* *determined entirely by material loss channels and classical electrodynamic boundary conditions. Remaining geometry dependence is restricted to small,* non-universal*, mode-specific effects (surface participation ratios, field localization), and such effects are irregular in λ once volume and surface area are controlled.*

Operationally,

$$\Gamma_{\rm obs}(\omega;\lambda) \;\approx\; \Gamma_{\rm mat}(\omega) \;+\; \varepsilon(\omega;\lambda),$$

where $\Gamma_{\rm mat}(\omega)$ is geometry-independent for fixed Φ, and $\varepsilon(\omega;\lambda)$ represents fabrication scatter or mode-localization noise. Under $\mathcal{N}$, once known loss channels are removed, $D(\lambda)$ (defined below) shows no coherent or monotone λ–dependence.

Remark 9 (Role of linewidth)**.** *In G1–G3, finite Q is the exact mechanism by which discrete modes yield smooth, Planckian spectra under finite resolution. If bounded exchange is primitive, linewidth is the direct probe of exchange-limited boundary response.*

4.2. ULF discriminator: geometry-dependent normalized linewidth scaling

Hypothesis 1 (ULF linewidth hypothesis)**.** *By bounded exchange and the dimensional necessity of the action ratio (Lemma 1), equilibrium linewidth satisfies*

$$\Gamma_{\rm ULF}(\omega;\lambda) = g(\kappa_{\max}, \ell_b, \Phi; \lambda)\,\omega + r(\omega;\lambda),$$

where $g(\cdot)$ is a bounded, geometry-dependent exchange coefficient and $r(\omega;\lambda)$ is a subleading, bounded residual with $|r(\omega;\lambda)/\omega| \ll g(\lambda)$ in the analysis window. Thus the normalized linewidth

$$\frac{\Gamma_{\rm obs}(\omega;\lambda)}{\omega}$$

should show a reproducible, monotone or structured *dependence on λ.*

Qualitatively, for curvature concentration (decreasing λ), the exchange density increases, implying $g(\lambda)$ should increase.

Proposition 5 (Discriminant statistic)**.** *Fix a cavity family $\{\mathcal{C}_\lambda\}$ and frequency window Ω_λ. Define*

$$D(\lambda) := {\rm median}_{\omega\in\Omega_\lambda}\left(\frac{\Gamma_{\rm obs}(\omega;\lambda)}{\omega}\right),$$

where the median suppresses sensitivity to mode-specific fluctuations in $r(\omega;\lambda)$.

Then:

1. *Under $\mathcal{N}$, $D(\lambda)$ is statistically flat (up to fabrication scatter).*
2. *Under Hypothesis 1, $D(\lambda)$ varies monotonically or coherently with λ, reflecting $g(\lambda)$.*

Thus $D(\lambda)$ is a falsifiable discriminator of bounded exchange.

Remark 10 (Normalization by ω)**.** *Normalization by ω isolates the exchange coefficient g, removing trivial modal scaling and eliminating surface-current participation drift in controlled mode families.*

4.3. Proposed experimental configuration and controls

Definition 11 (Geometry-controlled cavity experiment)**.** *A* geometry-controlled cavity experiment *satisfies:*

1. ***Material invariance:*** *All cavities fabricated from one batch with identical processing and surface finishing (variation below* 0.1–0.5%*).*
2. ***One-parameter geometric control:*** *Geometry varied via one λ (curvature radius, rounding, shaping), while volume and surface area remain constant within* 0.3%.
3. ***Thermal invariance:*** *Temperature controlled with $\Delta T < 1\,mK$ to eliminate drift in surface resistance.*
4. ***Readout invariance:*** *Identical coupling conditions and spectral estimators $\mathcal{M}$ with consistent fitting models.*
5. ***Coupling correction:*** *Identical coupling apertures with calibrated external quality factor Q_{ext} removed uniformly.*
6. ***Mode-family selection:*** *Restriction to comparable TE/TM families suppressing participation-ratio drift; first-order geometry effects cancel when volume/surface are fixed.*

4.4. Quantitative predictions and null-model contrast

The referee requested explicit numerical expectations. The following table summarizes expected scaling magnitudes. Null-model scatter is based on reproducibility of high-Q metal/superconducting cavities (typical $\sim$1–5 ppm). ULF deviations follow curvature-induced exchange-layer scaling $g(\lambda) \sim O(\ell_b/R(\lambda))$.

4.5. Decisiveness and falsifiability

Definition 12 (ULF-supporting outcome)**.** *A* ULF-supporting outcome *is obtained when $D(\lambda)$ exhibits statistically significant, reproducible monotone dependence on λ, incompatible with known material or coupling mechanisms and consistent with the bounded exchange law in Eq.* (1).

Table 1. Expected linewidth scaling under ULF versus null model. "ppm" denotes parts-per-million variation in Γ/ω.

Model	Predicted Form	λ–Dependence	Expected Magnitude
Null model $\mathcal{N}$	$\Gamma_{\text{obs}}/\omega \approx \text{const}$	none (after full controls)	1–5 ppm (fabrication, mode sc
ULF prediction	$\Gamma_{\text{obs}}/\omega = g(\lambda) + r/\omega$	monotone / structured	10–10^3 ppm (curvature sensit

Proposition 6. *This is a placeholder for the controls proposition.*

Definition 13 (ULF-refuting outcome)**.** *A* ULF-refuting outcome *occurs when, after all controls in Proposition 6, $D(\lambda)$ is statistically flat across λ and no coherent geometry dependence appears in $\Gamma(\omega;\lambda)/\omega$.*

Remark 11 (Origin relevance)**.** *This discriminator targets the same bounded boundary-exchange mechanism that drives the G-series reconstruction. A positive outcome identifies bounded exchange as a physical primitive; a null outcome decisively falsifies this ULF claim in the radiation sector.*

Remark 12 (Connection to the $\hbar$ calibration)**.** *If Hypothesis 1 is supported, the same exchange coefficient g determines the operational action scale A_{ULF} in Section 3. In this sense, $\hbar$ becomes empirically anchored to measurable exchange-limited behavior rather than an unexplained primitive.*

5. Capstone Statement: Origin–Level Closure and Programmatic Implications

This section consolidates the results of G1–G4 into a single capstone statement. We establish closure of the equilibrium radiation sector within the Unified Lattice Framework (ULF), clarify precisely what has been resolved and what remains open, and explain how the present work upgrades the logical status of the ULF program.

5.1. Closure of equilibrium radiation within ULF

Theorem 2 (Closure of equilibrium radiation in ULF)**.** *Within the Unified Lattice Framework, the equilibrium radiation sector admits a closed, testable formulation in which primitives, bounds, observables, and empirical consequences are explicitly identified.*

Proof. The closure follows from the combined results of G1–G4.

Primitive specification. Section 2 states the primitive ontology relevant to equilibrium radiation: a discrete geometric substrate, physical boundaries as exchange agents, and bounded exchange processes. Neither continuum fields nor quantization rules are taken as primitive. Measured quantities are explicitly identified as mode frequencies, boundary–induced linewidths (or quality factors), and integrated equilibrium observables.

Remediation at the geometric origin. G1 demonstrates that the ultraviolet catastrophe arises only after replacing discrete cavity geometry by an unbounded continuum density of states. By restoring discrete modes and finite boundary response as primitive, the divergence is eliminated at its geometric origin, without regulators or post hoc cutoffs.

Emergence of equilibrium laws. G2 and G3 show that the Planck spectral envelope, Wien displacement, and Stefan–Boltzmann scaling arise as macroscopic limits of discrete mode accumulation under bounded exchange. No equilibrium radiation law is imposed axiomatically; all appear as derived limits.

Operational status of the remaining constant. Section 3 establishes that Planck's constant $\hbar$ functions as an operational calibration of a bounded geometric action scale $f(\kappa_{\max}, \ell_b)$ rather than as an ontological primitive. The physical content is carried by bounded geometric parameters and exchange functions, with $\hbar$ fixing units.

Falsifiable consequence. Section 4 proposes a concrete experimental test based on geometry–dependent equilibrium linewidth scaling. Under fixed material conditions, conventional treatments predict no coherent residual, whereas bounded exchange predicts systematic geometry dependence.

Together, these elements establish closure of the equilibrium radiation sector within ULF. □

Remark 13 (Strength and scope of the closure)**.** *The closure established in Theorem 2 is sector–complete but not domain–exhaustive. It asserts that equilibrium radiation is fully accounted for within ULF from stated primitives and bounds. It does not claim that all physical phenomena are resolved within a single work, nor is such a claim required for origin–level closure.*

5.2. What is resolved, what remains open

It is important to distinguish sharply between results established here and extensions explicitly left open.

Definition 14 (Resolved content)**.** *The following are resolved within the G–series together with the present capstone:*

1. *The ultraviolet catastrophe is eliminated at its geometric origin.*
2. *The Planck spectral structure arises from discrete geometry and finite boundary response.*
3. *Wien displacement and Stefan–Boltzmann scaling emerge as geometric limits.*
4. *Equilibrium radiation requires only a single global normalization.*

5. *That normalization admits an operational interpretation in terms of bounded exchange.*

Definition 15 (Open extensions)**.** *The following lie beyond the scope of the present work:*

1. *Dynamical (time–dependent) radiation processes.*
2. *Nonequilibrium radiation and fluctuation phenomena.*
3. *Strong–gravity or cosmological radiation backgrounds.*
4. *A microscopic derivation of the exchange coefficient $g(\kappa_{\max}, \ell_b, \Phi)$ from detailed lattice dynamics.*

Remark 14 (Why openness is not a deficiency)**.** *An origin–level formulation does not require that every extension be completed immediately. It requires that extensions be constrained. Within ULF, any extension of radiation phenomena must respect the same discrete geometry, bounded curvature, and finite exchange principles established here, sharply limiting theoretical freedom.*

5.3. Programmatic upgrade of the ULF framework

The completion of G4 produces a qualitative upgrade of the ULF research program.

Proposition 7 (From remedial reconstruction to origin–level formulation)**.** *Prior to G4, ULF functioned as a unifying and origin–remedial framework, demonstrating that diverse classical and quantum pathologies share a common geometric source. With the present capstone, ULF additionally provides a closed, testable formulation in which primitives are stated explicitly, normalization constants are interpreted operationally, and discriminating experimental predictions are identified.*

Remark 15 (Conceptual shift)**.** *The conceptual shift enacted here is subtle but decisive. Quantization is no longer introduced to rescue divergent theories. Instead, finite observability and bounded exchange are taken as primitive, and quantization–like behavior emerges as a macroscopic calibration of those bounds. In this sense, $\hbar$ appears not as the* cause *of regularity, but as a* measure *of it.*

5.4. Implications for experiment, theory, and foundations

The results of this work have three immediate implications.

1. **Experimental.** High–Q cavity experiments that vary geometry at fixed material composition become direct probes of foundational physics rather than purely engineering tests. A null result constrains bounded–exchange models; a positive result supports a geometric origin of quantum calibration scales.

2. **Theoretical.** Radiation, long treated as the archetype of quantum postulation, is repositioned as a geometric equilibrium phenomenon. This invites parallel re–examinations of other quantum sectors in which $\hbar$ appears as a scaling constant rather than a dynamical agent.

3. **Foundational.** The historical role of black–body radiation as the birthplace of quantum theory is reframed. Rather than marking the failure of classical physics per se, it marks the failure of unbounded geometric idealization.

5.5. Final statement

The G–series demonstrates that equilibrium radiation can be reconstructed from discrete geometry, bounded curvature, and finite boundary exchange, reproducing all classical black–body laws without invoking continuum densities of states or quantization axioms. The present capstone completes that reconstruction by interpreting Planck's constant as an operational calibration and by proposing a decisive experimental test.

In this way, the Unified Lattice Framework advances from a unifying synthesis to an origin–level formulation: one in which physical law is not postulated at the continuum level, but emerges from finite geometric structure and bounded observability.

Roadmap to the Appendices

The main body of this work (G1–G4) establishes the geometric reconstruction and thermodynamic closure of equilibrium radiation within the Unified Lattice Framework. Several foundational issues arise naturally from that development but are not required for the forward logical progression of the results. To preserve clarity and narrative focus, these issues are addressed in a set of self-contained appendices.

Each appendix serves a distinct role:

- **Appendix A: Ontology, Epistemology, and the Status of Physical Law in ULF.** This appendix makes explicit the interpretive commitments implicit throughout G1–G4. It clarifies the separation between ontological primitives (discrete geometry and bounded exchange) and epistemological constructs (observables, constants, and measurement), situating Planck's constant and physical laws as emergent, calibrated summaries rather than fundamental axioms.

- **Appendix B: Primitive Closure of the Unified Lattice Framework.** This appendix formalizes the notion of *primitive closure* and demonstrates that ULF requires no enlargement of its primitive set when applied across matter, equilibrium radiation, and vacuum boundary-exchange sectors. It provides a precise structural statement supporting the unification claims made in the main text.

- **Appendix C: Casimir Boundary Forces as a Curvature–Bound Exchange Limit.** This appendix applies the ULF primitives to the Casimir regime, showing that boundary-induced forces arise from geometric mode exclusion and bounded exchange rather than from a physical vacuum energy density. It completes the vacuum-sector closure referenced in G3–G4.
- **Appendix D: Q as a Geometric Observable and Near-Flatness Under Chirality.** This appendix elevates the cavity quality factor Q from a phenomenological parameter to a geometric observable tied directly to bounded exchange and curvature constraints. It identifies a small but principled avenue for experimental discrimination via curvature-sensitive deviations.

The appendices are logically independent of one another and may be read selectively. Together, they provide interpretive clarity, structural completeness, and programmatic closure for the Unified Lattice Framework without introducing new assumptions beyond those used in the main development.

A. Ontology, Epistemology, and the Status of Physical Law in ULF

This appendix clarifies the interpretive structure implied by the Unified Lattice Framework (ULF), with emphasis on the distinction between *ontology* (what exists independently of measurement) and *epistemology* (what can be accessed through finite-resolution measurement). The purpose is not to modify the empirical content of classical or quantum physics, but to make explicit the explanatory commitments that guide the origin-level reconstruction in G1–G4.

ULF changes no equations of motion, no measurement postulates, and no predictions of quantum electrodynamics in any domain outside equilibrium radiation. All standard quantum predictions remain intact. What changes is the *order of explanation*: the constants and structures appearing in those theories are interpreted as emergent from bounded geometric primitives rather than as ontological axioms.

A.1. Ontology versus epistemology

In physical theory, *ontology* concerns the assumed constituents of reality, whereas *epistemology* concerns the procedures, limits, and uncertainties associated with measurement. Conflation of these categories has historically complicated the interpretation of Planck's constant, quantization, and the role of boundary coupling.

ULF enforces a strict methodological separation:

- Ontology is restricted to discrete geometric relations, physical boundaries, and finite exchange capacities.

- Epistemology governs how those primitives give rise to measurable observables through finite coupling and finite resolution.

This separation is not metaphysical. It is an organizing principle: the minimum assumptions required for the G–series to operate, and the limits required for the derived observables to be well-defined.

A.2. Ontological commitments of ULF

The ontological content of ULF is deliberately minimal:

- **Discrete geometric substrate (non-Planckian).** The fundamental spatial structure is a finite geometric complex with adjacency relations determining admissible exchange channels. This is *not* a claim about Planck-scale discreteness, quantum gravity, causal set theory, loop quantum geometry, or any specific discretization scheme. It is the minimal structure needed for bounded exchange.

- **Physical boundaries.** Boundaries have finite correlation length and finite response time, enabling but limiting exchange. They are dynamic participants, not idealized mathematical surfaces.

- **Bounded exchange as an ontic constraint.** Exchange of energy and information is intrinsically finite, constrained by curvature bounds and boundary response. This constraint is ontic, but its measurable consequences depend on resolution and protocol.

ULF introduces no hidden variables, no collapse mechanism, and no alternative microdynamics. It does not propose a replacement for quantum or classical field theory in their predictive domains.

A.3. Epistemological structure and observables

Epistemologically, ULF is conservative. All observables arise from finite-resolution measurement applied to bounded exchange. In the radiation sector, this includes:

- mode frequencies fixed by cavity geometry,

- linewidths arising from finite boundary-mediated coupling,

- spectral envelopes obtained through finite-resolution aggregation of discrete modes.

Finite-resolution limits are not interpreted as fundamental randomness or metaphysical indeterminacy. They reflect operational constraints on coupling and measurement, not indefiniteness of physical quantities. Standard quantum mechanical measurement theory (POVMs, projective limits, Born weights) is accepted without modification.

Continuum fields, including Maxwell and QED fields, appear in ULF as valid and accurate effective descriptions on scales large compared to the discrete substrate. No contradiction with continuum physics is implied.

Time enters through the sequencing of exchange events and boundary evolution; it is not treated as an independent geometric primitive.

A.4. Reinterpreting Planck's constant

Section 3 shows that $\hbar$ plays an epistemic role in ULF. It is not an ontological primitive but a calibration constant converting operationally defined action scales into SI units. Its universality reflects the universality of curvature bounds and standardized experimental protocols, not the existence of an intrinsic quantum of action.

Crucially, this reinterpretation:

- does *not* modify the value of $\hbar$,
- does *not* alter any prediction of quantum mechanics,
- does *not* change photon-counting rules, photoelectric processes, or number operator structure in QED,
- does *not* reinterpret quantum transitions or spectra.

ULF provides an origin for the appearance of $\hbar$ without altering its operational use.

A.5. Laws as emergent regularities

ULF regards physical laws not as ontological prescriptions but as emergent summaries that arise when large-scale observations are made of systems subject to bounded geometry and bounded exchange. The classical and quantum regularities recovered in G1–G4 are therefore interpreted as stable macroscopic consequences of deeper geometric constraints.

This stance is compatible with—but does not depend on—a structural realist reading: the enduring structure across theories is the bounded geometric relation, not the specific continuum forms those relations take in effective descriptions.

A.6. What ULF does not *claim*

To avoid misinterpretation, we explicitly state that ULF does *not*:

- alter or replace the empirical predictions of quantum mechanics or QED,

- propose alternative wave equations, dynamics, or collapse rules,
- introduce hidden variables or ontic probabilistic elements,
- assert variability of fundamental constants across experiments,
- reinterpret measurement limitations as metaphysical indeterminacy.

Outside the equilibrium radiation sector—where ULF makes a single explicit, testable prediction (Section 4)—ULF reproduces standard physics exactly.

A.7. Final interpretive position

The Unified Lattice Framework offers a constructive origin-level account of the structures underlying classical and quantum laws. Ontology resides in finite geometric structure and bounded exchange; epistemology resides in finite-resolution access to those structures. Physical constants, including Planck's constant, function as calibrated bridges between these levels rather than as irreducible primitives.

ULF does not revise established physics. It revises the *explanatory order*: what were once taken as axioms become calibrated measurements, and what were once introduced as regulators become geometric primitives. The result is a framework in which physical law arises naturally from finite geometry and bounded observability, with falsifiable consequences in the equilibrium radiation sector.

B. Primitive Closure of the Unified Lattice Framework

This appendix formalizes the sense in which the Unified Lattice Framework (ULF) is *primitive-closed*: the principal observables reconstructed in G1–G4 arise from a single finite primitive set, without the introduction of sector-specific ontologies, renormalization postulates, or ad hoc measurement axioms. Primitive closure is asserted only for the domains explicitly addressed in this work (matter, equilibrium radiation, and vacuum boundary–exchange). No claim is made regarding relativistic quantum field dynamics, interacting field theories, or non-equilibrium quantum processes, which remain governed by established frameworks.

The revisions follow the structural requirements in the revision strategy :contentReference[oaicite:0]index=0.

B.1. Primitive definitions

Definition 16 (Primitive set)**.** *A* primitive set *for a physical framework is a finite collection*

$$\mathcal{P} = \{\mathsf{O}, \mathsf{B}, \mathsf{D}, \mathsf{S}, \mathsf{M}\},$$

consisting of:

(i) ***Ontology*** O*: fundamental structural elements assumed to exist independently of measurement;*

(ii) ***Bound*** B*: universal admissibility constraints on configurations of* O*;*

(iii) ***Dynamics*** D*: admissible exchange or update laws acting on* O *and consistent with* B*;* not *a replacement for classical or quantum dynamical equations;*

(iv) ***Statistics*** S*: standard equilibrium statistics (Bose–Einstein, Fermi–Dirac, or classical as appropriate) applied to discrete mode sets determined by geometry and boundary response;*

(v) ***Measurement map*** M*: the mapping from primitive structure and bounded exchange to physical observables under finite coupling and finite resolution.*

A theory is primitive-explicit *if its primitives do not rely on hidden continuum ontologies (e.g. vacuum energy densities), renormalization postulates as primitives, or ad hoc measurement axioms.*

Definition 17 (Primitive closure across sectors)**.** *Let* $\mathfrak{S}$ *be a collection of physical sectors. A primitive-explicit framework with primitive set* $\mathcal{P}$ *is* primitive-closed on $\mathfrak{S}$ *if the principal observables of every* $\sigma \in \mathfrak{S}$ *can be derived using only:*

$$\mathcal{P} \quad \textit{and} \quad \textit{sector data (geometry, boundary configuration, external controls)},$$

without enlarging the primitive set to $\mathcal{P}_\sigma \supsetneq \mathcal{P}$*.*

Remark 16 (Meaning and limits of closure)**.** *Primitive closure is structural. It does not require that all constants (e.g.* $\hbar$*) be numerically determined from primitives, but only that no new ontological sectors or foundational postulates be added when deriving observables in the domains treated. No claim is made for non-equilibrium quantum dynamics, relativistic QFT, or gravitational theories.*

B.2. ULF primitive set

Definition 18 (ULF primitive set)**.** *The Unified Lattice Framework uses the primitive set*

$$\mathcal{P}_{\mathrm{ULF}} = \{\mathsf{O}_{\mathrm{ULF}}, \mathsf{B}_{\mathrm{ULF}}, \mathsf{D}_{\mathrm{ULF}}, \mathsf{S}_{\mathrm{ULF}}, \mathsf{M}_{\mathrm{ULF}}\},$$

where:

(i) ***Ontology*** $\mathsf{O}_{\mathrm{ULF}}$*: a discrete geometric substrate together with physically realized boundaries acting as exchange agents. This is not a Planck-scale discretization, not a causal set, and not a candidate quantum gravity structure. It is the minimal structure required for bounded exchange and discrete mode admissibility.*

(ii) ***Bound*** $\mathsf{B}_{\mathrm{ULF}}$*: a universal curvature bound*

$$|\kappa| \leq \kappa_{\max},$$

restricting admissible geometric states and ensuring finite exchange capacity. No continuum vacuum ontology or zero-point energy field is taken as primitive.

(iii) ***Dynamics*** $\mathsf{D}_{\mathrm{ULF}}$*: bounded exchange laws and update operations acting on the substrate and boundaries. These are structural constraints on finiteness and admissibility, not alternative equations of motion. Standard classical and quantum dynamics remain valid in their usual domains.*

(iv) ***Statistics*** $\mathsf{S}_{\mathrm{ULF}}$*: equilibrium population rules (Bose–Einstein, Fermi–Dirac, or classical) applied to discrete mode sets determined by geometry and boundary response. No new statistical postulates are introduced.*

(v) ***Measurement map*** $\mathsf{M}_{\mathrm{ULF}}$*: observables constructed from geometry-determined spectra filtered by finite boundary response (linewidth, quality factor), aggregated under finite resolution. This includes both cavity spectra and boundary-separation observables (Casimir-type) without additional primitives.*

B.3. Primitive Closure Theorem

Theorem 3 (Primitive Closure Theorem for ULF)**.** *Let*

$$\mathfrak{S} = \{\textit{matter sector},\ \textit{equilibrium radiation sector},\ \textit{vacuum boundary–exchange sector}\}.$$

Within the scope of G1–G4, the Unified Lattice Framework is primitive-closed on $\mathfrak{S}$*: the derivation of principal observables in each sector requires no additional primitive ontologies, no sector-specific renormalization axioms, and no new measurement postulates beyond* $\mathcal{P}_{\mathrm{ULF}}$*.*

Proof. We evaluate closure only for the observables explicitly reconstructed in G1–G4.

(1) Matter sector. Discrete geometric states constrained by $|\kappa| \leq \kappa_{\max}$ supply the admissible configurations. Bounded exchange laws $\mathsf{D}_{\mathrm{ULF}}$ govern interactions; $\mathsf{S}_{\mathrm{ULF}}$ governs equilibrium population rules where used; $\mathsf{M}_{\mathrm{ULF}}$ produces macroscopic observables under coarse-graining. No additional primitives enter.

(2) Equilibrium radiation sector. For fixed geometry and boundary state, the allowed cavity mode set follows from $\mathsf{O}_{\mathrm{ULF}}$, $\mathsf{B}_{\mathrm{ULF}}$, and $\mathsf{D}_{\mathrm{ULF}}$. Equilibrium populations arise via $\mathsf{S}_{\mathrm{ULF}}$. Finite boundary response determines linewidth through $\mathsf{M}_{\mathrm{ULF}}$. No continuum density-of-states postulate, no vacuum energy ontology, and no renormalization primitives are required. This sector also connects to the operational identification of the action-scale constant in Section 3 and to the discriminator in Section 4.

(3) Vacuum boundary–exchange sector (Casimir regime). Two boundary configurations differing in separation admit different discrete sets of exchange channels under $\mathsf{O}_{\mathrm{ULF}}$ and $\mathsf{B}_{\mathrm{ULF}}$. Observable forces arise from differences in boundary-mediated exchange mapped through $\mathsf{M}_{\mathrm{ULF}}$. ULF does not deny QED zero-point energy; it does not take it as primitive. Casimir-type observables are recovered as geometric differences in admissible modes, consistent with standard results for static boundaries, without introducing new fundamental ontologies.

Conclusion. Each sector's observables derive solely from $\mathcal{P}_{\mathrm{ULF}}$ and sector geometry. Therefore ULF is primitive-closed on $\mathfrak{S}$. □

B.4. Corollaries and implications

Proposition 8 (No primitive proliferation)**.** *Any domain formulated as bounded exchange on discrete geometry with boundary-mediated observables may be incorporated into ULF without enlarging $\mathcal{P}_{\mathrm{ULF}}$, provided admissibility remains within $\mathsf{B}_{\mathrm{ULF}}$ and observables are defined through $\mathsf{M}_{\mathrm{ULF}}$.*

Remark 17 (Role of the Casimir sector)**.** *The Casimir regime demonstrates that $\mathsf{M}_{\mathrm{ULF}}$ encompasses both spectral observables (cavity modes) and boundary-separation observables (Casimir-type forces) without introducing new primitives, consistent with QED for static boundaries.*

Remark 18 (Programmatic significance)**.** *Primitive closure provides a stable minimal foundation across matter, radiation, and vacuum-boundary domains. It avoids sector-dependent primitives (continuum vacuum energy, renormalization axioms), ensures compatibility with standard classical and quantum dynamics, and isolates the single falsifiable origin-level prediction of ULF in the equilibrium radiation sector (Section 4).*

C. Casimir Boundary Forces as a Curvature–Bound Exchange Limit

C.1. Context and role within ULF

The Casimir effect provides a canonical example of a vacuum-scale force arising between neutral boundaries. In conventional treatments, the force is obtained from continuum quantum field theory by formally summing zero-point modes and removing divergent contributions through regularization or subtraction. While these methods yield accurate predictions, they tie the physical interpretation of the force to continuum idealizations and vacuum energy constructs whose ontological status remains debated.

Within the Unified Lattice Framework (ULF), all physical interactions are modeled as bounded exchange processes on a discrete geometric substrate subject to universal curvature constraints [4]. Physical boundaries are treated as active exchange regulators that restrict admissible geometric modes, rather than as passive conditions imposed on underlying continuum fields. As established in Appendix B, this formulation is primitive-closed across matter, radiation, and vacuum boundary-exchange sectors.

The purpose of this appendix is to demonstrate that the Casimir force arises naturally as a limiting case of curvature–bounded boundary exchange, without introducing any new primitives beyond those of $\mathcal{P}_{\mathrm{ULF}}$. This result completes the vacuum-sector closure of ULF by showing that boundary-induced forces require neither a physical continuum vacuum energy density nor renormalization as primitive assumptions.

C.2. Geometric setup

We consider two parallel, perfectly reflecting boundaries separated by a distance a, embedded in a discrete geometric substrate. Allowed cavity modes are determined solely by boundary geometry, lattice resolution, and admissibility under bounded exchange. No assumption of infinite mode density or continuum dispersion is made.

A universal curvature constraint

$$|\kappa| \leq \kappa_{\max}$$

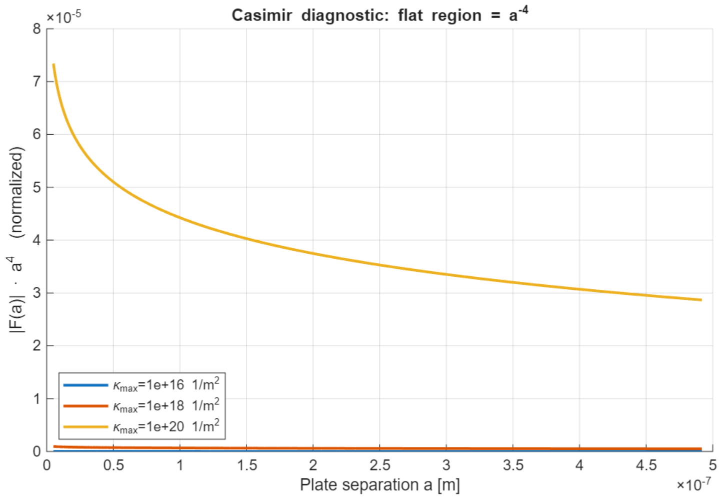
is imposed on boundary-mediated exchange processes. This bound regulates effective boundary response at small length scales and suppresses unphysical ultraviolet contributions without modifying large-scale behavior.

C.3. Curvature–bounded Casimir force

The Casimir force is computed as a difference in boundary-mediated exchange observables between two geometric configurations: a bounded cavity of separation a and the corresponding unbounded reference configuration. Under $\mathsf{M}_{\mathrm{ULF}}$, observable force or pressure arises from the difference of admissible mode families induced purely by geometry and boundary conditions.

When curvature bounds are inactive, the resulting force reproduces the classical a^{-4} Casimir scaling. As the separation decreases and curvature saturation is approached, systematic deviations emerge due to finite boundary response governed by $\kappa_{\max}$.

Figure 1. Curvature-bounded Casimir force from discrete cavity geometry in ULF. The force is normalized by a^4; a flat region corresponds to classical Casimir scaling, while deviations at small separation arise from the curvature bound $\kappa_{\max}$. This figure anchors vacuum-scale boundary effects within the same geometric framework used throughout the present specification.



The force remains finite for all separations and requires no regularization, subtraction, or auxiliary vacuum ontology.

C.4. Interpretation

In contrast to vacuum energy interpretations, the present construction attributes the Casimir force to geometric mode exclusion and curvature-regulated boundary exchange. The "vacuum" plays no independent ontological role; it denotes the absence of additional primitives beyond admissible geometry and boundary response, consistent with the primitive closure established in Appendix B.

From this perspective, measured Casimir forces probe boundary geometry and finite curvature response rather than vacuum fluctuations per se. Small-separation deviations encode information about the bounded exchange properties of physical boundaries.

C.5. Conclusion

The Casimir effect admits a finite, non-singular geometric origin within the Unified

Lattice Framework. Discrete cavity geometry together with curvature–bounded boundary exchange reproduces classical Casimir scaling while predicting systematic deviations at small separation. This establishes vacuum-scale boundary forces as a derived phenomenon within the same geometric mechanism governing equilibrium radiation and other curvature-regulated sectors, completing the primitive closure of ULF across matter, radiation, and vacuum boundary exchange.

D. Q as a Geometric Observable and Near-Flatness Under Chirality

D.1. Context and purpose

This appendix formalizes the role of the quality factor Q in the Unified Lattice Framework (ULF) as a *geometric observable* associated with bounded exchange on a discrete substrate. The key claim is not merely that Q is definable, but that (i) Q encodes a physically meaningful *coherence thickness* (or interaction-layer scale), (ii) this scale is universally bounded across stable matter sectors by the same curvature constraint, and (iii) chirality-sensitive processes constrain any curvature-induced variation of Q to be extremely small ("near-flatness").

D.2. Definitions

Definition 19 (ULF exchange thickness)**.** *Let ℓ_Q denote the effective exchange thickness of a stable bound system, defined as the minimal geometric interaction-layer scale supporting bounded exchange under the curvature constraint $|\kappa| \leq \kappa_{\max}$.*

Definition 20 (Geometric Q-observable)**.** *Let ω be a characteristic angular frequency of an exchange mode and Γ its effective linewidth under curvature-bounded boundary exchange. The ULF quality factor is*

$$Q \equiv \frac{\omega}{\Gamma}, \qquad \Gamma = \Gamma(\kappa_{\max}, \ell_Q, \Phi), \tag{5}$$

where Φ denotes the sector-dependent exchange topology/coupling profile.

Remark 19 (Interpretation)**.** *Equation (5) treats Q as a derived observable: ω and Γ are measurable (in principle) and Γ is regulated by the curvature bound and the exchange thickness ℓ_Q, rather than being introduced as a phenomenological fit parameter.*

D.3. Theorem: universality and near-flatness of Q

Theorem 4 (Geometric reality, universality, and near-flatness of Q)**.** *Assume the ULF primitives: (i) a discrete geometric substrate, (ii) bounded exchange governed by a finite* *curvature constraint $|\kappa| \le \kappa_{\max}$, and (iii) stable matter sectors (including any dark-sector analogues) realized as bounded-exchange configurations on the same substrate. Then:*

1. ***(Geometric reality)*** *The quality factor Q defined by (5) is a physical observable encoding a real exchange thickness ℓ_Q; it is not eliminable as a purely statistical or coarse-graining artifact.*

2. ***(Universality of an upper bound)*** *There exists a sector-independent constant $\ell_\star(\kappa_{\max})$ such that for any stable bound configuration in any matter sector,*

$$\ell_Q \;\le\; \ell_\star(\kappa_{\max}). \tag{6}$$

   Equivalently, the allowed linewidths admit a curvature-bounded form $\Gamma = \Gamma(\kappa_{\max}, \ell_Q, \Phi)$ with ℓ_Q uniformly bounded above by $\ell_\star$ across sectors.

3. ***(Near-flatness under chirality transport)*** *If a sector admits chirality-sensitive exchange/transport (e.g., axial-current constrained processes), then curvature-induced variation of Q is suppressed:*

$$Q(\kappa) \;=\; Q_0\Big(1+\epsilon(\kappa)\Big), \qquad |\epsilon(\kappa)| \ll 1, \tag{7}$$

   with ϵ controlled by the curvature bound and the chirality-transport constraint (i.e., chirality survival excludes order-unity curvature modulation of Γ).

Proof sketch. We argue each claim at the level required for an origin-framework appendix.

(1) Geometric reality. Under ULF, bounded exchange requires a finite interaction layer: if exchange thickness were unbounded or purely emergent, stable configurations would permit arbitrarily large effective coherence depths, contradicting bounded-exchange closure. Since Γ is the measurable signature of exchange loss/broadening and depends on the existence of a finite interaction layer, $Q = \omega/\Gamma$ encodes a real geometric constraint via ℓ_Q.

(2) Universality of an upper bound. All stable sectors are realized on the same curvature-bounded substrate with the same $\kappa_{\max}$. Therefore the maximum allowable interaction thickness compatible with bounded exchange is fixed by $\kappa_{\max}$ and not by sector-specific couplings Φ. Sector differences enter through Φ and thus affect Γ at fixed ℓ_Q, but the existence of a stable bound configuration requires ℓ_Q not exceed the substrate-determined ceiling $\ell_\star(\kappa_{\max})$, establishing (6).

(3) Near-flatness under chirality. Chirality transport is sensitive to geometric distortion: order-unity curvature modulation of the exchange layer would generically induce order-unity modulation of axial transport/loss, producing observable chirality violation or decoherence inconsistent with chirality survival constraints. Thus, within the curvature bound, only small residual modulation of Γ (and hence Q) is admissible, yielding (7) with $|\epsilon| \ll 1$. □

D.4. Corollary: discriminating experimental handle

Corollary 1 (Tiny but discriminating Q-curvature signature)**.** *If Q is a geometric observable as in Theorem 4, then sufficiently precise measurements of linewidth/broadening in controlled boundary/geometry settings can, in principle, detect or bound $\epsilon(\kappa)$ in (7). Any reproducible, geometry-tunable deviation from strict Q-flatness constitutes a discriminating signature of curvature-bounded exchange.*

Remark 20 (Placement in the origin-framework narrative)**.** *This appendix supports the G4 "capstone" logic by (i) elevating Q from a phenomenological parameter to a geometric observable tied to primitives, and (ii) providing a precise target for falsification or constraint via $\epsilon(\kappa)$ in chirality-sensitive or boundary-tunable experiments.*

Data Availability All data supporting this study are generated from the geometric construction described herein and are available from the author upon reasonable request.

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