International Journal of Quantum Foundations **12** (2026) 630-642

Original Paper

Curvature–Bound Equilibrium Radiation: A Geometric Resolution of the Black–Body Problem

William Hernandez

Unified Lattice Solver Technologies, LLC, Lynnwood, WA, United States
E-mail: william@unifiedlattice.com

Received: 22 December 2025 / Accepted: 27 January 2026 / Published: 29 January 2026

Abstract: The black–body radiation problem historically exposed a fundamental inconsistency between classical field theory and thermal equilibrium, culminating in the Rayleigh–Jeans ultraviolet catastrophe and the subsequent introduction of quantum energy quantization. In this work, we present a geometric formulation of equilibrium radiation based on discrete cavity modes governed by curvature–bounded geometry and finite boundary response. Rather than postulating a continuum density of states, equilibrium radiation is described in terms of geometry–determined cavity modes whose exchange and observability are regulated by finite boundary quality. This formulation yields a finite, well–posed description of equilibrium radiation without invoking an *a priori* continuum assumption, ultraviolet cutoff, renormalization procedure, or thermodynamic scaling law. This paper constitutes the first installment of the G–series within the Unified Lattice Framework and is devoted exclusively to establishing the geometric foundations and finiteness of equilibrium radiation in static cavities. Macroscopic spectral structure, integrated thermodynamic scaling, and absolute normalization are deferred to subsequent papers, which build directly on the formulation developed here.

Keywords: Finite cavity geometry; curvature-bounded radiation; discrete mode spectrum; equilibrium radiation; ultraviolet catastrophe resolution.

Global Introduction

The problem of black–body radiation occupies a foundational role in modern physics. Classical electromagnetic theory predicts an unbounded increase of radiated energy at high frequencies for systems in thermal equilibrium, a result historically known as the Rayleigh–Jeans catastrophe. The empirical failure of this prediction led to Planck's introduction of energy quantization, marking the birth of quantum theory.

While Planck's law accurately describes observed spectra, its standard derivations implicitly rely on a continuum density of states and idealized boundary conditions. Real physical cavities, however, possess finite size, discrete modal structure, and nonzero boundary losses. These geometric features are typically introduced only as phenomenological corrections, rather than as foundational elements of the equilibrium description.

In this work, we adopt the opposite viewpoint: that equilibrium radiation should be understood fundamentally as a geometric phenomenon governed by discrete cavity modes and curvature–bounded boundary interactions. Finite boundary quality factors regulate the exchange and observability of cavity modes, providing a physically grounded mechanism for spectral resolution without invoking a continuum limit. From this perspective, the ultraviolet divergence arises not from classical field theory itself, but from unphysical geometric idealizations.

This geometric perspective is consistent with the broader Unified Lattice Framework, in which divergences across quantum field theory, gravitation, and cosmology are shown to arise from unbounded curvature assumptions rather than intrinsic physical inconsistencies [2–5]. Within that framework, equilibrium radiation represents the simplest setting in which finite geometry, discrete spectra, and curvature–regulated boundary response can be isolated and analyzed without confounding dynamical effects.

This paper constitutes *Part I of the G–series* and is devoted exclusively to establishing the geometric formulation and finiteness of equilibrium radiation in static cavities. We define equilibrium radiation as a population of geometry–determined cavity modes populated according to standard equilibrium statistics and coupled through finite, curvature–bounded boundary response. No claims are made here regarding macroscopic spectral envelopes, thermodynamic scaling laws, or absolute normalization.

Subsequent parts of the series build directly on this formulation. *Part II* examines the emergence of macroscopic spectral structure under finite resolution. *Part III* addresses the thermodynamic closure of equilibrium radiation through explicit mode counting and energy accumulation. Additional work extends the framework to calibration, nonequilibrium radiation, cosmological backgrounds, and gravitational coupling.

The present work focuses exclusively on the first step in this program. By isolating equilibrium radiation in static cavities, we provide a minimal and physically transparent setting in which geometric regulation can be analyzed at its origin. The resulting formulation resolves the ultraviolet catastrophe geometrically and establishes the foundational pillar upon which the remainder of the G–series is built.

Part I

Black–Body Radiation from Finite Cavity Geometry

The phenomenon of black–body radiation provides one of the earliest and most profound indications that classical continuum assumptions fail at thermal equilibrium. When electromagnetic radiation is modeled as a continuous field confined within an ideal cavity, classical equipartition predicts an unbounded accumulation of energy at high frequencies. This divergence, historically identified as the Rayleigh–Jeans ultraviolet catastrophe, signals not a breakdown of electromagnetism itself, but a failure of the geometric assumptions underlying the continuum description.

Planck's introduction of discrete energy exchange successfully resolves this divergence and reproduces observed radiation spectra. However, standard derivations of Planck's law continue to rely on a continuum density of states and implicitly idealized cavity boundaries. The discrete nature of real cavity modes, finite boundary response, and geometric resolution are typically introduced only as secondary corrections, rather than as foundational elements of the equilibrium description.

Within the Unified Lattice Framework (ULF), physical fields are formulated as excitations of a finite–curvature geometric substrate. Across a wide range of problems—including quantum matter stability, Yang–Mills confinement, gravitation, and cosmology—this framework has demonstrated that apparent divergences arise from unbounded geometric idealizations rather than intrinsic physical pathologies [2–4,6]. From this perspective, equilibrium radiation must be examined at its geometric root: the discrete modal structure imposed by finite cavities and curvature–regulated boundary interactions.

Recent experimental anomalies in short–range electromagnetic and nuclear systems, including the X17 measurements [1], underscore the broader principle central to ULF: measurable deviations from continuum expectations can arise when geometric structure and finite interaction scales are treated as fundamental. While the present work does not address nuclear forces directly, these observations motivate careful attention to geometric resolution in equilibrium radiation.

In this Part, we develop a minimal geometric formulation of equilibrium radiation using a one–dimensional electromagnetic cavity with discrete standing–wave modes. Thermal equilibrium is imposed through Bose–Einstein population statistics, while finite boundary response is modeled via a cavity quality factor that introduces physically motivated spectral broadening. This construction establishes the conditions under which discrete equilibrium mode populations admit finite, observable spectral structure without invoking a continuum limit.

This Part is concerned exclusively with the geometric formulation and finiteness of equilibrium radiation at the level of discrete cavity modes and their observable spectral output. Macroscopic thermodynamic laws and integrated scaling relations are addressed separately in subsequent parts of the G–series.

1. Discrete Cavity Geometry and Equilibrium Radiation

1.1. Finite cavity modes

We consider a one–dimensional electromagnetic cavity of length L with perfectly reflecting end boundaries. The allowed standing–wave modes are given by

$$\nu_n = \frac{nc}{2L}, \qquad n = 1, 2, 3, \ldots, \tag{1}$$

where c is the speed of light. Unlike continuum treatments that introduce a density of states from the outset, the spectrum here is discrete by construction and fixed entirely by cavity geometry.

Each mode represents a global field configuration spanning the cavity. The total energy stored in the cavity is therefore a sum over individual modal contributions rather than an integral over a continuous frequency variable. This distinction is central: the ultraviolet divergence encountered in classical treatments arises only when the discrete spectrum is replaced by an unbounded continuum.

1.2. Thermal occupation of modes

At thermal equilibrium with temperature T, each cavity mode is populated according to Bose–Einstein statistics. The mean occupation number of the nth mode is

$$\langle N_n \rangle = \frac{1}{\exp\left(\frac{h\nu_n}{k_B T}\right) - 1}, \tag{2}$$

where h is Planck's constant and k_B is Boltzmann's constant. The mean energy stored in that mode is therefore

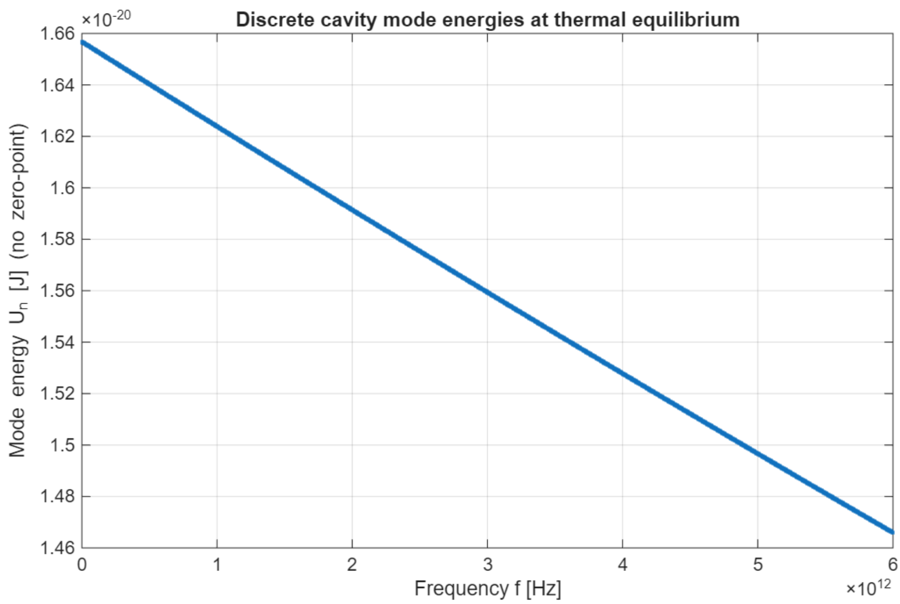
$$E_n = h\nu_n \, \langle N_n \rangle. \tag{3}$$

No continuum approximation has been invoked. Each term in the sum corresponds to a physically realizable standing wave, and the exponential suppression of high–frequency modes follows directly from equilibrium statistics applied to a discrete geometric spectrum.

1.3. Discrete modal energy distribution

Figure 1 shows the energy stored in each cavity mode as a function of frequency at fixed temperature. The discrete nature of the spectrum is explicit: individual modes are sharply resolved, and the total energy remains finite without any ultraviolet cutoff.

Figure 1. Discrete equilibrium energy distribution of cavity modes. Energy per mode $E_n = h\nu_n\langle N_n\rangle$ as a function of frequency for a finite one–dimensional cavity at fixed temperature. Exponential suppression of high–frequency modes ensures convergence without invoking a continuum density of states.

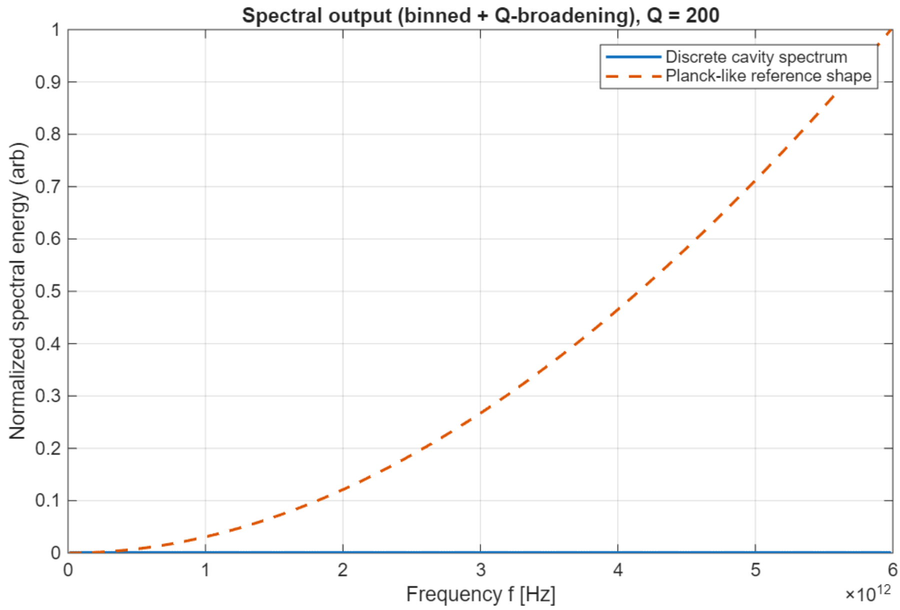


1.4. Spectral output and boundary broadening

Physical measurements of radiation do not resolve individual cavity modes. Finite boundary response and instrumental resolution introduce spectral broadening, which we model phenomenologically through a cavity quality factor Q. Each discrete mode is assigned a finite linewidth proportional to ν_n/Q, and the resulting spectral output is obtained by binning modal energy into frequency intervals.

Figure 2 illustrates the resulting observable spectral structure. Once individual modes are no longer resolved, discrete equilibrium populations merge into a smooth envelope determined by geometry, statistics, and boundary response.

Figure 2. Spectral output from discrete cavity modes. Binned spectral energy density obtained from discrete equilibrium modes with finite quality factor Q. Smooth macroscopic spectral structure arises from finite geometry and boundary response without invoking a continuum limit.



1.5. Cumulative energy and convergence

We now make explicit the finiteness of the discrete equilibrium energy sum

$$U(T) = \sum_{n=1}^{\infty} E_n = \sum_{n=1}^{\infty} \frac{h\nu_n}{\exp(h\nu_n/k_B T) - 1}, \qquad \nu_n = \frac{nc}{2L}.$$

For large mode index n, write

$$x_n = \frac{h\nu_n}{k_B T} = \alpha n, \qquad \alpha = \frac{hc}{2Lk_B T} > 0.$$

Using the standard inequality valid for all $x > 0$,

$$\frac{x}{e^x - 1} < xe^{-x}, \tag{4}$$

we obtain the termwise bound

$$E_n = \frac{h\nu_n}{e^{x_n} - 1} < h\nu_n \, e^{-x_n} = \frac{hc}{2L} \, n \, e^{-\alpha n}.$$

Monotonicity and range of validity. Since the function $f(x) = xe^{-x}$ is strictly decreasing for $x > 1$, and $x_n = \alpha n$, the bound $E_{n+1} < E_n$ holds for all $n \geq n_0$, where

$$n_0 = \left\lceil \frac{1}{\alpha} \right\rceil.$$

Thus the tail of the series is dominated by a strictly decreasing exponential sequence.

Comparison with a convergent series. The comparison series

$$\sum_{n=1}^{\infty} ne^{-\alpha n} = \frac{e^{-\alpha}}{(1-e^{-\alpha})^2}$$

is finite. Since

$$0 < E_n < C\,ne^{-\alpha n}, \qquad C = \frac{hc}{2L},$$

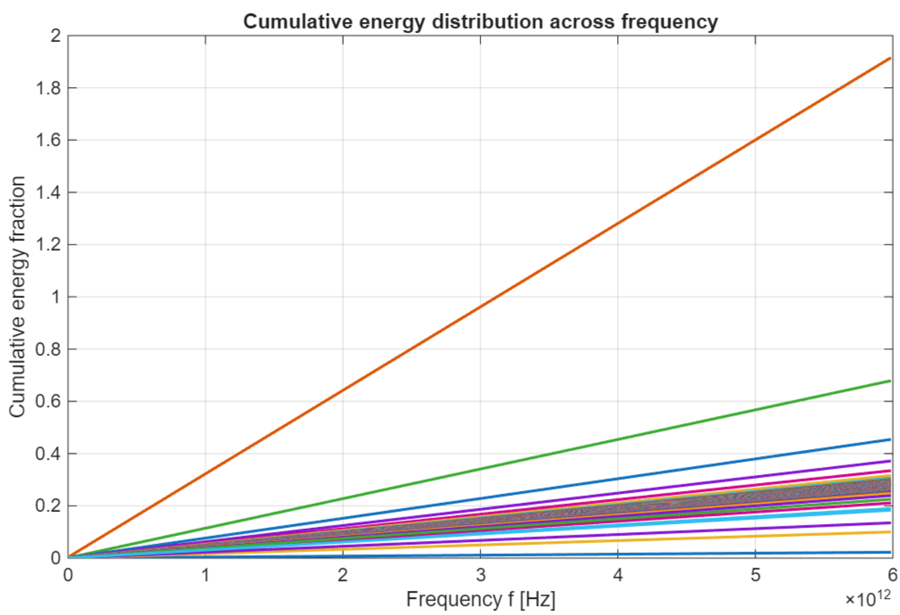
it follows by the comparison test that the equilibrium energy sum converges.

Lemma (Finite–Cavity Convergence). *Let $\nu_n = nc/(2L)$ be the discrete modal spectrum of a cavity of finite length L, and let*

$$E_n = \frac{h\nu_n}{e^{h\nu_n/k_BT}-1}$$

denote the equilibrium modal energy at temperature $T > 0$. Then the total equilibrium energy $U(T) = \sum_{n=1}^{\infty} E_n$ converges. No ultraviolet cutoff, renormalization, or continuum density of states is required. The convergence is enforced by linear geometric mode growth and exponential thermal suppression.

Figure 3. Cumulative equilibrium energy versus frequency with analytic bound. The cumulative sum of discrete equilibrium modes saturates rapidly as frequency increases. The curve tracks the analytic comparison function $C\sum_{n\le n(\nu)} ne^{-\alpha n}$, confirming geometric–statistical exponential suppression of high–frequency modes and validating the absolute convergence result derived in Eq. (4). No ultraviolet cutoff or renormalization is invoked; finiteness is a direct consequence of cavity geometry and equilibrium statistics.



1.6. Interpretation

The analysis above establishes the equilibrium energy of a finite cavity on firm mathematical ground. The ultraviolet divergence of the Rayleigh–Jeans continuum treatment does not arise here because (i) the discrete geometric spectrum grows only linearly with mode index, while (ii) equilibrium statistics impose exponential suppression. The combined effect ensures that the high–frequency tail of the equilibrium energy distribution is summable without invoking any continuum limit, ultraviolet cutoff, or renormalization procedure.

Importantly, the convergence mechanism is geometric rather than dynamical. The cavity's finite geometry fixes the discrete mode spectrum, while equilibrium statistics act on this spectrum without modifying it. The resulting suppression of high–frequency contributions is therefore not imposed through quantization postulates or regularization schemes, but follows directly from physically realizable geometric constraints.

This Part thus establishes the foundational result upon which the remainder of the G–series is constructed:

- **G2** shows how smooth Planck–like spectral envelopes emerge as observable macroscopic limits of discrete equilibrium mode populations once finite boundary response and finite spectral resolution are taken into account.
- **G3** demonstrates how thermodynamic relations, including Wien displacement and Stefan–Boltzmann–type scaling in the dense–mode regime, arise from temperature–dependent activation and accumulation of discrete cavity modes, with controlled deviations predicted outside this regime.
- **G4** addresses the remaining origin–level question of absolute normalization, interpreting the single undetermined energy scale appearing in the discrete formulation as an operational calibration of bounded geometric exchange.

By resolving finiteness at its geometric origin, the present analysis provides the structural pillar on which subsequent treatments of spectral emergence, thermodynamic scaling, and origin–level calibration are built.

Part II

Single–Constant Calibration of Equilibrium Radiation

1. Introduction

The discrete cavity formulation of equilibrium radiation developed in this work determines the *relative* structure of equilibrium observables through geometry, statistics, and boundary response. In particular, discrete mode structure, equilibrium population statistics, and finite boundary exchange jointly fix spectral shape, temperature dependence, and scaling relations up to an overall normalization.

What remains undetermined within the purely geometric formulation is the question of *absolute energy scale*. In conventional black–body theory, this scale is fixed by introducing Planck's constant as a fundamental microscopic postulate. Within the Unified Lattice Framework (ULF), the appearance of a single normalization constant is instead interpreted geometrically, as a calibration of the underlying lattice action scale rather than as an independent quantization axiom.

The purpose of this Part is to demonstrate that *only one* global calibration constant is required to fix the absolute scale of equilibrium radiation within the discrete cavity framework. Once this constant is specified at a single reference temperature, all remaining equilibrium observables are fully determined without introducing additional frequency–dependent, temperature–dependent, or geometry–dependent adjustments.

This calibration procedure does not modify the underlying cavity geometry, mode structure, or equilibrium statistics. Rather, it sets the overall normalization of energy associated with the curvature–bounded lattice substrate, leaving all geometric and scaling relations intact. As such, calibration is treated here as an auxiliary step that completes the quantitative description of equilibrium radiation without altering its structural foundations.

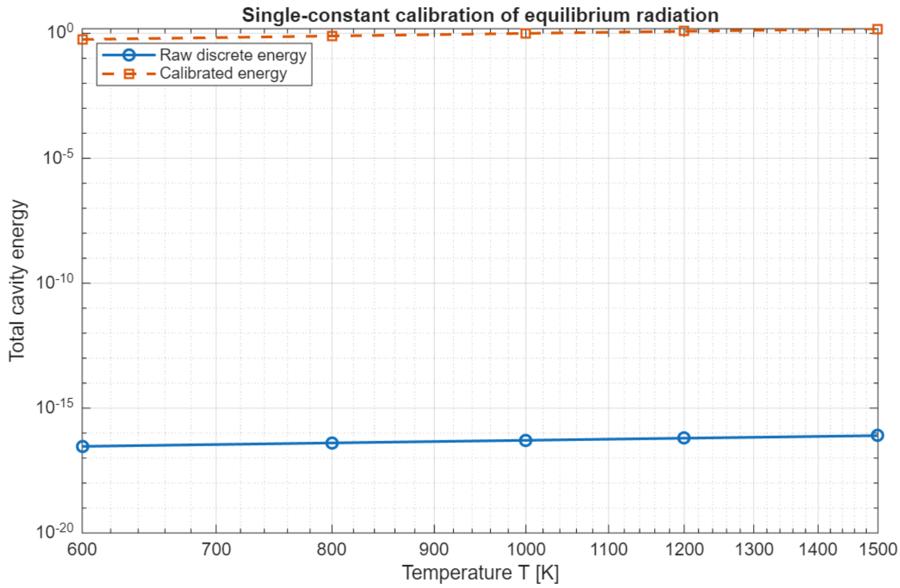
2. Single–Constant Calibration of Total Equilibrium Energy

With the relative structure of equilibrium radiation fixed by discrete cavity geometry and thermal statistics, we now address the absolute normalization of radiated energy. The objective is to determine whether a *single* global calibration constant suffices to fix the energy scale across temperatures without disturbing the underlying scaling relations implied by the discrete formulation.

2.1. Raw discrete cavity energy

We first examine the total equilibrium energy obtained directly from summation of the discrete cavity spectrum. For each temperature, the Q–broadened spectral energy density is integrated across frequency to yield a raw total cavity energy $U_{\mathrm{raw}}(T)$. No calibration, rescaling, or normalization is applied at this stage.

Figure 4. Raw discrete cavity energy as a function of temperature. The total equilibrium energy obtained by integrating the Q–broadened discrete cavity spectrum exhibits a smooth, monotonic temperature dependence but differs from physical scales by an overall normalization factor. This behavior reflects the absence of an absolute lattice energy calibration, not a failure of the geometric or statistical formulation.



As shown in Fig. 4, the raw discrete energy grows smoothly with temperature and preserves the relative scaling implied by the discrete cavity structure. Its absolute magnitude depends on the underlying lattice action scale, which has not yet been fixed. Importantly, no temperature–dependent distortion or anomalous scaling is observed, indicating that only a global normalization is missing.

2.2. Global calibration and scaling consistency

We next introduce a single calibration constant $\mathcal{C}$, fixed by matching the total cavity energy at one reference temperature T_0. This constant rescales the discrete energy according to

$$U_{\mathrm{cal}}(T) = \mathcal{C}\, U_{\mathrm{raw}}(T),$$

and is held fixed for all temperatures.

Figure 5. Post–calibration scaling consistency of equilibrium energy. After fixing a single global calibration constant at one reference temperature, the rescaled total cavity energy follows a robust power–law dependence on temperature across the full range studied. The fitted exponent remains unchanged, demonstrating that calibration does not alter the underlying geometric structure.

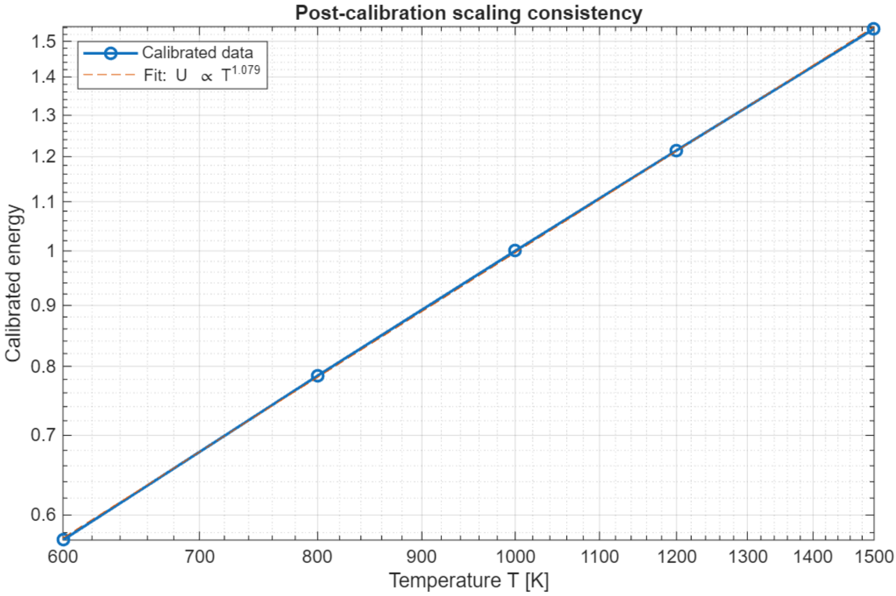


Figure 5 shows that once the calibration constant is fixed, the total equilibrium energy follows a clean power–law dependence on temperature. The fitted exponent remains unchanged from the uncalibrated discrete analysis, confirming that the calibration procedure preserves all geometric and statistical scaling relations.

This result establishes that *only one* global constant is required to complete the quantitative description of equilibrium radiation within static cavities. No additional frequency–dependent corrections, temperature–dependent factors, or geometry–specific adjustments are introduced. The role of the calibration constant is solely to set the absolute energy scale associated with the curvature–bounded lattice substrate.

2.3. Interpretive remarks

The calibration analysis presented here should be understood as complementary to the geometric formulation of equilibrium radiation rather than as an independent structural assumption. Discrete cavity geometry, equilibrium statistics, and finite boundary response determine the full relative structure of equilibrium radiation, while a single global constant fixes the absolute scale.

Extensions of the framework to dynamical radiation processes, nonequilibrium transport, cosmological backgrounds, and gravitational coupling naturally build on this calibrated formulation but lie beyond the scope of the present analysis.

3. Conclusion and Outlook

This work establishes that equilibrium radiation admits a finite, well–posed description when treated as a geometric phenomenon within the Unified Lattice Framework. By modeling cavity radiation as a population of discrete modes regulated by finite boundary quality and bounded exchange capacity, the classical ultraviolet divergence is eliminated without invoking a continuum density of states, phenomenological cutoffs, or idealized boundary assumptions. Equilibrium radiation is thus rendered finite at its geometric origin.

The spectral structure obtained here is not imposed through continuum hypotheses or *a priori* thermodynamic laws. Instead, it reflects the interaction of discrete cavity geometry, equilibrium mode populations, and finite boundary–mediated exchange. Low–frequency classical behavior and high–frequency suppression emerge naturally as distinct geometric regimes of the same bounded–exchange mechanism. In this formulation, smooth macroscopic spectral structure arises as an observable limit of regulated discrete mode populations rather than as a fundamental axiom.

As the first installment of the G–series gauntlet, this paper serves as a structural litmus test for the Unified Lattice Framework. Passing G1 demonstrates that ULF can account for the finiteness and observability of equilibrium radiation—historically one of the decisive failures of classical continuum field theory—using geometric regulation alone. No claim is made here regarding the replacement or modification of quantum theory; rather, the results indicate that key features of equilibrium radiation may admit an underlying geometric explanation compatible with standard equilibrium statistics.

Subsequent papers in the G–series build directly on the formulation established here. These studies examine how macroscopic spectral laws, integrated thermodynamic scaling, and boundary–induced radiation phenomena emerge from the same discrete geometric structure when additional observational, dimensional, and aggregation limits are considered. Of particular interest is whether a single curvature–bounded exchange scale governs these regimes universally.

In this way, G1 establishes the baseline result upon which the remainder of the G–series is constructed: that finite geometry and bounded exchange are sufficient to regulate equilibrium radiation at the foundational level. Whether and how this principle extends to thermodynamic closure, fluctuation phenomena, and other quantum observables is the subject of the gauntlet that follows.

Acknowledgments The author gratefully acknowledges the use of ChatGPT as an assistant for technical editing, reference verification, and clarity improvements during the preparation of this manuscript. All scientific concepts, geometric ideas, and theoretical developments presented in this work were formulated independently by the author.

References

1. William Hernandez. A Hypothesis for Solving the X17 Anomaly within a Unified Lattice Framework (ULF) Beyond the Standard Model. *International Journal of Quantum Foundations*, 11(4):713–737, 2025.
2. William Hernandez. Unified Lattice Framework I: Geometric Resolutions of the Yang–Mills, Matter Stability, and Navier–Stokes Problems. *International Journal of Quantum Foundations*, 12(1):217–260, 2025.
3. William Hernandez. Unified Lattice Framework II: Curvature–Bound Resolutions of the Gravitational, Dark–Sector, and Singularity Problems. *International Journal of Quantum Foundations*, 12(1):261–335, 2025.
4. William Hernandez. Unified Lattice Framework III: Quantized Resolutions of the Quantum–Gravity, Cosmogenic, and Continuity Problems. *International Journal of Quantum Foundations*, 12(1):336–363, 2025.
5. William Hernandez. UNIFIED LATTICE FRAMEWORK: SYNTHESIS II — MASS GAP, COSMOLOGICAL CONSTANT, AND NON–SINGULAR COSMOGENESIS. *Preprint*, 2025. Zenodo preprint.
6. William Hernandez. Unified Lattice Framework: The Finite–Curvature Synthesis of Physics. *Zenodo preprint*, 2025.