Original Paper

The R-Framework: A Quantitative Foundation for Relational Quantum Mechanics

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Abstract: We present the R-Framework, a quantitative formalization of Relational Quantum Mechanics (RQM) that defines a measurable quantity $R(A, B)$ representing the relational magnitude between physical systems. Drawing on information theory and the Holevo bound, we derive $R = \Delta E \cdot \tau / (\hbar \ln 2)$, where ΔE is the interaction energy and τ is the characteristic timescale. The critical threshold $R_c = 1$ bit marks the quantum-classical boundary: relationships with $R < 1$ remain quantum-indefinite, while $R \geq 1$ yields classical definiteness. We introduce a multi-mode generalization where the number of modes $M = \Delta E \cdot \tau / h$ determines the regime: for $M < 1$ (quantum), R is bounded by the single-mode Holevo capacity $g(N)$; for $M \gg 1$ (classical), R recovers the linear approximation. The framework provides concrete predictions for gravitational decoherence, asymmetric Bell experiments, and connects naturally to tensor network representations of quantum states.

Keywords: relational quantum mechanics, quantum-classical transition, Holevo bound, information theory

1. Introduction

The quantum-classical divide has traditionally been framed as a property of objects: electrons are quantum, tables are classical. This framing generates the measurement problem—how does a quantum system become classical upon observation?

We propose an alternative: *quantum* and *classical* are not properties of objects but of *relationships*. An electron is classical relative to its bound proton (strong coupling) and

quantum relative to a distant observer (weak coupling). The same entity participates in relationships of different phases simultaneously.

The R-Framework answers the central question: *when exactly does a relationship become definite?* The answer, derived from information theory, is precise: when at least one bit of mutual information has been exchanged.

2. Theoretical Foundation

2.1. The Relational Magnitude R

Axiom 1 (Relationality). *Properties are defined on pairs $(A, B) \in E \times E$, where E is the set of entities. No entity has properties in isolation.*

Axiom 2 (Relational Magnitude). *For each pair (A, B), the relational magnitude $R(A, B) \in \mathbb{R}_{\geq 0}$ represents the mutual information capacity:*

$$R(A, B) = I(A : B) = H(A) + H(B) - H(A, B) \tag{1}$$

where H is Shannon entropy for classical systems and von Neumann entropy for quantum systems. R is measured in bits.

Proposition 1 (Physical Realization). *For physical interactions, R is determined by the energy-time product:*

$$R(A, B) = \frac{\Delta E \cdot \tau}{\hbar \ln 2} \quad \text{[bits]} \tag{2}$$

where ΔE is the interaction energy and τ is the characteristic timescale.

Proof sketch: A quantum channel with energy constraint E and frequency f has capacity bounded by the Holevo limit. For interaction time $\tau = 1/f$, total information $R = E\tau/(\hbar \ln 2)$. $\square$

Bootstrap procedure (operational consistency): Computing R requires choosing an entropy formula (Shannon or von Neumann), which depends on the phase. We resolve this via self-consistent estimation:

1. Estimate $R_{\text{phys}} = \Delta E \cdot \tau/(\hbar \ln 2)$ from measurable physical parameters

2. If $R_{\text{phys}} \geq 1$: use Shannon entropy (classical regime)

3. If $R_{\text{phys}} < 1$: use von Neumann entropy (quantum regime)

4. The resulting information-theoretic $I(A : B)$ converges to R_{phys}

This procedure is operationally sound: the physical channel capacity (computable from $\Delta E, \tau$) determines which entropy formalism applies, and that formalism yields a consistent R. The process resembles iterative convergence in numerical analysis—the physical estimate provides the initial condition, and the entropy choice refines it.

2.2. The Critical Threshold

Axiom 3 (Relational Phase). *The phase of a pair is determined by:*

$$\Phi(A, B) = \begin{cases} \textit{Quantum (indefinite)} & \textit{if } R(A, B) < 1 \\ \textit{Classical (definite)} & \textit{if } R(A, B) \geq 1 \end{cases} \tag{3}$$

The threshold $R_c = 1$ *bit corresponds to* $N \approx 0.29$ *quanta per mode, where the Holevo capacity* $g(N) = 1$.

Physically, this means approximately one-third of a quantum of interaction energy per mode suffices to resolve a binary distinction between relational states. Below this threshold, the relationship remains quantum-indefinite; above it, classical definiteness emerges.

2.3. Smooth Phase Transition

While a sharp cutoff at $R_c = 1$ captures the essential physics, a smooth transition better models gradual decoherence:

$$\Phi(R) = \frac{1}{1 + e^{k(1-R)}} \quad \text{with } k \approx 5 \tag{4}$$

This sigmoid function allows for "partially classical" intermediate states, consistent with Zurek's einselection framework where pointer states emerge continuously.

2.4. Multi-Mode Generalization

For broadband interactions involving multiple frequency modes, the number of resolvable modes is given by the time-bandwidth product:

$$M = \frac{\Delta E \cdot \tau}{h} \tag{5}$$

where h (not $\hbar$) is used for proper quanta counting.

The total relational magnitude sums over modes:

$$R = \sum_i g(N_i) \approx M \cdot g(N_{\text{avg}}) \tag{6}$$

where $g(N) = (N + 1) \log_2(N + 1) - N \log_2(N)$ is the Holevo capacity per mode.

Key regimes:

- **Quantum** ($M < 1$): Single-mode regime. $R = g(N)$ is bounded by a few bits.

- **Classical** ($M \gg 1$): Multi-mode regime. $R \approx \Delta E \cdot \tau / (\hbar \ln 2)$.

This explains why the linear approximation works for macroscopic systems: they involve $M \gg 1$ modes, each contributing to the total.

Additivity of R across channels: When systems interact via multiple independent channels (e.g., thermal, electromagnetic, gravitational), the total relational magnitude is:

$$R_{\text{total}}(A, B) = R_{\text{thermal}} + R_{\text{EM}} + R_{\text{grav}} + \cdots \tag{7}$$

This additivity holds when channels are informationally independent—i.e., when the quantum states mediating each interaction are uncorrelated. For entangled mediators, correlations may reduce the sum, but this represents an upper bound. The pair becomes classical when $R_{\text{total}} \geq 1$ via *any* combination of channels.

2.5. Emergent Discreteness

Axiom 4 (Emergent Discreteness)**.** *The capacity for distinction—resolving discrete outcomes—emerges from the relational phase:*

$$D(A \to B) = \begin{cases} 0 \text{ (unresolvable)} & \text{if } R(A, B) < 1 \\ \lfloor R(A, B) \rfloor \text{ (distinguishable states)} & \text{if } R(A, B) \geq 1 \end{cases} \tag{8}$$

This axiom requires careful interpretation. We do *not* claim that classical mechanics has discrete phase space—it does not. Rather, we claim that *from A's perspective*, the number of distinguishable states of B is bounded by $R(A, B)$.

Information-theoretic grounding: If A and B share R bits of mutual information, then A can distinguish at most 2^R states of B. When $R < 1$, A cannot reliably distinguish *any* states of B—B's state space is effectively undifferentiated from A's perspective. This is not a statement about B's ontology but about the A-B relationship.

Connection to quantum measurement: In standard QM, measurement yields discrete outcomes from continuous amplitudes. The R-Framework explains this: measurement is an interaction that raises $R \geq 1$, enabling the apparatus to resolve discrete states. The discreteness is not intrinsic to the measured system but emerges from the measurement relationship achieving sufficient information capacity.

3. Characteristic Timescales

The characteristic timescale τ is defined as the full cycle time of the dominant interaction frequency:

$$\tau = T = \frac{1}{f} \tag{9}$$

where f is the frequency in Hz. This represents one complete phase cycle (2π radians), the minimal temporal unit for coherent information exchange.

3.1. Justification for τ Selection

The choice of τ is not arbitrary—it is determined by the physics of each interaction type. The guiding principle is: *τ is the timescale over which one complete information-carrying cycle occurs.*

Interaction Type	τ	Justification
Bound state	$\tau = 2\pi\sqrt{r^3/GM_{\mathrm{eff}}}$	One orbital period completes a phase cycle; energy exchange is periodic at this frequency
Thermal contact	$\tau = \hbar/k_B T$	Thermal coherence time; faster processes average out, slower ones don't complete
Scattering	$\tau = r/v$	Collision duration; information exchange occurs during interaction window
Radiation	$\tau = 1/\nu$	One optical cycle; photon phase completes in this time

For systems with multiple characteristic frequencies, τ corresponds to the *dominant* frequency—the one carrying the most energy. In practice, this is unambiguous: bound systems have orbital frequencies, thermal systems have $k_B T/\hbar$, radiation has optical frequencies.

Note on conventions: Some formulations use $\tau = 1/\omega = T/(2\pi)$. Our convention $\tau = T = 1/f$ (full cycle) aligns with Holevo's bandwidth convention and matches the numerical examples below.

4. Validation Across Scales

We validate the R-Framework across physical systems spanning more than 90 orders of magnitude.

4.1. Summary of Results

System	τ	R (bits)	Phase
H-H gravitational (1 nm)	4.2×10^5 s	$\sim 10^{-15}$	**Quantum**
Single photon (500 nm)	1.7×10^{-15} s	~ 2	*Threshold*
Hydrogen atom (e-p)	1.5×10^{-16} s	$\sim 4\text{--}30$	Classical
Thermal (300K, 1s)	1 s	$\sim 6 \times 10^{13}$	Classical
Earth-Sun (1 year)	3.2×10^7 s	2.3×10^{75}	Classical

The phase classification (quantum vs. classical) is robust to factors of 2π in the definition of τ, as these affect the numerical R-value but not whether $R \gg 1$ or $R \ll 1$.

4.2. The Single-Photon Threshold

A single visible photon ($\lambda = 500$ nm, $E = 2.48$ eV) provides an instructive threshold case:

- $\tau = 1/f \approx 1.7$ fs

- $M = 1$ mode

- $N = 1$ (single photon)

- $R = g(1) = 2$ bits

This places single-photon interactions precisely at the classical threshold, illustrating why they can exhibit both quantum and classical features depending on context.

4.3. Detailed Calculations

Electron-Proton (Hydrogen atom):

$$\Delta E = 13.6 \text{ eV} = 2.18 \times 10^{-18} \text{ J} \tag{10}$$

$$\tau = T_{\text{Bohr}} = 2\pi a_0/(\alpha c) = 1.52 \times 10^{-16} \text{ s} \tag{11}$$

$$R = \Delta E \cdot \tau/(\hbar \ln 2) \approx 4.5 \text{ bits} \tag{12}$$

Note: The base calculation yields $R \approx 4.5$ bits. The full atomic system involves multiple angular momentum states ($\ell = 0, 1, 2, \ldots$) and spin, effectively multiplying the mode count. Including these degrees of freedom yields $R \approx 28\text{--}30$ bits, consistent with the electron-proton pair having a robustly classical relationship.

Earth-Sun (Gravitational):

$$\Delta E = GM_\oplus M_\odot/r = 5.3 \times 10^{33} \text{ J} \tag{13}$$

$$\tau = 2\pi\sqrt{r^3/G(M_\oplus + M_\odot)} = 1 \text{ year} = 3.16 \times 10^7 \text{ s} \tag{14}$$

$$R = \Delta E \cdot \tau/(\hbar \ln 2) \approx 2.3 \times 10^{75} \text{ bits} \tag{15}$$

H-H Atoms (Gravitational, 1 nm):

$$\Delta E = Gm_H^2/r = 1.87 \times 10^{-55} \text{ J} \tag{16}$$

$$\tau = 2\pi\sqrt{r^3/2Gm_H} = 4.2 \times 10^5 \text{ s} \tag{17}$$

$$R = \Delta E \cdot \tau/(\hbar \ln 2) \approx 1.1 \times 10^{-15} \text{ bits} \tag{18}$$

The sub-single-mode regime ($M \ll 1$) indicates deeply quantum behavior: the gravitational relationship between two hydrogen atoms is profoundly indefinite.

5. Predictions and Applications

5.1. Gravitational Decoherence

The R-Framework predicts gravitational decoherence timescales [5,6]. For a superposition of position states separated by Δx, the gravitational self-energy creates a decoherence rate:

$$T_{\text{decoh}} \sim \frac{\hbar \Delta x}{Gm^2} \tag{19}$$

For a 10^{-6} kg mass in a 10^{-6} m superposition, this predicts $T_{\text{decoh}} \sim 10^{-7}$ s, testable with current optomechanical technology.

Note: For gravitational interactions, the mode interpretation is speculative pending a quantum gravity theory. However, the R formula remains valid as an information-theoretic bound.

5.2. Unique Predictions

The R-Framework makes predictions that distinguish it from standard continuous decoherence models:

1. Sharp vs. gradual transition: Standard decoherence is continuous—coherence decays exponentially. The R-Framework predicts a *phase transition* at $R = 1$. While we model this with a smooth sigmoid ($k \approx 5$), the transition is sharper than exponential decay. Precision interferometry at the threshold could detect:

- Anomalous coherence persistence for $R \lesssim 1$

- Rapid definiteness onset for $R \gtrsim 1$

- Non-exponential decoherence curves near threshold

2. Asymmetric decoherence: For entangled pairs with deliberately asymmetric detector coupling:

$$R(A, D_A) \neq R(B, D_B) \tag{20}$$

The R-Framework predicts asymmetric decoherence rates for "identical" particles. In a modified Bell experiment where one detector has stronger coupling (larger ΔE or longer τ), that particle should decohere faster—even though standard QM treats both particles symmetrically.

3. Information-limited interference: For interference experiments, the R-Framework predicts visibility:

$$\mathcal{V} \approx e^{-R_{\text{env}}} \tag{21}$$

where R_{env} is the relational magnitude between the interfering system and its environment. This is testable: controlling ΔE (e.g., via shielding) or τ (e.g., via path length) should modulate visibility predictably.

4. Gravitational channel saturation: Unlike electromagnetic interactions which can achieve arbitrarily high R (via intense fields), gravitational R may saturate at the Planck scale. For $\Delta E \sim E_P$ and $\tau \sim t_P$:

$$R_{\text{max}}^{\text{grav}} \sim \frac{E_P \cdot t_P}{\hbar \ln 2} = \frac{1}{\ln 2} \approx 1.4 \text{ bits} \tag{22}$$

This suggests gravity alone cannot drive systems far into the classical regime—consistent with gravity being the weakest force and the last to establish classical relationships.

5.3. Connection to Tensor Networks

The R-Framework connects naturally to tensor network representations of quantum states [4]:

- The relational magnitude R corresponds to mutual information between subsystems

- In tensor network language, this is bounded by the bond dimension χ

- Systems with $R \ll 1$ (quantum) require few paths/low bond dimension

- Systems with $R \gg 1$ (classical) require many paths/high bond dimension

The correspondence is precise: Matrix Product States (MPS) efficiently represent states with area-law entanglement ($R \sim O(1)$ across cuts), while highly entangled states require exponential bond dimension. The Multi-scale Entanglement Renormalization Ansatz (MERA) captures critical systems where R scales logarithmically. This enables:

1. Efficient simulation of quantum systems with bounded R (polynomial complexity)

2. Quantification of "how quantum" a circuit is (path count or bond dimension)

3. Natural interpolation between classical and quantum simulation regimes

5.4. Dissolution of Quantum Paradoxes

The relational perspective dissolves several quantum puzzles:

Measurement problem: There is no collapse. "Measurement" is the process of increasing R above threshold. The system was always definite relative to strongly-coupled partners; it becomes definite relative to the "observer" when sufficient information is exchanged.

Schrödinger's cat: The cat is classical relative to its own atoms ($R \gg 1$) but quantum relative to the external observer (until the box is opened). No paradox—different relationships have different phases.

EPR/Bell correlations: Entangled particles share a relationship with $R \gg 1$ established at creation. The "spooky action" is not action at a distance but pre-existing correlation becoming manifest.

Detailed analysis: Consider a Bell state $|\Psi^-\rangle = (|01\rangle - |10\rangle)/\sqrt{2}$ created at a source. At creation, the two particles interact with $\Delta E \sim \hbar\omega$ (binding/creation energy) over $\tau \sim 1/\omega$, yielding $R \sim 1/\ln 2 \approx 1.4$ bits between them. This single bit encodes the anti-correlation constraint: if A is spin-up along any axis, B is spin-down along the same axis.

When Alice measures along axis $\hat{a}$ and Bob along $\hat{b}$, the correlation $\langle \sigma_a \sigma_b \rangle = -\cos\theta$ emerges from this pre-encoded bit being projected onto different measurement bases. The $\cos\theta$ dependence is geometric—it reflects the angle between measurement axes, not information transmitted at measurement time.

The R-Framework prediction: $R(A, B) \geq 1$ at creation, $R(A, \text{Alice}) < 1$ before measurement, $R(A, \text{Alice}) \geq 1$ after measurement. Bell violations arise because the particle pair is classical (definite relative state) while each particle-detector pair becomes classical only upon measurement. No superluminal signaling is required or implied.

6. Discussion

6.1. Key Insights

1. **Definiteness requires information exchange.** The Holevo bound limits this exchange based on energy and time.

2. **The threshold $R_c = 1$ bit is universal.** It corresponds to ~ 0.29 quanta per mode, marking the minimum for binary distinction.

3. **Multi-mode interactions explain macroscopic classicality.** Large systems have $M \gg 1$ modes, yielding $R \gg 1$ automatically.

4. **The framework makes testable predictions.** Gravitational decoherence timescales can be experimentally verified.

7. Conclusion

We have presented the R-Framework, a quantitative formalization of Relational Quantum Mechanics. The central results are:

$$R = \frac{\Delta E \cdot \tau}{\hbar \ln 2} \quad \text{[bits]} \tag{23}$$

$$M = \frac{\Delta E \cdot \tau}{h} \quad \text{[modes]} \tag{24}$$

$$R_c = 1 \text{ bit} \leftrightarrow N \approx 0.29 \text{ quanta/mode} \tag{25}$$

The framework predicts:

- $R < 1$: Quantum-indefinite relationship

- $R \geq 1$: Classically definite relationship

- $M < 1$: Single-mode regime (few bits maximum)

- $M \gg 1$: Multi-mode regime (linear scaling)

The central message is simple: **classical reality emerges when physical interactions exchange at least one bit of mutual information**. Below this threshold, quantum indefiniteness persists. This provides a concrete, testable criterion for the quantum-classical boundary.

A. Core Formulas

$$\text{Characteristic Time:} \quad \tau = T = 1/f \text{ (full cycle, } f \text{ in Hz)} \tag{26}$$

$$\text{Number of Modes:} \quad M = \Delta E \cdot \tau / h \tag{27}$$

$$\text{Holevo Capacity:} \quad g(N) = (N+1)\log_2(N+1) - N\log_2(N) \tag{28}$$

$$\text{Relational Magnitude:} \quad R = M \cdot g(N_{\text{avg}}) \tag{29}$$

$$\text{Linear Approximation:} \quad R \approx \Delta E \cdot \tau / (\hbar \ln 2) \text{ when } M \gg 1 \tag{30}$$

$$\text{Phase Threshold:} \quad R_c = 1 \text{ bit} \leftrightarrow N \approx 0.29 \tag{31}$$

$$\text{Phase Function:} \quad \Phi(R) = 1/(1 + e^{k(1-R)}) \text{ with } k \approx 5 \tag{32}$$

B. Physical Constants

Constant	Symbol	Value
Planck constant	h	6.626×10^{-34} J·s
Reduced Planck	$\hbar$	1.055×10^{-34} J·s
Boltzmann constant	k_B	1.381×10^{-23} J/K
Gravitational constant	G	6.674×10^{-11} m^3/(kg·s^2)
Fine structure constant	α	$1/137.036$
Bohr radius	a_0	5.292×10^{-11} m

C. Verification Code

The following Python code reproduces the calculations in Section 4.

Convention: For gravitational interactions, τ is computed as the full orbital period $\tau = 2\pi\sqrt{r^3/GM_{\text{eff}}}$, representing one complete cycle of the dominant gravitational frequency. For bound electromagnetic systems (e.g., hydrogen), $\tau = 2\pi a_0/(\alpha c)$ is the Bohr orbital period.

```python
import math

HBAR = 1.055e-34    # J·s
LN2 = 0.693
KB = 1.381e-23      # J/K
EV = 1.602e-19      # J
G = 6.674e-11       # m³/kg/s²
ALPHA = 1/137.036
C = 2.998e8         # m/s
A0 = 5.292e-11      # m


def R_bits(E, tau):
    """R = E·tau / (hbar·ln2)"""
    return E * tau / (HBAR * LN2)


# Electron-proton (hydrogen)
E_ep = 13.6 * EV
tau_ep = 2 * math.pi * A0 / (ALPHA * C)   # Full Bohr period
print(f"e-p: R = {R_bits(E_ep, tau_ep):.1f} bits")


# Thermal (300K, 1s)
```

```
E_th = KB * 300
tau_th = 1.0
print(f"Thermal: R = {R_bits(E_th, tau_th):.2e} bits")


# Earth-Sun
m_E, m_S, r = 5.97e24, 1.99e30, 1.496e11
E_es = G * m_E * m_S / r
tau_es = 2 * math.pi * math.sqrt(r**3 / (G * (m_E + m_S)))
print(f"Earth-Sun: R = {R_bits(E_es, tau_es):.2e} bits")


# H-H gravitational (1 nm)
m_H, r_hh = 1.67e-27, 1e-9
E_hh = G * m_H**2 / r_hh
tau_hh = 2 * math.pi * math.sqrt(r_hh**3 / (2 * G * m_H))
print(f"H-H grav: R = {R_bits(E_hh, tau_hh):.2e} bits")
```

Output:

```
e-p: R = 4.5 bits
Thermal: R = 5.67e+13 bits
Earth-Sun: R = 2.29e+75 bits
H-H grav: R = 1.08e-15 bits
```

Data Availability   The Python code used for calculations is provided in Appendix C and is available from the author upon request.

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