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The Fractal Corresponds with Light and Foundational Quantum Problems

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Abstract: Nearly one hundred years after its origins, foundational quantum mechanics remains one of the greatest unexplained mysteries in physics today. During this period, chaos theory and its geometry—the fractal—have developed. In this paper, the propagation behaviour of a simple iterating fractal—the Koch Snowflake—was described, analysed and discussed. From an arbitrary observation point within the fractal set, the fractal propagates forward by oscillation, and, retrospectively—viewing it from behind—it grows exponentially from a point of origin. The fractal propagates a potentially infinite exponential sinusoidal wave of discrete triangular bits, exhibiting many characteristics of light and quantum entities. The fractal's wave speed is potentially constant, offering insights into the perception and a direction of time where, to an observer when travelling at the frontier of propagation, change, and thus time, may slow to a stop. In isolation, the infinite fractal is a superposition of component bits, in which position and scale pose a problem of localisation. In reality, this problem is experienced within isolated 'fractal landscapes' in which position is known only through the addition of information or markers. The quantum 'measurement problem', 'uncertainty principle', 'entanglement', and the quantum-classical interface are addressed; these are problems of scale invariance associated with isolated fractality. Dual forward and retrospective perspectives of the fractal model offer the opportunity to unify quantum mechanics with cosmological mathematics, observations, and conjectures. Quantum and cosmological problems may be different aspects of the one fractal geometry.

Keywords: Measurement Problem, Observer, Entanglement, Unification

1 Introduction

Notwithstanding the problem of unifying quantum mechanics with general relativity and of defining the quantum-classical divide, quantum mechanics, at its foundations, remains an enigma and a point of crisis for physicists nearly one hundred years after its origins. The enigma[1] includes—among others: its superposition and wave-particle behaviour, the measurement problem, Heisenberg uncertainty, and quantum (EPR) entanglement [2]. Established before quantum theory and its problems emerged, our classical understanding of light also posed a mystery. And it still does. These include: its wave behaviour, revealed by Young's wave experiment[3] and its logarithmic electromagnetic spectrum(EMS) propagation described by Maxwell's equations [4], [5], and its universal constant speed essential assumption of Einstein's special relativity[6]. Since the inception of quantum mechanics, scientists have addressed this problem from various approaches; none of which has fully satisfied the community. The question still remains: Is there a geometry that can explain these problems? Have we missed something?

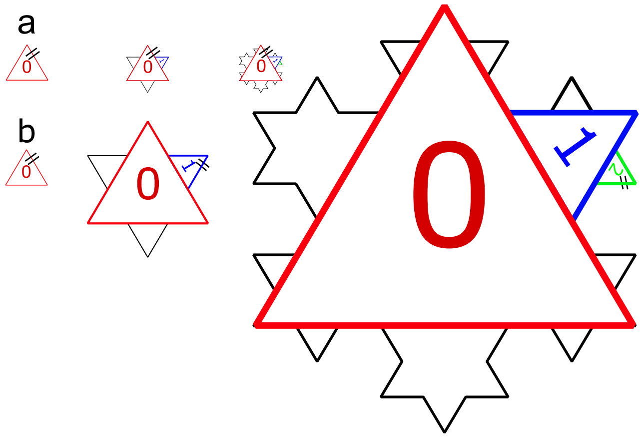
Within half the time quantum mechanics has existed, another independent geometry, and a pillar of 20th-century science, has emerged. It is the mathematics of chaos theory and its geometry, the fractal [7]. To mathematicians, fractals are an essential description of reality. We are surrounded by fractals, exemplified by clouds, coastlines, snowflakes, and, arguably, the small-scale distribution of galaxies in the universe. Mathematician Ian Stewart in his popular book: Does *God Play Dice? A New Mathematics of Chaos* [8] remarked: "Chaos was unknown in Einstein's day..." Both quantum mechanics and fractals independently share more profound similarities than their examples; these include: the use of complex numbers to define them, bifurcation [9], a concept of infinity, the potential of all possible events at all times, and they both counter the classical physics 'clockwork—deterministic—universe'. Can the geometry of fractals help make sense of the quantum and light enigma and offer an insight towards a unifying solution?

In this paper, the propagation behaviour of the iterating isolated fractal is described and analysed. This is followed by an application that assesses whether it aligns with the current understanding of foundational quantum mechanics. The Koch snowflake fractal was chosen for its quantitative and regular properties to best describe its behaviour. Building on the positive results, this paper aims to open a new line of inquiry into quantum foundations and to expose possibilities for future exploration.

2 Fractal Model and Assumptions

The foundational fractal, the Mandelbrot set, is a scale-invariant complex structure derived from a potentially infinite series on the complex plane by the iteration of a simple rule $z_n + 1 = z_n{}^2 + c$ [10]. In this way, the fractal corresponds to the ubiquitous regular-irregular (order-chaos) pattern observed in nature. The Koch Snowflake fractal [11], shares the same principles of emergent pattern growth and infinity as the Mandelbrot set without the irregularity or complexity— all bits are the same.

Figure 1 shows two perspectives of fractal growth: (a) the forward—progressive—looking, and (b) the retrospective.



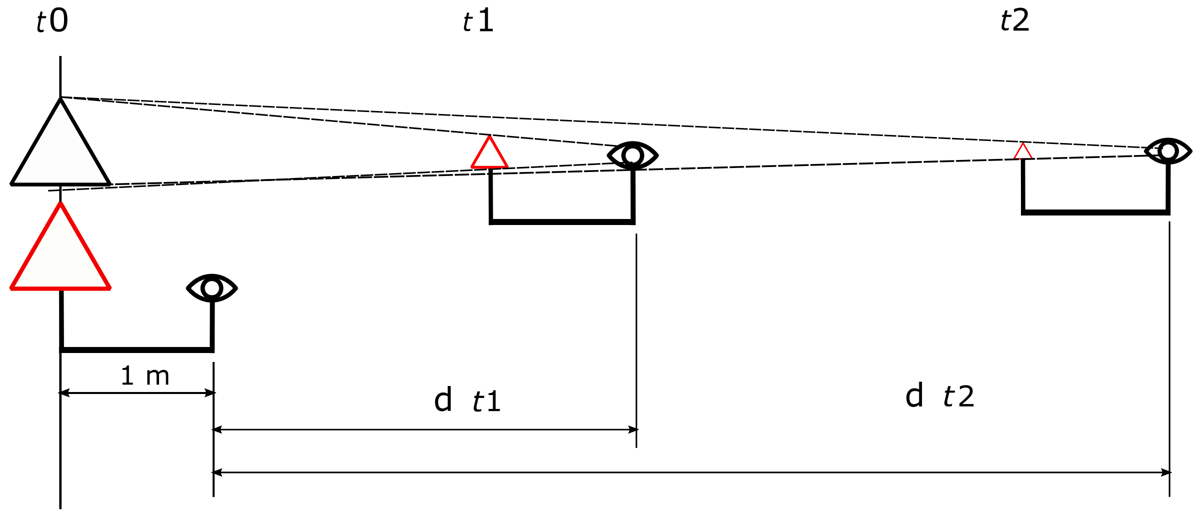
(a) the (classical) forward or evolving Snowflake perspective, where the standard sized thatched (iteration '0') is the focus, and the following triangles diminish in size from colour red iteration 0 to colour green iteration 2; and (b), the inverted retrospective perspective where the new (thatched) triangle is the focus and held at standard size while the original red iteration 0 triangle expands in the area—as the fractal iterates.

Figure 1: Dual Perspectives of (Koch Snowflake) Fractal Growth.

From an assumed observation point outside the structure, from a single triangle beginning, (a) shows the emergent 'snowflake' fractal structure converges to its snowflake shape in and around 7 ± 2 iterations. From here, the fractal continues to grow by a potential 'infinity' of discrete and identical—but diminishing-sized—equilateral triangle bits. The additional bits (blue 1 and green 2) are reduced by one-third the size, respectively, of the constant-sized red '0' thatched initial triangle bit. The development towards (snowflake) shape, from iteration to iteration, is a function of an arbitrary iteration rate or iterate-time (t). From a retrospective perspective from a position in or on the set, (b) bits grow rather than diminish with iteration. This is achieved by inverting the bit sizes and holding the new 'thatched' bit size constant (the same size as the original bit size '0') and allowing the older generations of bit sizes to grow with iteration. The size of the initial red iteration 0 triangle expands exponentially relative to the size of the new blue triangle.

2.1 The Spatial Distance Between Apparent Bit Sizes: Possible Inverse Square Law.

Zooming into the infinite snowflake fractal is akin to viewing an infinite tunnel. With this as an assumption, the diminishing triangle bit size with each iteration may correspond to a *spatial* property of the fractal, where the difference in bit size is the relative distance between each bit. If the triangle bits are assumed to be identical in shape and size, and the difference in size between them is due only to the spatial distance between the bits and a fixed observation location, then this distance may be measured. The smaller the bit, the farther away it appears from infinity. This apparent *spatial distance* between triangle bits can be measured by first holding at an arbitrary 1 metre from the eye, an iteration 0 bit-sized triangle to the iteration 0 bit-sized triangle (Figure 2).



Distance (d) is the distance between the red iteration-time bit sizes (*t*0 to *t*2) and the size of the initial (black) *t*0 bit size when the two sizes are observed equal in size to an observer (eye symbol). An arbitrary reach (1m) between the observer and the bit was set. As the bit size decreases, the distance increases by a factor of 3 with each iteration.
Figure 2: Fractal Spatial Distance Measurement.

Then repeated, holding the iteration-1 bit size constant and measuring the distance to the point at which the triangles eclipse or are equal in size for the observer. The process is repeated for iteration bit sizes 2, 3, 4 and so on. Using this method, the spatial distance between different iteration bit sizes increases by a factor of 3 as the apparent bit size decreases, suggesting an inverse-square law.

2.2 Time

Time, with respect to the development of the fractal, may be viewed from two perspectives: by the inherent iteration-time of the fractal—the assumption of this paper; or by an externally measured absolute clock time. The former's unit of time may be set to be equal to, for example, an arbitrary 1 beat per second or the Planck time.

3 Propagation

Figure 3 shows both plan and front elevation views of propagation and development of the fractal for the first 6 iteration times and for infinity (∞). The four levels, A, B, C, and D, describe wave properties of one branch of the fractal with respect to iteration: pristine, propagation, rotation, and wave, respectively.

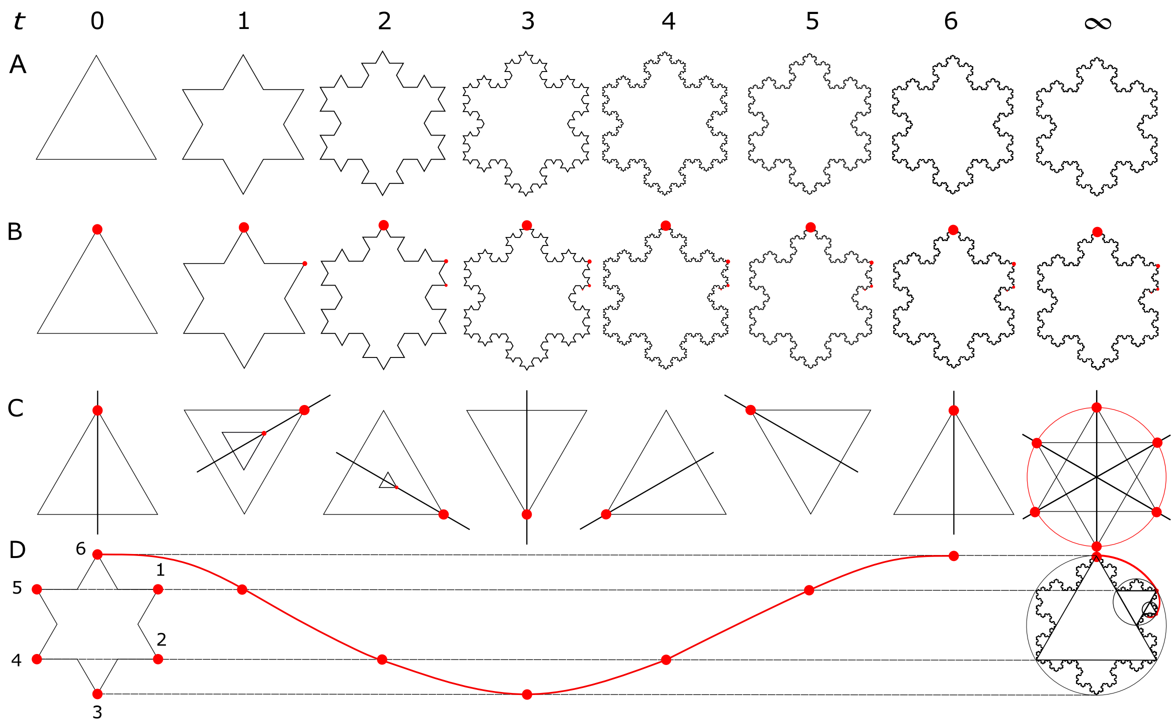


Figure 3: Fractal Logarithmic Sinusoidal Spiral.
(A) shows the development of the Koch Snowflake fractal from iteration (*t*) 0 to 6, and infinity (∞). (B) shows the transverse wave propagation of a 'red dot' on a fractal Koch Snowflake to iteration 6 and to superposition at infinity. (C) shows real triangle bit size (inside) and normalised bit size rotating clockwise through 360o as the fractal develops, and superposition at infinity. (D) shows a normalised Sin wave produced at each iteration-time and a logarithmic sinusoidal at infinity produced by a method of scribing a circular arc from respective triangle centre points.

3.1 Superposition of Bits

Figure 3(A∞) shows that there is perfect, pristine coherence among identical bits throughout the infinite set. The fractal is in perfect isolation, with no sense of scale or location. This is the superposition of identical bits. This also shows the perspective of an 'observer' outside the set: as if not yet 'observing' the fractal. The triangle bits that make up the fractal shape are identical and discrete: there are no positions—no 'half' iterations and no half bits—between the iteration positions.

3.1.1 Symmetry

The isolated identical triangle bits are in a 'coherent' state of all translational, rotational, and time symmetries at one time. It is a dihedral group of degree 3, order 6. These six iterations produce six permutations from the original generator. The rotations occur in both directions simultaneously. In this case of the fractal, supersymmetry relates to the super rotational symmetry of the component (triangle) bits before observation. Without observation, bits propagate in all directions, clockwise and anticlockwise; with observation, their propagation direction is set—either clockwise or anticlockwise. Each bit after iteration is a new symmetry generator.

3.2 Wave Propagation of Bits

When a change is made to a (triangle) bit, and the iteration is performed, wave propagation occurs, which can be analysed as follows. Figure 3(B) demonstrates an arbitrary change to a triangle that is iterated, where a 'red dot' marker is placed at the apex of every new pristine triangle bit from *t*0 to *t*6. The unmarked iterating bits for this demonstration remain pristine; however, potentially, information, in this case the dot, may propagate to all. If so, this dot would propagate as a vector in all (spatial) directions and distances by the exact method explained here. Figure 3(B∞) shows the marker propagation into infinity. This pattern is a standard oscillation wave with an exponentially increasing sinusoidal frequency (*f*) coupled with inversely decreasing wavelength (λ) and a possible universal constant speed of propagation (see 3.3).

Figure 3(C) shows the rotation of triangle bits—both real bit sizes (shown inside) and normalised (shown on the outside)—through 360 degrees. The bits rotate by discrete 60-degree increments per iteration, completing 360 degrees in 6 iterations. The 6th iteration bit is nearly impossible to discern from the 0th iteration position. Any bit size beyond this iteration is assumed, from a fixed observation position, to be unable to be discerned without 'zooming' into the set. If an observer zoomed into the set, they would observe the continuing propagation of dots. Figure 3 (C∞) show a superposition of the rotation.

Figure 3(D) shows the development of the spiral pattern produced with iteration. The geometrical 'plan view' schematic uses constant or normalised bit sizes. Figure 3 (D∞) shows that, with iteration time, the produced wave is a logarithmic sinusoidal—or helical—wave. Bits propagate such that the wave produces a frequency that increases logarithmically while simultaneously producing an amplitude and wavelength that decrease logarithmically. The wave period (T) is equal to 6 iterations; the frequency (*f*) = 1/6; the wavelength (λ) was assumed to decrease exponentially, inversely proportional to *f*. This fractal helical is produced by a method of scribing cycles from the centre points of respective triangle bits. The red curve appears continuous; however, it intersects discrete red dot points at the apexes of the discrete triangular bits; therefore, the curve is 'imaginary'. Positions on the line may be calculated with the use of complex (*i*) numbers, as conducted by quantum and light equations.

3.3 Wave Speed Measurement

A constant wave-propagation speed may be observed in the fractal. Here, three independent possible positions are described: wave speed adjusted for *spatial distance*, *iteration-beat*, and *frontier perspective*.

1. *Spatial Distance Wave Speed*: by multiplying the frequency of oscillations by the wavelength of oscillations ($v = f\,\lambda$) throughout the set. This method includes knowledge of the *spatial distance* of the fractal (2.1). Initially, without any adjustment for spatial distance, the exponential sinusoidal wave property suggests that the frequency increases and the wavelength decreases with increasing iteration times, which does not satisfy a constant group velocity. However, if this decreasing distance between points is adjusted for by the fractal spatial-distance (measured separately and shown for the Koch snowflake to increase by a factor of 3 between bits, this problem may be relieved, and a constant group velocity may prevail.
2. *Iteration-beat*: describes the effect the constant iteration-beat has on observations by an observer within the set. No matter the position of an observer in the fractal, the iteration beat is constant. The fractal will continue to emerge in front of the observer, corresponding to the iteration beat shown in Figure 3(A). This suggests that the fractal may transmit information at a constant speed over the entire propagation range. This beat speed, however, cannot be assumed to be constant

across the fractal propagation range for the problems described in 1 (above).

3. *Perspective from the Set-Frontier*: describes what an observer at the frontier of the fractal set would observe, constant speed while moving at the speed of iteration (the propagation speed), and the place where new bits come into existence as part of the production rule. On the frontier of the iterating set, a constant speed may be observed, with a trade-off that all other senses of motion or iteration time to the observer will be lost, as there is no change. It may also be argued or interpreted that the observer at this frontier position, at constant iteration beat, will always see new bits forming ahead. They will observe a fractal structure in front of them, satisfying a constant speed, no matter the speed of the observer in the set.

4 A Problem of Scale and Position on the Isolated Fractal

A problem of knowing the location and knowing the scale of constituent bit sizes on the infinite superposition fractal set before an observation or measurement presents itself in Figure 3(A).

This ‘measurement problem’, where the scale and the location of an observer are undefined before ‘measurement’, is a continuation of the properties of the isolated fractal (above). Scale is revealed with the addition of another ‘known’ object—or ‘marker’—of known scale, that is positioned on or near the fractal. This action constitutes an observation. With the addition of a marker to the bit, the history of wave propagation of the superposition of bits to produce the fractal structure ends. From the observation position, an observer, without any technology to magnify, can view a finite 7 ± 2 iterations forward into the fractal and behind. From an ‘observed’ or ‘measured’ position, independent or separated constituent triangle bits of the fractal, that make up the total (snowflake) fractal, may be interpreted, on their own, as not being part of or independent from a total system. These bits are, however, part of the wave process in forming this fractal structure, and these bits may be interpreted as being a part of wave propagation.

4.1 Wave-speed to Position Measurement Trade-off

The transition from a superposition fractal (Figure 3. $A\infty$)—with a possible defined wave speed—to a ‘measured’ marked location of a single bit within the set, as described in 4It is mutually exclusive. Superposition wave-propagation properties, including wave speed, are traded off for a position within the set. Conversely, from the marked ‘measured’/’known’ position, speed and the history of propagation are traded off and not available.

4.2 Measurement Between Locations of Varying Distance

The isolated fractal system—without any marker (Figure 3. $A\infty$)—may instantaneously have its scale or other information revealed between different observers at different locations within a fractal set. The following describes the measurement paradox across different locations within a fractal set. Two viewing ‘portal’ locations (location 1 and location 2) are positioned within the fractal set at an arbitrary distance of greater than one iteration-time apart from each other. These portals are locations of observation. Specific configurations or assumed scenarios; pristine, instantaneous change, and propagated change (configurations A, B, and C, respectively) are created and analysed with respect to the two observers at the two portals.

A. *Pristine*, coherent, superposition set: All bits are identical or coherent at all places and all iteration times within the infinite set, as demonstrated in Figure 3A.
B. *Instantaneous Change*: An arbitrary instantaneous change is made to the set by a change to all pristine triangle bits (as described in configuration A). In this case, an

arbitrarily 'red-dot' is added to the apex of the pristine bit (Figure 3. B∞). This change would be found to be coherent (the same at all instants) throughout the set, at all locations and at all iteration times.

C. *Propagated Change*: Converse or contrary to 'configuration B', the arbitrary change is propagated by iteration and is not instantaneously observed throughout the set at the same time. The arbitrary addition of the red dot to the apex of the triangle is propagated by iteration at the 'production (or iteration) speed' of the fractal (as by Figure 3. B *t*0 to ∞).

When the fractal is initially set to configuration A, a state of fractal superposition and fractal supersymmetry, before observation, the set is viewed as the same by both observers, regardless of their distance apart. If a configuration B—instantaneous arbitrary change—is made to the set endogenously, or by the observer at location 1, the effect of this change will be recorded/measured by the observer in location 2. The change could be a change in the colour of the bits—for example, to the colour yellow—or the symmetric pattern or typography of the fractal. In this case, the observers at different locations observe the same fractal pattern. If a configuration B change is made exogenously to both observers before observation, then both observers at different locations shall observe the same fractal pattern after observation.

If, however, a configuration C-non-instantaneous propagated—change is made, whether endogenous or exogenous, the change will not be observed instantaneously by each observer. The information will instead propagate between the portal locations at the fractal propagation speed. As the fractal iterates at a constant speed, the information will not be shared between the two portals. Observers at the two portals will obtain or observe different results.

5 Fractal Model Application on Light and the Quantum

The following presents an application and discussion of the isolated iterating fractal model, as applied to the description of light and other quantum entities in physics. Does the fractal correspond to the problem of the quantum? The discussions will follow the same order as the model development.

5.1 Constant Light Speed

The process of fractal development by iteration and propagation of bits of information corresponds to and is consistent with how light behaves and how light theory and the EMS are described (3.3). Although the model does not demonstrate the unification of dual electromagnetic-wave propagation, it may at least offer insight into the process. Outside the scope of this investigation, the fractal can be shown to exhibit dual processes, which may have implications for electromagnetic unification. The fractal process is not linear propagation but rather a helical wave. The fractal oscillates and varies logarithmically in frequency and size (wavelength) as it propagates. This corresponds with the EMS, first described by Maxwell. From this fractal observation, it may be that the structural geometry and the mathematics surrounding light are the geometry and mathematics of the fractal; that light is a fractal by nature. The proposed constant-wave-speed deductions—spatial-distance wave speed, iteration-beat, and on the frontier—of the fractal, open avenues for further investigation and modelling. It may be that the fractal can demonstrate Einstein's universal constant speed of light, no matter the speed of the observer.

5.2 Time

The fractal may offer direct insight into the foundational question of 'what is time. If the iteration rate is set to the production rate of light photons, the model may correspond to reality.

More on this in 5.8. Without any external observer or timekeeper, the iteration of bits is the only measure of time in the model; time appears to be emergent. The passage of time may also be assumed to be tied to changes or additions of reference points, or markers, in the model. This can be demonstrated in practice using fractal landscapes (see 5.5). With no observation in a landscape, or reference points—or changes of them—time is not discernible. No time is experienced. Similarly, when the iteration rate is 0, the model has no concept of time, but external time may still be present. There appears to be a duality here, characteristic of fractals, which warrants further investigation.

That the fractal propagates information (bits) in one direction, forward and ahead of an observer, may correspond with the 2nd Law's 'arrow of time'. This 'arrow' may also have direct insight into the perception of time to the observer; that reality is a standing wave.

The fractal may also correspond with Einstein's theory of special relativity. If light is assumed to be a fractal and an observer is assumed to be moving constantly at the *frontier* of the fractal set where the first bit comes into existence—the speed of light—this observer would observe no emergent fractal shape ahead of them, and a concept or perception of time would be absent.

Other insights into time can be drawn from the fractal, but are, however, outside this aspect of the fractal.

5.3 Demonstrating Quantum Discrete 'Particle' and Wave Propagation

The model (3.1 and 3.2) offers a direct window into electromagnetism and the quantum world where bits—particles—and waves of all different frequencies are, when unobserved, in a superposition with no position until 'observed'. The discrete bits that make up the fractal shape act as or may be akin to how discrete photons, electrons or other quantum entities are described by quantum theorists—Planck [12], Einstein and others. As with quantum entities, there are no half or other positions between the iteration positions. The wave is made up of discrete particles.

Complex numbers (i) by the Euler Formula, (equation 1) may be used to locate positions—scribed during the emergence of the fractal helical— between these, the discrete point positions.

$$e^{i\theta} = COS^{\theta} + iSIN^{\theta}. \qquad (1)$$

Coupled with this use of complex numbers, as the fractal invokes oscillation and thus Pi (π), and is a convergent series of diminishing-sized bits, invoking e. The fractal may also lend practical credence to Euler's Identity (equation 2) in practice.

$$e^{i\pi} = -1 \qquad (2)$$

5.3.1 Quantum Wave Properties and Fractality

The scale-invariant properties of fractal objects in reality, and maybe of 'classical' reality itself, (see fractal landscapes 5.5 below) may directly pertain to quantum theory's predictions and claims. The claim that reality—on all scales—can be (paradoxically) described by quantum wavefunctions may be resolved by an understanding of fractality, where aspects of reality are part of the (same) greater fractal wave-function. Position within the fractal set may also be best described by the use of the de Broglie 'pilot-wave' [13], the Schrödinger wavefunction equation [14] and Borne probably function.

As the fractal also develops by decreasing wavelength and increasing frequency with increasing iteration time (Figure 3), the Fast Fourier Transform may be used to characterise this fractal propagation, as it is currently used to describe the propagation of quantum wavefunctions. This would suggest the (Koch snowflake) fractal is a compressed summary of all the wave activity in the fractal system. If so, this discovery may offer insights into the fractality of reality via the Fast Fourier Transform and all its applications.

5.3.2 Photon Emission

That the propagation of information is in all directions (3.2) may correspond and offer insight to the (spontaneous) emission of photons with electromagnetism theory. Every new bit after iteration spawns a wave propagation of this parent in all directions.

5.4 Demonstrating Wave and Particle Duality

Just as the 'point' positions of electrons on an atom are described as being—both at the same time—a particle, and 'a smeared out' wave; here, with the fractal, we see a corresponding analogy. For the isolated fractal, this 'strange' property can be demonstrated. The isolated fractal demonstrates a wave comprised of identically propagated bits. The set can potentially be described as a (duality of) wave and point particles as posited by quantum mechanics. And just as with quantum mechanics, the fractal can demonstrate entities are both wave and particle at the same time until reference or 'observation' is made.

5.4.1 Complementarity

Quantum complementarity may be a result of a property of fractals. Fractals seem to demonstrate duality, of everything. The best examples are order and chaos, and the regular irregularity of the geometry; even as the fractal grows, it develops. These dualities may pertain directly to Bohr's quantum complementary principle.

5.5 The Heisenberg uncertainty principle

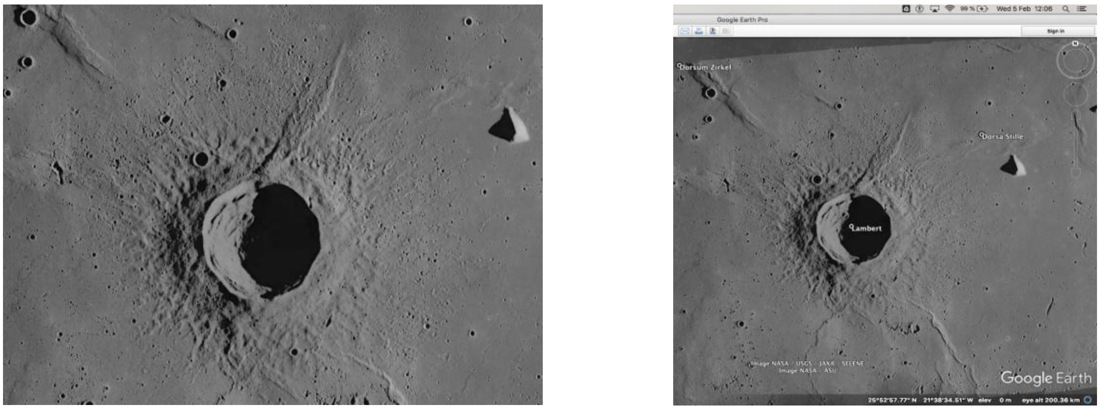
The Heisenberg uncertainty principle [15] may also (crudely) be demonstrated (4.1) by the fractal. Both position and wave velocity cannot be known simultaneously. When the fractal is in a superposition state, its scale and propagation speed are unknown; however, when reference points are added, the position and scale are known simultaneously, and the propagation speed is determined. The converse of this is true; before the reference or measurement is made, the position is not known, only the speed.

5.6 Addressing the Measurement Problem by Fractal Landscapes and Reference Points

The fractal demonstration (section 4) corresponds with the quantum observation or measurement problem: before 'measurement', the fractal bits are in a (superposition) state of all positions. The addition of a known reference point to the isolated fractal, or the observation of the fractal from an outside observer, acts as a measurement. This object or reference locks in the scale of a fractal bit-size and also—potentially—gives scale to the total fractal structure itself. The reference point may be another complex fractal object of known size; however, if the reference point is scale-invariant, this may introduce additional positional issues. More on this in the observer section. The act of measurement also corresponds to the quantum-mechanical phenomenon known as wavefunction collapse. When the marker is applied or an 'observation' or 'measurement' is made, there is a collapse in the infinite superposition of identical bits, corresponding directly to quantum theory and observation. Insight from fractal observation indicates that the bits are local: they are related to one another through the emergence of the fractal structure.

As fractal mathematics is a legitimate description and property of the macro 'classical' world, this fractal 'measurement' 'collapse' property also reveals itself in—classical—reality. The phenomenon of fractal non-location is observed in instances where there is only one uniform repeating pattern; a scale-free space where no scale can be discerned. Such spaces may be termed 'fractal-landscapes'. Natural real-life instances of fractal landscapes include clouds,

trees and forests, waves on water, sand dunes, and snow and snowdrifts. All of these are common examples of fractals. In such fractal landscapes, one—a conscious observer—can only discern position when a reference point is given or made. If one finds oneself in a total fractal landscape with no reference points, one will be disoriented or lost. Taking this further, if the complex layers of reality were stripped away, one at a time, until there is only one fractal landscape, one will experience the quantum—scale-free problem of non-location. Only when a reference point is added to the fractal landscape do these (scale-invariant) problems of scale, time, and location disappear, and position becomes known.
To demonstrate this fractal measurement problem, the following moonscape fractal landscape (Figure 4 below) is analysed.



(a) An arbitrary image of a scale-invariant crater with no references.

(b) screenshot from Google Earth of the same crater (actually Earth's Moon, Lambert crater) with altitude, data, and other references.

Figure 4: Isolated Fractal 'Crater' Landscape.

Figure 4 (a) is of a cratered fractal landscape. Without prior knowledge or technology, we are unable to discern—at least—size, position, or time. Although the crater landscape is not time-invariant (i.e., it does not iterate with respect to time), the craters conform to a fractal power-law distribution and thus constitute a classic example of a fractal. A property of fractals is that large-scale fractal objects are similar in shape and are described in the same way as their 'smaller' counterparts or examples. In the crater image, Figure 4, it could be that the craters are at an exact scale of 1:1 to the observer; however, they could also be, in principle—as with large-scale galaxy clusters and quasar groups—the scale of the largest structure in the observable universe. In contrast, Figure 4(b), demonstrates the addition of multiple reference points. With these, position, scale and other knowledge are revealed and secured. We now know—better—what it is that we are observing. The 'reference points' show the image is taken from Google Earth software and is of the Earth's Moon crater called Lambert. This information alone, given the internet age technology, gives a (relative) time reference point as to when this image was taken. In more detail, the lower-left information indicates that the image scale is not 1:1; it was acquired from an altitude of 200.36km above the Moon's surface, making the crater in question approximately 30,000m in diameter. The (English) language and the number system are also time reference points, as is—among others—the compass north direction, the Google Earth brand, the shadow, the longitude and the latitude. With these additional reference points, scale and position are determined.

5.6.1 Quantum-Classical Interface

The fractal reveals an important insight into the 'quantum-classical interface', the line where quantum becomes classical reality. The isolated fractal suggests—in agreement with some quantum theory interpretations—that this transition is not only a problem of the 'micro' quantum world (versus the 'macro' classical reality) but is, however, an ever-present problem of isolated systems, and this is best demonstrated and understood by examining the isolated fractal systems. The Moon crater(s) fractal-landscape (Figure 4) analysis was an attempt to demonstrate this. This is not to say, for instance, that the (quantum) atom behaves the same as something we experience in the classical world; it is only to reinforce that the property of fractal scale-invariance can occur at all levels. It is universal. In instances of total isolation, monotonic fractal landscapes lack a distinction between the macro and the micro. Even if there are differences between the objects, scale and location will be lost until reference is made. The problem is a universal property of fractality and can only be solved with reference or measurement.

5.6.2 The Observer

These discussions surrounding 'measurement' of the fractal reveal themselves to share, once again, the same problem of the role of the observer as it is discussed within quantum mechanics. Who exactly is the observer? What is it to know? And, what is consciousness? These questions are also open regarding the fractal; further discussion is outside the scope of this investigation; however, they are not beyond the bounds of possible understanding. It may be that, with the fractal, there is direct insight into knowing, in which position and measurement also stand for a universal point of understanding. It would follow that it takes a complex—maybe fractal-based—mind to understand and to know. Fractals may lead to a sense of consciousness.

5.7 Fractal Demonstrating Entanglement

The fractal demonstrates a parallel problem known as the EPR entanglement paradox. The result (4.2) is not a prominent feature of the fractal, and this solution would not have arisen unless it had already been identified as a problem in quantum mechanics. The initial property assumptions of *configuration A* align with the quantum entanglement postulates that photons are first 'loaded' or 'entangled' before observation. With observation at different parts of the fractal system—*configuration B*—there are no 'hidden' or 'spooky' variables that are involved; it is a property of an isolated superposition fractal. The 'observed' change was instantaneous and non-local. If the assumptions of fractal isolation were broken, the consequential slow propagation speed of iteration is akin to the 'slow' and not instantaneous speed of light that Einstein highlighted and his 'spooky action at a distance' argument. This demonstration of the fractal entanglement shows it is feasible to demonstrate entanglement in a 'warm' setting, as opposed to current low-temperature near absolute zero Kelvin environments. This may open an opportunity to warm quantum computing research.

5.8 Opportunities

There are many questions, issues and opportunities arising from this finding, all of which, at this point, are beyond the scope of this investigation; however, not beyond the scope of future possibilities. It is important to note that, while this work shows that light may share aspects or properties with fractals, it does not imply that the true nature of light or the atom is precisely like the regular snowflake fractal described. Fractals come in a finite number of forms; however, all fractals share the described characteristics. A final word on this: it may be that the atom is an inverted fractal structure, described by the retrospective iteration (section 2).

The fractal set demonstrates three forms of symmetry—time, translational and rotational. This invokes Noether's Theorem, where for every symmetry there is a conservation law. This, in turn, via time symmetry, may point to a connection between fractality and energy (E) and mass (M).

While the different interpretations of quantum mechanics are not directly addressed in this paper, it does, however, offer an explanation of quantum mechanics and why we might consider these interpretations. They are there to be thought about; however, they may be an underlying distraction from the true mechanics. The opportunity to further discuss these interpretations with respect to a fractal foundation would be interesting, but is outside the scope of this paper. It is reasonable to think that these (fractal-based) interpretations would also be parallel to the current arguments. For instance, the branching superposition fractal also offers direct insight into the Everettian—'Many-Worlds'—conjecture. There is an opportunity, from a different aspect of the fractal, to solve this conjecture.

The fractal may, with further work, also give direct insight into the double-slit experiment, as it can be demonstrated that the fractal, in a different context, behaves very similarly with respect to demonstrations of (fractal) pattern development over time [16].

The opportunity also arises to investigate and model the opposite 'retrospective' perspective of fractals: the view from an observer within the set, looking back rather than forward. This perspective already returns an exponential behaviour where the original bit grows and expands exponentially from an arbitrarily small field [17]. This perspective directly points to current cosmological observations and conjectures, including a single-point beginning and the current galaxy distribution; the Hubble-Lemaître expansion; accelerated expansion with a time-varying acceleration rate; and a direct solution to the Vacuum Catastrophe. When the (fractal) bit size is set to the Planck area size and the fractal is allowed to grow, it reaches the arbitrary area size of 1 in 72 iterations. If this iteration speed is set to correspond with the light photon propagation speed, we may have proof that the universe and light behave as a fractal. All of this offers further opportunities for further modelling. It may be that the two problems—the point beginning exponential cosmos and the quantum—are different aspects of the same geometry—fractal geometry. While this paper does not offer any insight into gravity or general relativity, nor does it take from them. It may be that general relativity is independent. This paper presents an opportunity to advance the mathematics of the unification of cosmology and quantum mechanics.

6. Discussions and Conclusions

In this paper, I first describe the development of the isolated iterating fractal and then apply it to light and the quantum problem. Independent of any notion of quantum foundations, a geometry conceived after the development of quantum mechanics, corresponds with how physicists describe (photon/wave propagation) light and other quantum entities and their quantum problem. This analysis neither added to nor detracted from the quantum enigma. Notwithstanding that chaos and fractals already share similar properties, including unpredictability and the infinite potential of possible events, this investigation shows that the isolated fractal exhibits additional properties that are significant and parallel to those of light and the quantum. These properties are not directly related or obvious to chaos theory, but they are pertinent to the isolated fractal—chaos's geometry. They include: the demonstration of a superposition of identical bits; a wave propagation with a possible constant speed; extraordinary relativity time scenario possibilities; wave-particle duality of these bits; the collapse of the wave propagation after 'measurement', giving insight into the quantum—small-scale (quantum)/large-scale (reality)—interface; and finally, a demonstration of quantum entanglement. The fractal model shows there is no large-scale/small-scale macro/micro divide and suggests this effect is related to the degree of isolation of (fractal) reality from other (fractal) objects.

The fractal model does not directly offer insight into gravity or general relativity; however, it does offer direct insight into the structure and origin of the cosmos through its dual perspective. A key insight of the fractal is its complementary dual perspectives from a position of observation. Looking forward, the fractal oscillates; looking behind, from a position within, it exponentiates. Forward is the quantum perspective, and looking back or behind the fractal expands exponentially with iteration time and has been shown, by experimental models, to correspond to cosmological observations and conjectures: Hubble-Lemaître, the lambda term, and the inflationary expansion, emergent from a single point, a single bit, an origin.

The dual problems of quantum foundations and (Lambda CDM) cosmology also appear to be fractal problems, and an understanding of the fractal best addresses these problems. Not images of fractals but the geometry itself. They—'the problems'—are both different aspects of the same geometry.

Through this elementary demonstration, the fractal reduces quantum behaviour to a known form of modern geometry. Fractal geometry, the geometry of our time, also offers a possible solution to the unification of quantum mechanics and cosmology. Indeed, by studying the fractal, as described, one can predict the quantum and the cosmos. With different hands, the fractal model may be mathematically described, complementing what is already established in the standard model to form an overall solution akin to great geometric solutions of the past.

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References

1. Drummond B. Understanding quantum mechanics: a review and synthesis in precise language. Open Phys. 2019;17: 390–437. doi:10.1515/phys-2019-0045
2. Einstein A, Podolsky B, Rosen N. Can Quantum-Mechanical Description of Physical Reality Be Considered Complete? Phys Rev. 1935;47: 777–780. doi:10.1103/PhysRev.47.777
3. Young T. II. The Bakerian Lecture. On the theory of light and colours. Philos Trans R Soc Lond. 1802;92: 12–48. doi:10.1098/rstl.1802.0004
4. Maxwell JC. VIII. A dynamical theory of the electromagnetic field. Philos Trans R Soc Lond. 1865;155: 459–512. doi:10.1098/rstl.1865.0008
5. Maxwell JC. XXV. On physical lines of force. Lond Edinb Dublin Philos Mag J Sci. 1861;21: 161–175. doi:10.1080/14786446108643033
6. Einstein A. Zur Elektrodynamik bewegter Körper. Ann Phys. 1905;322: 891–921. doi:10.1002/andp.19053221004
7. Isaeva OB, Kuznetsov SP. Approximate Description of the Mandelbrot Set. Thermodynamic Analogy. arXiv:nlin/0504063. 2005 [cited 15 Feb 2021]. Available: http://arxiv.org/abs/nlin/0504063
8. Stewart I. Does God Play Dice?: The New Mathematics of Chaos. 2Rev Ed edition. London, England; New York, N.Y.: Penguin; 1997.
9. Magnitskii NA. Bifurcation Theory of Dynamical Chaos. Chaos Theory. 2018 [cited 15 Feb 2021]. doi:10.5772/intechopen.70987

10. Mandelbrot BB. FRACTAL ASPECTS OF THE ITERATION OF z →Λz(1- z) FOR COMPLEX Λ AND z. Ann N Y Acad Sci. 1980;357: 249–259. doi:10.1111/j.1749-6632.1980.tb29690.x

11. Weisstein EW. Koch Snowflake. Wolfram Research, Inc.; [cited 27 Mar 2021]. Available: https://mathworld.wolfram.com/KochSnowflake.html

12. Planck M. Ueber das Gesetz der Energieverteilung im Normalspectrum. Ann Phys. 1901;309: 553–563. doi:10.1002/andp.19013090310

13. Broglie LD. Recherches sur la théorie des Quanta. Ann Phys. 1925;10: 22–128. doi:10.1051/anphys/192510030022

14. Schrödinger E. Quantisierung als Eigenwertproblem. Ann Phys. 1926;384: 361–376. doi:10.1002/andp.19263840404

15. Heisenberg W. Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik. Z Für Phys. 1927;43: 172–198. doi:10.1007/BF01397280

16. The Royal Institution. Fractals: The Geometry of Chaos - Christmas Lectures with Ian Stewart. 2018. Available: https://www.youtube.com/watch?v=nNMRiHaJxEA

17. Macdonald B. In Situ Experiment on Fractal Corresponds with Cosmological Observations and Conjectures. 2021. Available: https://www.researchgate.net/publication/338127694_In_Situ_Experiment_on_Fractal_Corresponds_with_Cosmological_Observations_and_Conjectures