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Original Paper

Interpretation of Quantum State Contraction Due to Time-Retrograde Waves in Observation Process

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Abstract: This study interprets the observation process in quantum mechanics as involving quantum states that propagate forward and backward in time. The retrogression in time follows from the energy-time uncertainty relation, according to which if a quantum state's energy is fixed by a measurement, its time evolution may fluctuate. The resulting time-reversed state was represented by a bra vector, which was the complex conjugate of the original state's ket vector. The time-reversed bra vector and time-evolving original ket vector interacted along the time axis to generate a 'bra-ket state', which was time-invariant and indicates the determination of the state. Among all time-reversed states, the one with the least energy dispersion reaches the furthest into the past, which results in this state being established and the other states disappearing. This is why a single state is determined by observation. Furthermore, while time fluctuation also occurs in the future, the relevant state vectors do not exist beyond the furthest future point. Consequently, once the present exceeds this fluctuation time, only a previously generated time-invariant bra-ket state exists. The existence of a single state after the furthest time in the past and future explains how observations induce quantum state contraction.

Keywords: Quantum optics, entangled state, Aspect's experiment, time reversal, uncertainty relationship between time and energy, retro-causality, theory of relativity

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Introduction

In quantum mechanics, the phenomenon of quantum state contraction following observation is a long-standing mystery. While a mathematical understanding of the phenomenon can be acquired from the von Neumann projection operator[1], which extracts a single state from a superposition of state vectors obtained through measurements, the underlying physical principles remain elusive. To fill this gap in our understanding, the current study sought to explain the phenomenon by using quantum time reversal. To date, several theories have been proposed to elucidate quantum-mechanical principles in terms of forward and backward propagation in time. These theories include the transactional interpretation of quantum mechanics (TI) by Cramer et al.[2], the two-state vector formalism (TSVF) of Aharonov et al.[3], and the absorber theory (AT) proposed by Pegg[4].

In the TI framework, anterograde and retrograde waves are exchanged between the observed wave emitter and absorber. During this exchange, waves lying outside the emitter and absorber vanish owing to the superposition of opposite phases, while in-phase waves lying between them interfere constructively. This constructive interference leads to the contraction of states. However, in this framework, the mechanism triggering the propagation of the confirmation wave in the negative time direction upon the arrival of the offer wave at the absorber remains unknown.

The TI theory is rooted in the framework of the Wheeler-Feynman radiation theory [5], which involves the two solutions of the classical electromagnetism propagation equation: one representing forward propagation and the other describing backward propagation in time. Notably, if this concept is to be extended to general quanta, the quanta must possess two solutions, one directed forward and the other directed backward in time. However, since the Schrödinger equation, which governs non-relativistic quantum mechanics, is a first-order differential equation with respect to time, it has only one type of solution in the time domain. For the acquisition of dual-time-directed solutions in general quantum mechanics, considerations based on relativistic quantum mechanics are necessary[6].

In relativistic quantum mechanics, both the Klein-Gordon equation, describing Bose particles, and the Dirac equation, modelling Fermi particles, come into play. While the former, being a second-order differential equation with respect to time, permits the simultaneous existence of solutions directed forward and backward in time, the latter, being a first-order differential equation with respect to time, has only one type of solution in the time domain.

Nonetheless, the Dirac equation has four simultaneous solutions associated with charge and spin, and its antiparticle solution could be considered to describe time reversal. However, in this framework, overlapping particle and antiparticle waves traveling backward in time do not reinforce each other but rather annihilate each other.

Cramer's transactional interpretation of quantum mechanics posits that confirmation waves traveling towards the past, similar to electromagnetic waves, are not always pre-existing waves, but are rather generated upon receipt of offer waves propagating forward in time. However, the mechanism underlying the formation of these incoming confirmation waves remains unknown.

Notably, the TSVF uses both past and future states to ascertain the current state, and it determines the probability of acquiring a weak measurement that is incapable of inducing system decoherence. However, the theory does not elaborate on the process through which the state vector is influenced across the period extending from the future to the past.

The AT of Pegg shares similarities with the interpretations of Aspect et al.'s experiment[7] provided in a previous study[8]. The theory represents an application instance of the Wheeler-Feynman radiation theory, and it considers the consistent presence of advanced photon waves traveling backward in time. However, the applicability of this theory is restricted to photons. Notably, Einstein-Podolsky-Rosen correlations similar to those observed in Aspect et al.'s experiment have been confirmed in experiments involving general quanta such as protons[9]. Consequently, relying solely on the photon theory, which proposes the existence of a retrograde wave solution, is inadequate.

Against this background, the current study posits that time retrogression is a property permitted by the energy-time uncertainty relation, and it considers that if the energy of a system is determined, the system's temporal evolution may exhibit fluctuations, enabling time travel into the past.

1. Interpretation of the quantum state contraction using time-reversed waves

1.1. The uncertainty relation between energy and time

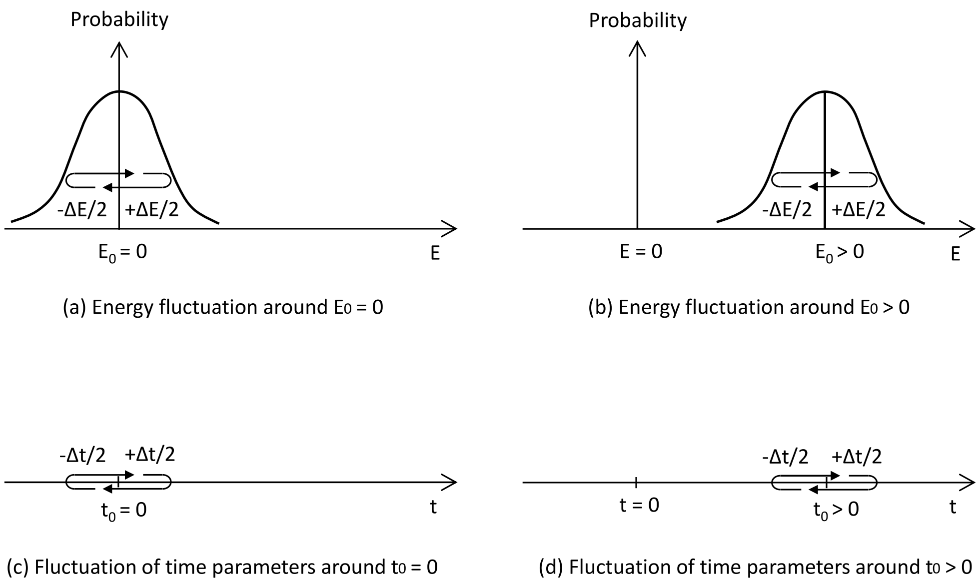
The uncertainty relation between energy and time ($\Delta E \Delta t \geq \frac{\hbar}{2}$) is derived from the uncertainty relation between position and momentum ($\Delta q \Delta p \geq \frac{\hbar}{2}$). Thus, the former relation is not considered to represent a fundamental principle.

Furthermore, this relation does not imply the existence of the following canonical conjugate relation between time and energy: $[E, t] = i\hbar$. Notably, Pauli refuted this canonical conjugation[10], and this implies that time functions as a parameter and not as an observable. Consequently, determining the eigenvalue of time becomes impossible. In particular, energy cannot be constrained to negative values.

Various interpretations have been provided for the uncertainty of energy and time [11]. The widely accepted interpretation of the energy-time uncertainty relation is that involving the time scale of state change, proposed by Mandelstam and Tamm [12].

However, Arai showed that the following canonical commutation relation exists between a symmetric operator T and a self-adjoint operator H[13]:

Figure 1. Fluctuations of energy and time parameters



$$[T, H] = i \tag{1}$$

Let H and T are on a complex Hilbert space $\mathcal{H}$, $\sigma(T)$ be the spectrum of T, and $\mathbb{C}$ is the set of complex numbers. Then, the following statements hold.

(i) If H is bounded below, then $\sigma(T)$ is either $\mathbb{C}$ or $\{z \in \mathbb{C} \mid \mathrm{Im}z \geq 0\}$.
(ii) If H is bounded abobe, then $\sigma(T)$ is either $\mathbb{C}$ or $\{z \in \mathbb{C} \mid \mathrm{Im}z \leq 0\}$.
(iii) If H is bounded, then $\sigma(T)$ is either $\mathbb{C}$.

Here, owing to the positive definiteness of the energy, following Pauli[10], we adopt condition (i). The time that we are aware of in this world is a real number. Hence, the spectrum contains the real axis of time. However, this is not sufficient to overcome Pauli's original objection, which remains valid: in standard quantum mechanics, time is not represented by a self-adjoint operator conjugate to the Hamiltonian. Therefore, the following discussion does not treat time as an observable, but as a parameter with fluctuations, and does not require that equation (1) hold.

The energy-time uncertainty relation $\Delta E \Delta t \geq \frac{\hbar}{2}$ suggests that if $\Delta t < 0$, then $\Delta E < 0$. Furthermore, fluctuations in ΔE around $E_0 = 0$ would lead to situations with $E < 0$ (Figure 1(a)).

However, when Δt and ΔE are considered as fluctuation widths, that is, $\Delta t > 0$ and $\Delta E > 0$, and fluctuations occur around E_0 such that $E_0 > \frac{\Delta E}{2} > 0$, the resulting energy

E, including the fluctuations, can be expressed as $E = E_0 \pm \frac{\Delta E}{2} > 0$. Clearly, this expression yields positive E values (Figure 1(b)).

In the context of time, the sign lacks intrinsic significance, except for the assumption of a positive value for the current time. From this perspective, $t_0 = 0$ serves as a relative coordinate on the time axis (Figure 1(c)). Notably, when time fluctuates around the centre, we hypothesise the possibility of a very small retrogression of time around $t_0 > 0$ (Figure 1(d)).

Thus, the energy-time uncertainty relation permits the reversal of time without necessitating negative energy values. According to the uncertainty relation $\Delta E \Delta t \geq \frac{\hbar}{2}$, the determination of the energy through measurement leads to a corresponding fluctuation in time, denoted by Δt, implying that the time can shift in the negative direction by approximately half of this fluctuation. For such a negative fluctuation of $\frac{\Delta t}{2}$, the quantum state undergoes time reversal.

1.2. Time reversal and physical state change by observation

In quantum mechanics, the Schrödinger equation is an equation of motion for the state vector:

$$i\hbar\frac{\partial}{\partial t}\left|\psi(\boldsymbol{x},t)\right\rangle = \hat{H}\left|\psi(\boldsymbol{x},t)\right\rangle \tag{2}$$

Under time reversal, the time derivative operator transforms as

$$\frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial(-t)} = -\frac{\partial}{\partial t}. \tag{3}$$

The complex conjugate of (2) is a time-reversed form of (2):

$$i\hbar\frac{\partial}{\partial t}\left\langle\psi(\boldsymbol{x},t)\right| = \left\langle\psi(\boldsymbol{x},t)\right|\hat{H}. \tag{4}$$

where

$$\left|\psi(\boldsymbol{x},t)\right\rangle^{\dagger} = \left\langle\psi(\boldsymbol{x},t)\right|. \tag{5}$$

Furthermore, we assume

$$\hat{H}^{\dagger} = \hat{H}. \tag{6}$$

Thus, time reversal is modeled by an antiunitary transformation[14], under which the state vector becomes the complex conjugate of the original. Here, if the original state vector is denoted by the ket vector $|\psi\rangle$, its complex conjugate is represented by the bra vector $\langle\psi|$.

The rationale behind the simultaneous requirement of the complex conjugation and the transposition of the state vector is as follows: First, within the realm of this study, the vector inner product is not merely a mathematical operation but regarded as an indication of a physical process. Furthermore, the generation of the bra vector essential for this inner product operation necessitates time reversal.

Generally, the energy of a quantum state $|\psi(t)\rangle$ is determined by the individual contributions of its eigenstates $|\psi_i(t)\rangle$. According to the uncertainty relation between time and energy, certaining a energy for the state $|\psi_i(t)\rangle$ introduces a fluctuation Δt in the time parameter, allowing for the conceptual possibility of the backward propagation of time, thus generating a time-reversed wave $\langle\psi_i(t)|$.

Notably, the time evolution of state $|\psi_i(t)\rangle$ can be modeled as a unitary transformation:

$$|\psi_i(t)\rangle = e^{-\frac{i\hat{H}t}{\hbar}} |\psi_i\rangle . \tag{7}$$

On the other hand, the time evolution of the time-reversed state $\langle\psi_i(t)|$ can be modeled as an antiunitary transformation:

$$\langle\psi_i(t)| = \langle\psi_i| e^{\frac{i\hat{H}t}{\hbar}} . \tag{8}$$

When $\langle\psi_i(t)|$ and $|\psi_i(t)\rangle$ combine along the time axis, they generate $\langle\psi_i(t)|\psi_i(t)\rangle$, which can be expressed as follows:

$$\langle\psi_i(t)|\psi_i(t)\rangle = \langle\psi_i| e^{\frac{i\hat{H}t}{\hbar}} e^{-\frac{i\hat{H}t}{\hbar}} |\psi_i\rangle = \langle\psi_i|\psi_i\rangle \tag{9}$$

Thus, a time-invariant state results from combining of the states. This principle can also be used to ascertain superposed state $|\psi(t)\rangle$ that has not been determined on the time axis:

$$\langle\psi_i(t)|\psi(t)\rangle = \langle\psi_i| e^{\frac{i\hat{H}t}{\hbar}} e^{-\frac{i\hat{H}t}{\hbar}} |\psi\rangle = \langle\psi_i|\psi\rangle = \langle\psi_i|\psi_i\rangle . \tag{10}$$

Thus, the established bra state $\langle\psi_i(t)|$ can be used to determine the ket superposition state $|\psi(t)\rangle$.

This study calls the state of equation (9) the 'bra-ket state'. This state becomes a static state without time variation because the time-evolving operator is cancelled by the operator that evolves in the opposite direction, as shown in equation (9). Also, by taking the inner product of the bra state and the ket state, the quantity of this bra-ket state becomes the C number. Each state in the superposition has a complex coefficient like as $a_i\,|\psi_i\rangle$. In this case, the bracket state is $|a_i|^2\,\langle\psi_i|\psi_i\rangle = |a_i|^2$, which corresponds to the probability of obtaining this by measurement as in the Born's interpretation.

These equations appear to simply express the stationary and orthonormal nature of the energy eigenstates that we are familiar with. However, this study argues that these are not just mathematical expressions of the quantum mechanical energy eigenstates that have been introduced so far, but that these processes of determining the states by such time-reversed bra vectors occur as physical phenomena.

This type of time reversal occurs when the quantum system interacts with some external system, such as a measuring device, as will be shown in the next section. At this time, energy fluctuations occur in the quantum system, and the time parameter fluctuates due to the uncertain relationship between energy and time. This fluctuation in the time parameter can occur in either the positive or negative direction, and a time parameter that fluctuates in the negative direction causes time reversal in the quantum system.

In this case, if an interaction with an external system occurs at $t = t_0$ and the fluctuation of the time parameters begins, the boundary condition for the time-reversed wave is $\langle\psi_i(t_0)| = \langle\psi_i| e^{\frac{i\hat{H}t_0}{\hbar}}$, and the boundary condition for the time-progressive wave is $|\psi_i(t_0)\rangle = e^{-\frac{i\hat{H}t_0}{\hbar}} |\psi_i\rangle$.

1.3. Quantum state contraction due to time-retrograde waves

By identifying the state generated by the time-reversed vector through observation, we can describe the mechanism of quantum state contraction due to the observation as follows. Let us suppose that before observation, the quantum state is in the superposition state

$$|\psi\rangle = a_1 |\psi_1\rangle + a_2 |\psi_2\rangle + \cdots + a_m |\psi_m\rangle + \cdots . \tag{11}$$

When we attempt to ascertain this state through observation, each component state $a_1 |\psi_1\rangle, a_2 |\psi_2\rangle, \cdots, a_m |\psi_m\rangle, \cdots$ begins to establish itself. This process reduces the energy fluctuation widths of the states, namely $\Delta E_1, \Delta E_2, \cdots, \Delta E_m, \cdots$. Ultimately, the state with the smallest energy fluctuation width is probabilistically determined.

Concurrently, the time fluctuation widths $\Delta t_1, \Delta t_2, \cdots, \Delta t_m, \cdots$ increase, in accordance with the energy-time uncertainty relation $\Delta t \geq \frac{\hbar}{2\Delta E}$. The state with the largest time fluctuation width is determined stochastically, according with the energy fluctuation width.

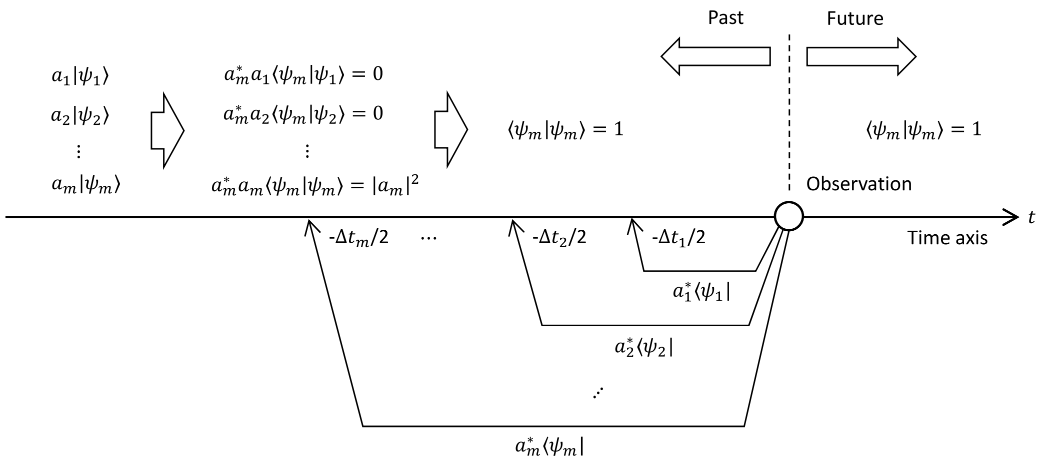
Depending on the time fluctuation width of each state vector, time reversal could occur, and the time-reversed vectors transform into the following bra vectors: ${a_1}^* \langle\psi_1|, {a_2}^* \langle\psi_2|, \cdots, {a_m}^* \langle\psi_m|, \cdots$. Among these bra vectors, the bra vector with the highest degree of energy certainty through measurement travels the farthest into the past. Figure 2 depicts this situation.

In this scenario, the bra vector ${a_m}^* \langle\psi_m|$ reaches its farthest point in the past at $-\Delta t_m/2$. This bra vector ${a_m}^* \langle\psi_m|$ then interacts with states existing in the past, namely $a_1 |\psi_1\rangle, a_2 |\psi_2\rangle, \cdots, a_m |\psi_m\rangle, \cdots$, and the following relations hold.

$${a_m}^* a_1 \langle\psi_m|\psi_1\rangle = 0,\ {a_m}^* a_2 \langle\psi_m|\psi_2\rangle = 0,\ \cdots,\ {a_m}^* a_m \langle\psi_m|\psi_m\rangle = |a_m|^2,\ \cdots \tag{12}$$

The above process can determine state $a_m |\psi_m\rangle$ with probability $|a_m|^2$, while the other states ($a_1 |\psi_1\rangle, a_2 |\psi_2\rangle, \cdots$) disappear. Although the bra vectors ${a_1}^* \langle\psi_1|, {a_2}^* \langle\psi_2|, \cdots$

Figure 2. Contrations of quantum state by a state vector reaching the farthest past



reach $-\Delta t_1/2, -\Delta t_2/2, \cdots$, their corresponding ket vectors $a_1 |\psi_1\rangle, a_2 |\psi_2\rangle, \cdots$ have already disappeared. Therefore, the establishment of these states is prohibited.

This is the mechanism of quantum state contraction through observation.

The aforementioned time fluctuation also occurs in the future, as depicted in Figure 3. Specifically, under this time fluctuation, states $a_1 |\psi_1\rangle, a_2 |\psi_2\rangle, \cdots, a_m |\psi_m\rangle, \cdots$ traverse into the future from the observation point; however, these state vectors do not exist beyond the furthest future point located at $+\Delta t_m/2$.

Consequently, once the 'present' exceeds this fluctuation time, only the time-independent state $\langle\psi_m|\psi_m\rangle$ exists with a probability of one, according to Equation (10).

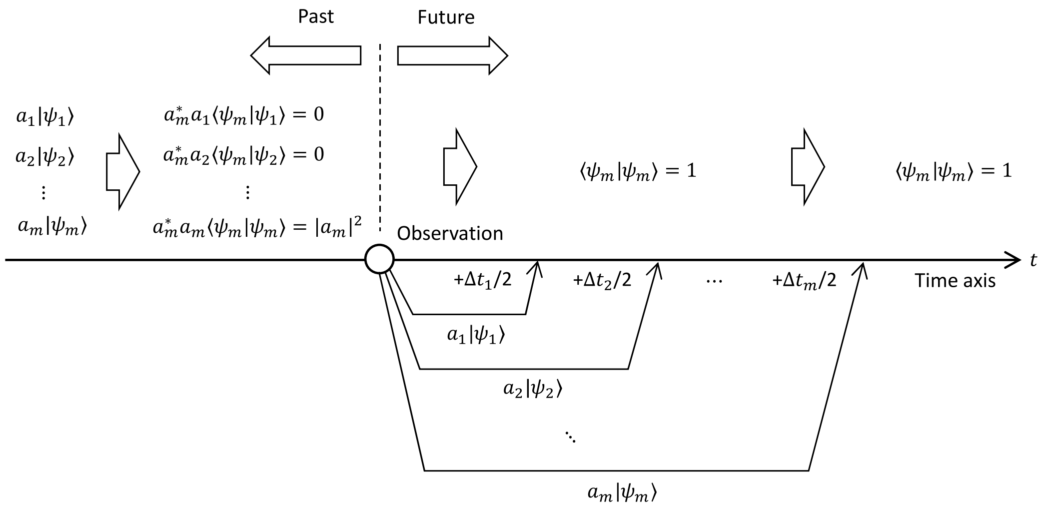
This completes the phenomenon of quantum state contraction or the so-called 'wave packet collapse'. These processes, which have been described mathematically by von Neumann[1], occur as physical processes as outlined above.

Here, 'measurement' is considered to be a physical operation that limits the dispersion of energy states. The operation limiting the dispersion of momentum also limits the dispersion of energy states. Similarly, the operation limiting the dispersion of position also limits the dispersion of energy states.

When electron collides with a photographic plate to form a bright spot and identifies its final point reached[15], the electrons' energy is ultimately lost and defined to be zero. Hence, this collision process is an operation limiting the dispersion of energy states. Such limiting of the dispersion of energy states also occurs when photons are introduced into a photomultiplier tube to generate a current signal, as the photon's energy is ultimately lost and defined to be zero.

Unlike previous measurement theories, it is not necessary to include measurement

Figure 3. State vectors reaching the future



systems that must be treated quantum mechanically. If we define 'measurement' as above, we can separate the process of observing a quantum system from its environment. Thereby, we can overcome the difficulties of previous observation theories, such as where to place the Heisenberg cut that separates quantum systems from classical systems and whether the effect of consciousness should be considered. If we define the measurement process in this way, we need not deal with the effects of the observation device, and therefore, open systems were not considered in this study.

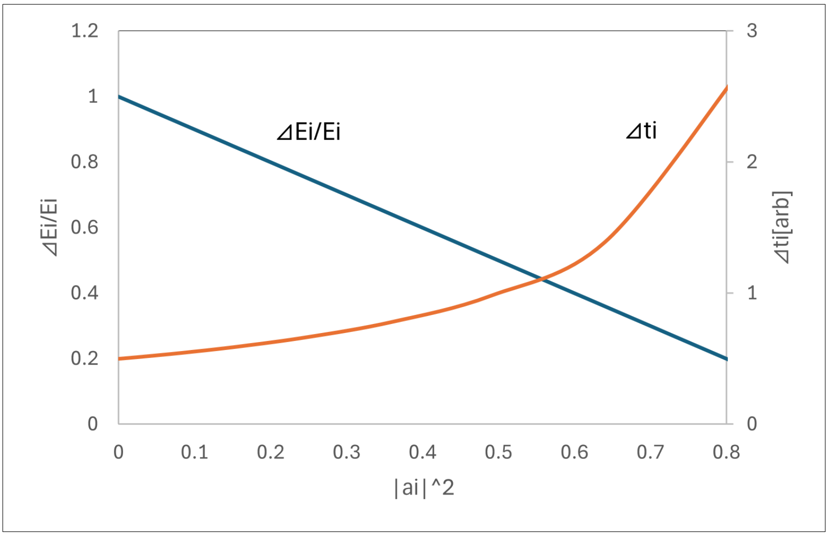
In Section 1.2, energy eigenstates are assumed. In energy eigenstates, the energy of each state is determined in advance. When measuring a system in which such energy eigenstates are superposed, it is necessary to consider how the energy fluctuations occur and how one state is determined by the Born's probability law.

The overlay state before measurement is expressed as equation (11). Measurements act to select states on the system. Once subjected to this state selection, each following individual states $a_1 |\psi_1\rangle , a_2 |\psi_2\rangle , \cdots , a_m |\psi_m\rangle , \cdots$ are given energy by a measuring device.

Each state has an oscillation according to equation (7), and the measurement device also has quantum fluctuations. Therefore, when each state interacts with the measurement device, fluctuations occur in the energy received from the measurement device. At this time, the phase relationship between each time-varying state and the measurement device is determined haphazardly. This determines the magnitude of the fluctuation in the energy received from the measuring device probabilistically.

As mentioned above, the state $a_m |\psi_m\rangle$ with the smallest energy fluctuation magnitude ΔE_m is the state determined by this measurement. Therefore, the state selected by the measurement is also determined probabilistically.

Figure 4. Square of expansion coefficients and fluctuations of energy and time



Measurement is not simply a disturbance, but acts to determine the state. According to quantum mechanics, the probability of a state being determined follows the Born rule. This study considers this in terms of the relationship between energy fluctuations and time fluctuations that occur during the measurement process.

The expectation value $\langle E_i \rangle$ of the energy of the state $|\psi_i\rangle$ can be written as follows using Born rule:

$$\langle E_i \rangle = |a_i|^2 E_i. \tag{13}$$

Here, E_i is the energy when the state is determined to be $|\psi_i\rangle$. Also, a_i is the expansion coefficient of E_i expanded in the energy basis, $0 \leq |a_i|^2 \leq 1$. During the measurement process, energy fluctuations occur in each state. Among these energy fluctuations, E_i is the maximum energy that the state $|\psi_i\rangle$ can have. Therefore, the width of the energy fluctuations ΔE_i that occur during the measurement process can be written as follows:

$$\Delta E_i = E_i - \langle E_i \rangle = (1 - |a_i|^2) E_i. \tag{14}$$

This relationship against $|a_i|^2$ and $\Delta E_i / E_i$ is shown by Figure 4. In this way, the energy fluctuation range ($\Delta E_i / E_i$) that can occur during the measurement process is inversely correlated, becoming smaller as $|a_i|^2$ becomes larger. Also, the time fluctuation width ($\Delta t_i \geq \hbar / 2\Delta E_i$) is inversely proportional to (ΔE_i), so it increases as $|a_i|^2$ increases. Therefore, the state $|\psi_m\rangle$ with the largest $|a_m|^2$ will have the largest time fluctuation Δt_m and is expected to become the final measurement state.

This study also assumed a superposition of pure states in which each state has an orthonormal relationship. However, it is difficult to keep a quantum system in a pure state, and there is a certain cost involved in obtaining information about the single state[16]. In general measurements, a quantum system in a superposition state may transform into a mixed system, or the quantum system may be a mixed system that is not in a pure superposition state to begin with. These observation processes are usually described using a mixed density matrix. This study also required such handling in general measurements, and consequently, there could be situations where the states do not converge to one. How to handle such cases is a future issue.

1.4. Consistency with previous theories and experiments

This study is consistent with the widely accepted interpretation of the energy-time uncertainty relation proposed by Mandelstam and Tamm[12], which involves the time scale of state establishment.

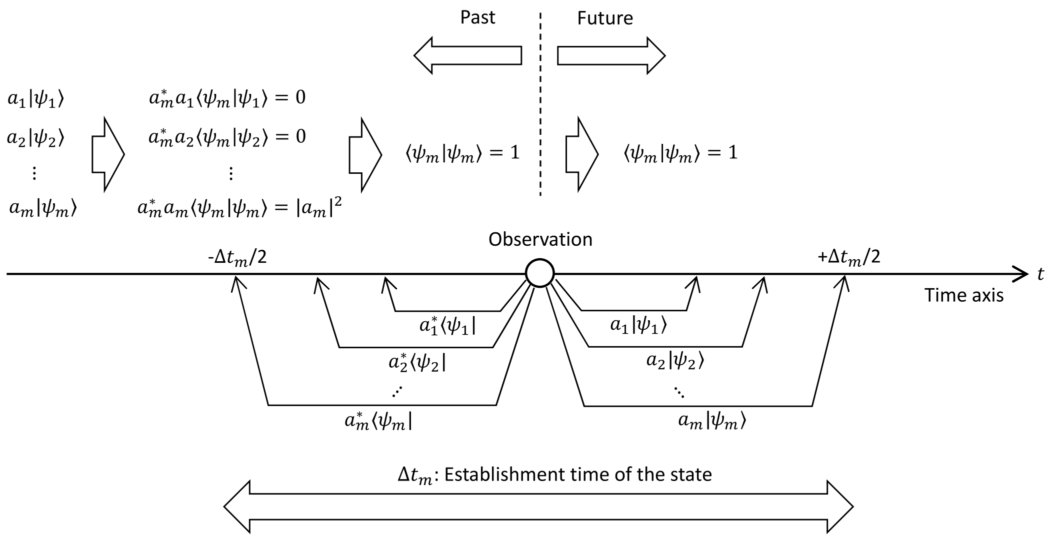
This situation is illustrated in Figure 5. As the energy is determined by observation, time fluctuation occurs, and the bra vector ${a_m}^* \langle\psi_m|$ reaches the furthest past $-\Delta t_m/2$. This ${a_m}^* \langle\psi_m|$ will become the final observation result. Such time fluctuation also occurs in the future. The state ket vectors $a_1 |\psi_1\rangle, a_2 |\psi_2\rangle, \cdots, a_m |\psi_m\rangle, \cdots$ reach their respective time fluctuation points. However, $a_1 |\psi_1\rangle, a_2 |\psi_2\rangle, \cdots$ do not go beyond $+\Delta t_m/2$, and hence, what remains in the end is only ${a_m}^* a_m \langle\psi_m|\psi_m\rangle$, which is already established in the braket state. Therefore, for the state to be established, a time duration Δt_m from $-\Delta t_m/2$ to $+\Delta t_m/2$ is required. This is consistent with the conventional interpretation above that the energy-time uncertainty relation indicates the time it takes for a state to be determined.

Recently, research into the measurement process of quantum systems has progressed through the use of weak measurements proposed by Aharonov et al.[17]. Weak measurements are made without destroying the superposition state. In weak measurements[17], the energy of the quantum system being measured is not completely determined, and hence, the energy fluctuation ΔE_i of each quantum state would be large, while the time fluctuation Δt_i would be small.

Nevertheless, this study predicts that the state $|\psi_m\rangle$ with the smallest energy fluctuation ΔE_m and the largest time fluctuation Δt_m would be the most likely to result from the weak measurement. However, in weak measurements, the difference between these fluctuations is small, and hence, the variation in the results from each measurement is large. Aharonov et al. applied a time-symmetric 'two-state vector formulation' [3] to weak measurement; in this formulation, future states as well as past states are to calculate the probability of the measurement result.

This study also considered elements that corresponded to Aharonov et al.'s theory. As

Figure 5. Reaching state vectors and estblishment time of the state

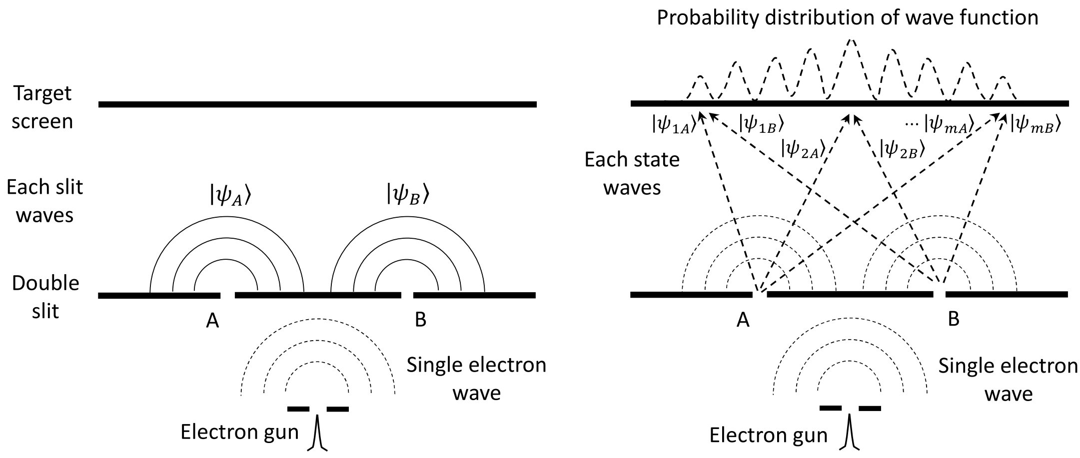


shown in Figure 5, for a certain range into the future from the time of measurement, there exist many states $a_1 |\psi_1\rangle, a_2 |\psi_2\rangle, \cdots$ in addition to $a_m |\psi_m\rangle$, and hence, the state is not determined until the limit of the future time that these states can reach. In other words, the contraction of the quantum state is not complete until the state is determined. This appears to be the case for the two-state vector form, where a range of future states also influence the current state. Nevertheless, in this study, the state has already been selected by $a_m{}^* \langle\psi_m|$ which reaches the furthest past. Thus, even if multiple states exist in the range extending slightly into the future from the measurement point, the state to be determined has been selected at a past stage.

However, if the measurement does not sufficiently determine the energy of the quantum system, multiple states begin to coexist again and the superposition state is maintained. Even in such a situation, the state with the smallest energy fluctuation can be determined from the weak value of the observation at each time, and there is a difference in the probability of obtaining the weak value. The measurement result obtained most frequently from multiple weak measurements will be consistent with the result from a strong measurement.

In systems using superconducting qubits, which have extremely small energy dissipation, the capability to return to the superposition state is high even when a weak measurement is performed. This makes it difficult to determine the state in the manner of this study. Researchers have succeeded in directly observing various quantum states without determining them[18][19].

Figure 6. Single electron interference in double slit apparatus



(a) Each slit waves (b) Each state waves on target screen

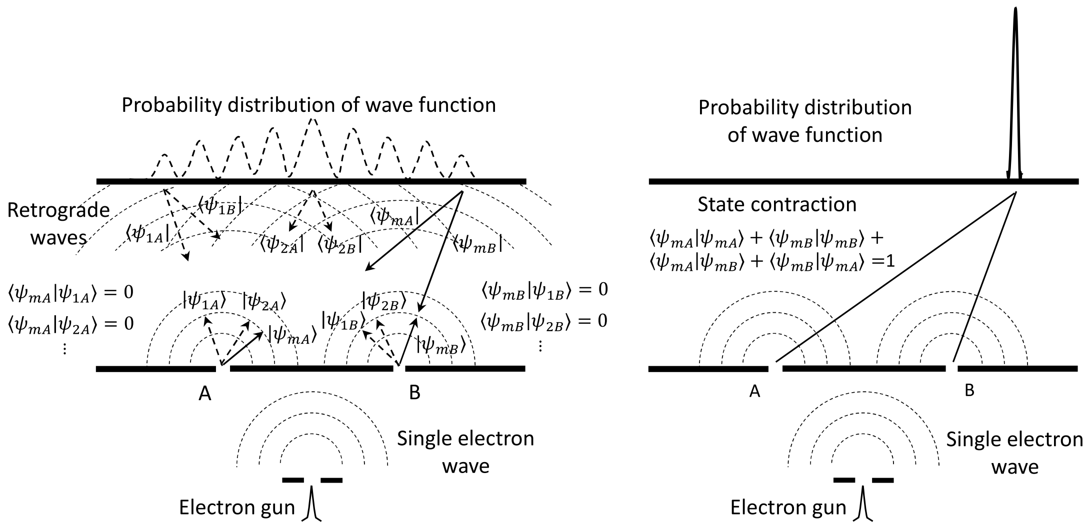
1.5. Wave packet collapse in single electron interference apparatus

The wave packet collapse mechanism in this study was determined by considering the famous single-electron interference at a double slit. Figure 6(a) shows a schematic of the experimental setup[20]. A single electron emitted from an electron gun travels through space as an electron wave, passes through the double slit, and splits into two electron waves $|\psi_A\rangle + |\psi_B\rangle$ which are omitted coefficients, that interfere with each other.

When this interference wave reaches the target screen, as shown in Figure 6(b), state waves $|\psi_{1A}\rangle + |\psi_{1B}\rangle$,$|\psi_{2A}\rangle + |\psi_{2B}\rangle, \cdots, |\psi_{mA}\rangle + |\psi_{mB}\rangle, \cdots$ are formed at the different arrival positions. These electron wave states interact with silver atoms on the target screen and determine their energy. Depending on the uncertainty range of this energy, namely ΔE_1,ΔE_2,$\cdots$,ΔE_m,$\cdots$, time fluctuations Δt_1,Δt_2,$\cdots$,Δt_m,$\cdots$ arise, in accordance with the energy-time uncertainty relation.

Since these state waves also fluctuate in the past direction, they generate time-reversed waves $\langle\psi_{1A}| + \langle\psi_{1B}|$, $\langle\psi_{2A}| + \langle\psi_{2B}|$, $\cdots$,$\langle\psi_{mA}| + \langle\psi_{mB}|$, $\cdots$ as shown in Figure 7(a). Among the time-reversed waves, the state with the smallest energy uncertainty ΔE_m has the largest time fluctuation Δt_m, and therefore this bra vector $\langle\psi_{mA}| + \langle\psi_{mB}|$ can reach the furthest into the past. As this bra vector $\langle\psi_{mA}| + \langle\psi_{mB}|$ combines with the components of the ket vectors $|\psi_{1A}\rangle + |\psi_{1B}\rangle$, $|\psi_{2A}\rangle + |\psi_{2B}\rangle$, $\cdots$,$|\psi_{mA}\rangle + |\psi_{mB}\rangle$, $\cdots$ at the furthest time point, the components orthonormal to it disappear, and finally $\langle\psi_{mA}|\psi_{mA}\rangle + \langle\psi_{mB}|\psi_{mB}\rangle + \langle\psi_{mA}|\psi_{mB}\rangle + \langle\psi_{mB}|\psi_{mA}\rangle = 1$ remains as shown in Figure 7(b). This represents the respective probability, and this is the physical mechanism by which the wave packet collapses upon observation.

Figure 7. Wave packet collapse in single electron apparatus



(a) Each retrograde waves

(b) State contraction by furthest past retrograde wave

In such an experimental setup, it may be possible to experimentally verify this theory, for example by observing the time-reversed waves coming from the target screen side in Figure 7(a). However, placing a measuring device on the path will block the time-progressing wave, so some ingenuity is required for observation.

1.6. Single photon detection in Mach-Zehnder interferometer

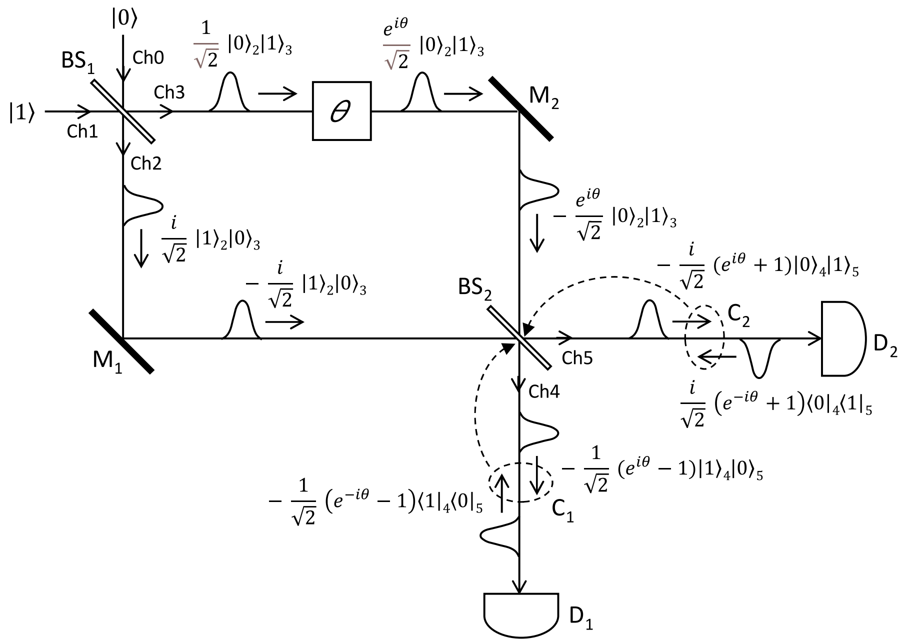
In the Mach-Zehnder interferometer shown in Figure 8[21], it is known that interference occurs when a single photon is split into several paths by a beam splitter and these paths are focused onto another beam splitter by a mirror. We used two symmetric and lossless beam splitters, BS_1 and BS_2, and let one photon enter from Ch1 of BS_1. Let this state be $|1\rangle$. Since zero photons are allowed to enter from Ch0, let this be $|0\rangle$. The unitary matrix of the beam splitter can be expressed as

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}. \tag{15}$$

The output of BS_1 is

$$|\mathrm{Ch2}\rangle = \frac{i}{\sqrt{2}} |1\rangle_2 |0\rangle_3 , \tag{16}$$

Figure 8. Single photon detection in Mach-Zehnder interferometer



$$|\mathrm{Ch3}\rangle = \frac{1}{\sqrt{2}} |0\rangle_2 |1\rangle_3 . \tag{17}$$

A phase converter is placed on Ch3, which results in the following equation:

$$|\mathrm{Ch3}\rangle = \frac{e^{i\theta}}{\sqrt{2}} |0\rangle_2 |1\rangle_3 . \tag{18}$$

$|\mathrm{Ch2}\rangle$ and $|\mathrm{Ch3}\rangle$ underwent phase inversion and a sign change because of the perfectly reflecting mirrors M_1 and M_2, and the reflected waves were output by the beam splitter BS_2 as follows.

$$|\mathrm{Ch4}\rangle = -\frac{1}{\sqrt{2}} \left(e^{i\theta} - 1\right) |1\rangle_4 |0\rangle_5 , \tag{19}$$

$$|\mathrm{Ch5}\rangle = -\frac{i}{\sqrt{2}} \left(e^{i\theta} + 1\right) |0\rangle_4 |1\rangle_5 . \tag{20}$$

When these output waves were detected by detectors D_1 and D_2, their energy dispersion was reduced and the time uncertainty width increased. This led to the generation of the following time-reversed waves of the bra vector:

$$\langle \mathrm{Ch4}| = -\frac{1}{\sqrt{2}} \left(e^{-i\theta} - 1\right) \langle 1|_4 \langle 0|_5 , \tag{21}$$

$$\langle \mathrm{Ch5}| = \frac{i}{\sqrt{2}} \left(e^{-i\theta} + 1\right) \langle 0|_4 \langle 1|_5 . \tag{22}$$

At C_1 and C_2, where these time-reversed bra vectors combined with the time-forward ket vectors, the following equations, which provide the probabilities of obtaining a photon, hold:

$$C_1 : \langle \mathrm{Ch4}|\mathrm{Ch4}\rangle = \frac{1}{\sqrt{2}}\left(1 - cos\theta\right), \tag{23}$$

$$C_2 : \langle \mathrm{Ch5}|\mathrm{Ch5}\rangle = \frac{1}{\sqrt{2}}\left(1 + cos\theta\right). \tag{24}$$

In conventional quantum theory, these probabilities were calculated by multiplying the ket vector by its bra vector as a mathematical technique. In this study, however, they were obtained by combining the time-reversed bra vector with the ket vector as a physical process. These showed that the wave packet disappeared upon observation.

In the case of a single photon, the photon is detected by either detector D_1 or D_2. Now, consider the case where a photon is detected by detector D_1. Here, whether the photon is detected at D_1 or not is determined when the photon's path is selected as Ch4 or not at BS_2. Therefore, the bracket state of equation (23) that determines the state is formed at the stage of BS_2. At this time, the position of C_1 regresses to BS_2. Then, the time-reversed wave $\langle \mathrm{Ch4}|$ travels to BS_2 and erases $|\mathrm{Ch5}\rangle$, which is orthogonal to it. In this way, the wave packet contraction is completed. The result is similar if the photon is detected at D_2, in which case the position of C_2 regresses to BS_2.

Even in this system, it may be possible to experimentally verify this theory. For example, when observing a photon at D_1 in Figure 8, it may be possible to observe the time-reversed photon of equation (21) returning from D_1. However, even in this case, it is necessary not to interfere with the time-forward photon of equation (19) coming from BS_2.

1.7. Bidirectional quantum time and unidirectional macroscopic time

Fluctuations in the quantum intrinsic time occurred both in the past and future directions and were symmetric with respect to time. However, as shown in Figures 5, the determined state evolved in unidirectional time. This was because the determined state did not fluctuate over time, and it followed the progression of 'macroscopic time'. However, why does macroscopic time progress in only one direction?

Dressel et al. investigated the statistical arrow of time for a quantum system being monitored by a sequence of measurements. For a continuous qubit measurement example, they demonstrated that time-reversed evolution is always physically possible, provided that the measurement record is also negated. Despite this restoration of dynamical reversibility, a statistical arrow of time emerged, and it could be quantified by the log-likelihood difference between forward and backward propagation hypotheses[22].

Manikandan et al. generalised Dressel et al.'s discussion of a statistical arrow of time for continuous quantum measurements, and they showed that backward probabilities could be computed from a process similar to retrodiction from the time reversed final state. Furthermore, they extended the definition of an arrow of time to ensembles prepared with pre- and post-selections, and they obtained a non-vanishing arrow of time in general[23].

In recent years, research on the origin of time has progressed. Rovelli argues that time is not a prerequisite for physical systems because time does not appear explicitly in quantum loop theory studies. He states that time is produced by state changes in spin networks in quantum loop theory and proposes that macroscopic time be treated as a thermodynamic state variable analogous to entropy[24]. Connes and Rovelli discussed the relationship between time and thermodynamics on the basis of a covariant quantum theory, and they introduced the concept of thermal time[25]. It is said that time does not originally exist, but is created by changes in physical systems.

Extending this idea, it is suggested that 'macroscopic time' arises from changes in macroscopic physical systems. Since such changes are statistically unidirectional, the macroscopic time associated with such changes is also thought to be unidirectional. This appears to be the reason for what we perceive as macroscopic time progressing in one direction.

2. Discussion

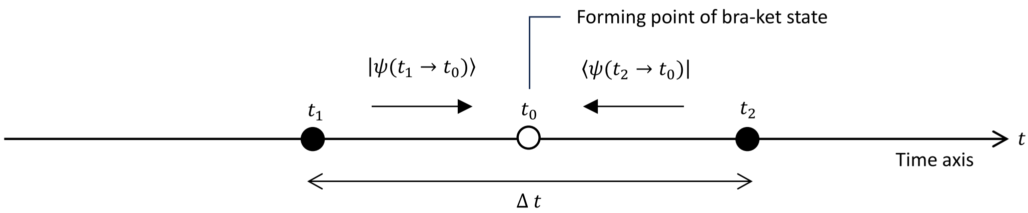
2.1. Definitions for non-standard terms

This study uses non-standard terms and concepts such as time-reversed bra vectors and bra-ket states. This section explains these definitions and concepts more precisely.

2.1.1. Time-reversed bra vector

If we apply a time-reversal transformation to the fundamental equations of quantum mechanics and take the complex conjugate of the whole, we get the same form as the original equations. The time-reversal vector is the solution of the time-reversal equation. This vector is the complex conjugate of the original and then transposed, changing the vector from a ket vector to a bra vector. The time evolution of the time-reversed bra vector is an antiunitary transformation (see equation (8)), describing the time evolution in the negative direction. The phase of the time-reversed bra vector is a complex conjugate of the phase of the original time-forward ket vector, so the sign of only the imaginary part is reversed, not the entire phase. This time-reversed bra vector is compatible not only with the non-relativistic Schrödinger equation, but also with the Dirac equation, which treats fermions relativistically, and the Klein-Gordon equation, which treats bosons relativistically, as shown in the author's previous paper[8]. Also, the validity of treating

Figure 9. Nearby forming point of bra-ket state



the behavior of a single photon with the Schrödinger equation was also mentioned in the author's previous paper[8] based on previous research.

2.1.2. Bra-ket state

The bra-ket state is defined as the inner product of quantum states that are complex conjugates of each other. In quantum mechanics, such inner products have been used as a method for calculating the probability of each quantum state. However, this study considers that some bra-ket states exist as quantum states in certain situations. This study particularly deals with the inner product of the time-forward ket vector $|\psi_i(t)\rangle$ and the time-reversed bra vector $\langle\psi_i(t)|$ as below.

$$\langle\psi_i(t)|\psi_i(t)\rangle = \langle\psi_i|\psi_i\rangle . \tag{25}$$

The process by which this state is formed can be considered as shown in Figure 9. The state vector that changes from t_1 to t_0 is expressed as follows:

$$|\psi_i(t_1 \to t_0)\rangle = e^{-\frac{i}{\hbar}\int_{t_1}^{t_0} \hat{H}tdt} |\psi_i(t_1)\rangle . \tag{26}$$

Similarly, the state vector that changes from t_2 to t_0 is

$$\langle\psi_i(t_2 \to t_0)| = \langle\psi_i(t_2)| e^{\frac{i}{\hbar}\int_{t_2}^{t_0} \hat{H}tdt}. \tag{27}$$

These form a bra-ket state at t_0 as follows:

$$\begin{aligned}
\langle\psi_i(t_2 \to t_0)|\psi_i(t_1 \to t_0)\rangle &= \langle\psi_i(t_2)| e^{\frac{i}{\hbar}\int_{t_2}^{t_0} \hat{H}tdt} e^{-\frac{i}{\hbar}\int_{t_1}^{t_0} \hat{H}tdt} |\psi_i(t_1)\rangle \\
&= \langle\psi_i(t_2)| e^{(\frac{i}{\hbar}\int_{t_2}^{t_0} \hat{H}tdt - \frac{i}{\hbar}\int_{t_1}^{t_0} \hat{H}tdt} |\psi_i(t_1)\rangle \\
&= \langle\psi_i(t_2)| e^{-\frac{i}{\hbar}\int_{t_1}^{t_2} \hat{H}tdt} |\psi_i(t_1)\rangle \\
&= \langle\psi_i(t_2)| e^{-\frac{i}{\hbar}(\hat{H}t_2 - \hat{H}t_1)} |\psi_i(t_1)\rangle \\
&= \langle\psi_i(t_2)| e^{-\frac{i}{\hbar}(\hat{H}\Delta t)} |\psi_i(t_1)\rangle .
\end{aligned} \tag{28}$$

If we set $\Delta t \to 0$, the above equation becomes

$$\begin{aligned} \langle\psi_i(t_2 \to t_0)|\psi_i(t_1 \to t_0)\rangle &= \langle\psi_i(t_0)|\,\hat{\mathbf{1}}\,|\psi_i(t_0)\rangle \\ &= \langle\psi_i(t_0)|\psi_i(t_0)\rangle \\ &= \langle\psi_i|\psi_i\rangle\,. \end{aligned} \tag{29}$$

The bra-ket state thus formed does not change over time.

2.2. How does the time-reversed state vector start and lead to state contraction?

2.2.1. Boundary conditions for the time-reversed state to propagate backward

This study supposes that the moment the energy dispersion decreases and the uncertainty of time increases, the quantum time of the object recedes. If that time is designated as t_m, the initial conditions can be determined at t_m. Since the state of this time-reversed quantum is the complex conjugate of the time-forward quantum, it starts from a state in which the sign of the imaginary part is reversed. In other words, if the phase of the time-forward quantum at t_m is θ $(0 \leq \theta \leq \pi)$, then the phase of the time-reversed quantum at t_m is $-\theta$ (Figure 10). Furthermore, the real parts, absolute values, and squared absolute values of the wave functions ψ of both at t_m are equal, and the magnitude and existence probability of the wave functions ψ are equal.

2.2.2. How does this lead to state contraction?

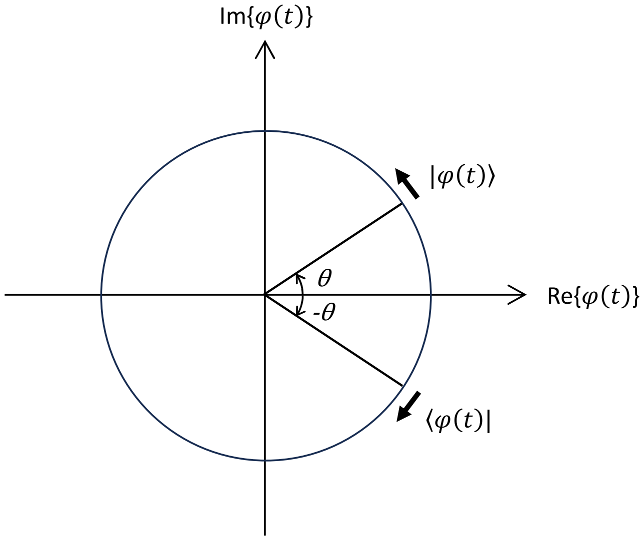
As shown in equation (11), when a superposition state represented by a linear combination of ket vectors is multiplied by a bra vector representing a single state as an inner product, due to the orthogonality between the states, only the inner product component between this bra vector and the ket vector that was originally in the same state remains. This allows us to describe the process by which one state remains from the superposition state as in equation (12). In this study, this process is considered as state contraction.

2.3. Experimental possibilities for distinguishing the other interpretations

2.3.1. Abbreviation of the concept in this study

To simplify the description, it may be better to give an abbreviation to the concept in this study. The essential concept of this study is time reversal due to time uncertainty.

Figure 10. Phases of bra vector and ket vector



However, there are other theories that use time reversal like TI and TSVF. These can be called TRVFs (time reversed vector formalisms) including this study. To distinguish this study from others, this study is called 'BKSF (bra-ket state formalism)' below.

2.3.2. Experimental predictions that distinguish them from TI

In the TI, the phase θ_1 of the time-progressive wave (confirmation wave) and the phase θ_2 of the time-regressive wave (response wave) that establishes the state by being superimposed on it are in phase with each other, i.e., $\theta_1 = \theta_2$. On the other hand, in this study (BKSF), the phase θ_1 of the time-progressive wave and the phase θ_2 of the time-regressive wave are in a complex conjugate relationship, with only the sign of the imaginary part being inverted, i.e., $\theta_1 = -\theta_2$. If we could somehow measure these phases θ_1, θ_2, or the phase difference $\theta_1 - \theta_2$, we could distinguish between these interpretations.

2.3.3. Experimental predictions that distinguish from TSVF

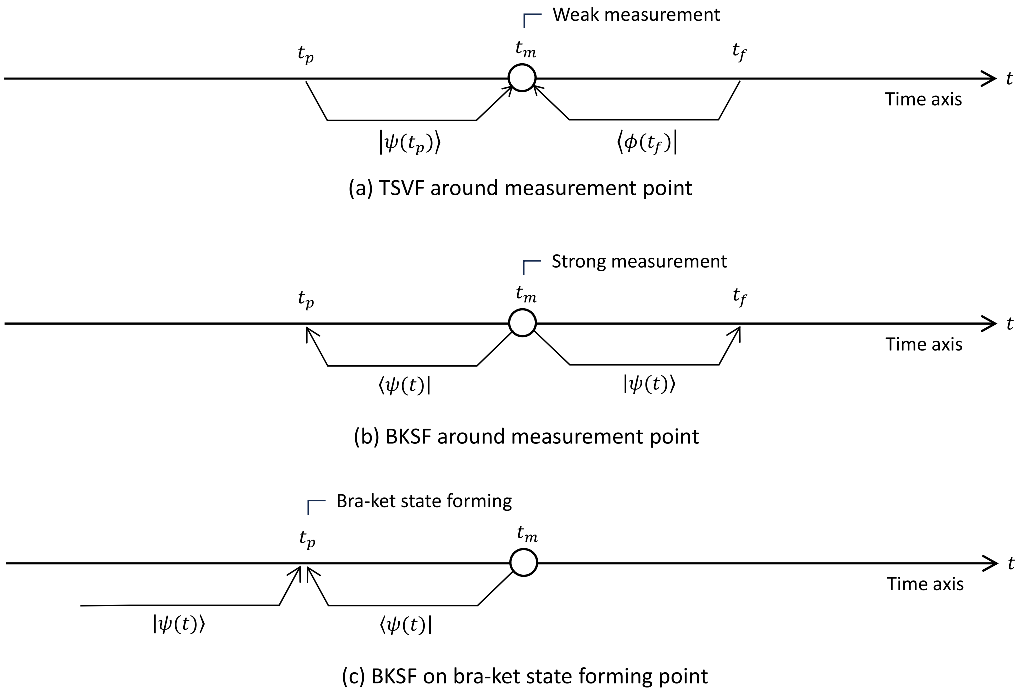
Figure 11 shows a comparison between TSVF and BKSF.

As shown in Figure11(a), TSVF defines the state vector at t_m(measurement point) by the state vector at t_p(past) and the state vector at t_f(future). TSVF calculates the expected value of the physical quantity A as follows:

$$A_w = \frac{\langle\phi|\, A \,|\psi\rangle}{\langle\phi|\psi\rangle}. \tag{30}$$

This result is consistent with the average of multiple weak measurements at t_m. In the

Figure 11. Comparison of TSVF and BKSF(this study)



BKSF shown in Figure 11(b), if a normal measurement (here called a strong measurement) is performed on t_m, the state vector can go either in the t_p direction or the t_f direction due to the time uncertainty.

Comparing the two formalisms, we can see that the two state vectors act in the opposite directions. The two state vectors of TSVF are acting on t_m from the past t_p and future t_f, and two state vectors of BKSF pointing from t_m to the past and future. However, these differences could not be directly distinguished experimentally. This is because, as shown in Figure 10(c), the state vectors of BKSF come from the past and future to form a bra-ket state at t_p. This is the same situation as TSVF. So, it seems difficult to distinguish between the two interpretations' vectors.

However, the two state vectors of the TSVF appear prominently in weak measurements, and it is possible to observe them in weak measurements that do not completely determine the state. On the other hand, the time-reverse waves in BKSF appear more clearly when strong measurements are made to confirm the state. Therefore, if time-reverse waves can be observed when strong measurements are made, it supports BKSF. However, when weak measurements are made, the situation shown in BKSF becomes closer to the weak measurement situation and may be indistinguishable from the time-reversing state vector in TSVF.

3. Summary

This study interprets the observation process in quantum mechanics as involving quanta that propagate backward in time. The retrogression of time is regarded as a property permitted by the energy-time uncertainty relation, according to which if the energy of a system is maintained constant, its time evolution may fluctuate. Specifically, time reversal is described by an antiunitary transformation, wherein the state vector emerges as the complex conjugate of the original state vector. Here, if the original state vector is a ket vector, its complex conjugate is a bra vector. When a time dependent bra vector and a ket vector interact along the time axis, they generate a time-invariant bra-ket state, whose state is established. The establishment of bra-ket state explains the phenomenon of quantum state contraction due to observation. The state with the lowest energy dispersion caused by observation exhibits the largest time dispersion, in accordance with the energy-time uncertainty relation. Accordingly, the state with the lowest energy dispersion can transmit a time-reversed bra vector to the farthest point in the past, resulting in the formation of a braket state and eliminating other states. Time fluctuation also occurs in the future, and once the 'present' exceeds the fluctuation time, only the time-invariant state exists. This process results in the contraction of the quantum state due to observation. This theory is consistent with the widely accepted interpretation of the energy-time uncertainty relation, which involves the time scale of state establishment.

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