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Original Paper

A Toy Model of Fermions as Toroidal Scalar Field Oscillators with Emergent Quantum Dynamics

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Abstract: This work is a geometric toy model which explores a geometric interpretation of an effective spinor based on toroidal excitation within a neutral, scalar field. The model attempts to derive the spinor from the gradient of the four component compactified toroidal scalar field, reconstructing the fermion Lagrangian without invoking gamma matrices. The rest mass arises from internal oscillation constrained by the toroidal geometry and can be generated via a Yukawa coupling. The framework introduces both global and local gauge-like symmetries arising from the geometry of the scalar field, suggesting that spinor behavior and gauge interactions may emerge from a deeper scalar field structure. This geometric approach offers a deterministic foundation for spin-½ dynamics and a novel path toward unifying spinor fields with scalar field. The Schrodinger operators emerge from toroidal dynamics of scalar field. This can potentially lead to a unified Lagrangian and it has the potential to solve cosmological constant problem.

Keywords: Toy model; Dirac Spinor; Unified Lagrangian; Scalar; Gravity

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1. Introduction

The Dirac equation provides a relativistic quantum description of spin-1/2 particles. However, its abstract spinor formalism lacks a concrete geometric interpretation. This work examines whether a toroidal oscillation in neutral, scalar geometrical field can reproduce an effective spinor. This can lead to a unified Lagrangian and can potentially solve cosmological constant problem.

2. Effective Spinor in a geometric Scalar field

The relativistic Klein-Gordon equation: $(\nabla^2 - \partial_t^2 - m^2)\,\varphi(x, t) = 0$.

Alternatively, the Klein Gordon equation can be written as, $(\square + m^2)\,\varphi = 0$ [1]

The Dirac equation is $(i\gamma^\mu\partial_\mu - m)\psi = 0$

In this article, it is hypothesised that the scalar $\varphi = (\varphi^x, \varphi^y, \varphi^z, -\varphi^t) \in R^4$, lives in the compactified toroidal space-time with dimensions $[M^1]$.

In natural units, $iD = i\gamma^\mu\partial_\mu$, where D is the Dirac operator

Squaring gives $(iD)^2 = (i\gamma^\mu\partial_\mu)^2 = -(\gamma^\mu\partial_\mu)^2$

By Clifford algebra of gamma matrices,

$(\gamma^\mu\partial_\mu)^2 = 1/2\{\gamma^\mu, \gamma_\mu\}(\partial_\mu)^2$ where $(\partial_\mu)^2 = \square$ and $(\gamma^\mu)^2 = 1$

$(iD)^2 = (i\gamma^\mu\partial_\mu)^2 = -\square$

The equation above is useful for further derivations.

By utilising the identity in differentiation,

$\partial^\mu\partial_\mu(\varphi^2) = 2\partial_\mu\varphi\,\partial^\mu\varphi + 2\varphi\,\square\,\varphi$,

$\square\,\varphi^2 = 2\partial_\mu\varphi\,\partial^\mu\varphi + 2\varphi\,\square\,\varphi$

By Klein Gordon equation, $\square\,\varphi = -\,m^2\varphi$, where φ is scalar field

Therefore, $\square\,(\varphi)^2 = 2\,\partial_\mu\varphi\,\partial^\mu\varphi - 2\,m^2\varphi^2$ and $¼\,\square\,(\varphi)^2 = 1/2\,\partial_\mu\varphi\,\partial^\mu\varphi - 1/2\,m^2\varphi^2$

The KG field Lagrangian may be denoted as *Lc* through-out the article and it is an on shell Lagrangian which satisfies KG equation of motion. Further derivations involve on shell lagrangian, as the symmetries are dealt with, in subsequent sections.

$Lc = ¼\,\square\,(\varphi)^2 = 1/2\,\partial_\mu\varphi\,\partial^\mu\varphi - 1/2\,m^2\varphi^2$

$Lc = ¼\,\square\,(\varphi)^2$

This is a total derivative which pushes the dynamics to the boundary or the toroidal surface, which perfectly fits for the toroidal model.

In the current model, another ansatz is needed,

$Lc = 1/2\,\partial_\mu\varphi\,\partial^\mu\varphi - 1/2\,m^2\varphi^2 - 1/4\lambda\varphi^4$

The kinetic term is exactly balanced by mass term and the lagrangian is negative and has very small energy density due to the self interaction potential.

$Lc = ¼ \square (\varphi)^2 = -1/4\lambda\varphi^4$

Let $K = ¼ \square (\varphi)^2 = -1/4\lambda\varphi^4$

Therefore $-1/4\, m^2\varphi^2 = 1/2\, \partial_\mu\varphi\, \partial^\mu\varphi - 1/2\, m^2\varphi^2$

$\mathscr{L} = 1/2\, \partial_\mu\varphi\, \partial^\mu\varphi - 1/2\, m^2\varphi^2$

$1/4\, m^2\varphi^2 = 1/2\, \partial_\mu\varphi\, \partial^\mu\varphi$

$\sqrt{K} = 1/2\, i\gamma^\mu\partial_\mu\varphi = i/2\sqrt{\lambda}\varphi^2$

$i\gamma^\mu\partial_\mu\varphi = i\sqrt{\lambda}\varphi^2$

In this toy model, the action of the gamma matrices to a four-component real scalar multiplet inside toroidal space is allowed. It will give idea about the toroidal oscillation modes and the gamma matrices act as internal linear maps, not like in Dirac operator algebra. The internal toroidal degrees of freedom makes it an effective spinor, though the field is scalar. The degrees of freedom includes helicity , direction of propagation, winding number and orientation of loops.

The author has proposed a toroidal model of fermion in the article "Fermion Spin Linked to Zitterbewegung" published in July edition of IJQF[2].In the geometrical model, it was proposed that fermions possess oscillation on the toroidal surface derived from a Hamiltonian combining rotational and translational dynamics. Chirality reflects the helical winding direction. The proposed toroid is a torus which is part of sphere with radius r such that major radius is 0.646 r and minor radius 0.354r.The anti fermion is hypothesized as having propagation opposite to direction of propagation of fermion. The fermion may be regarded as an oscillation in the toroidal field[3]

The scalar field is assumed to be real, neutral and oscillating as governed by Klein-Gordon equation. The oscillation is written in complex wave equation for convenience.

$\varphi = Ae^{ikx-wt}$

In case of toroidal model of fermion, the oscillation occurs in a closed loop on a toroid and A=r/2[r is radius of toroid] and wave number k= 1/r.

$i\gamma^\mu\partial_\mu\varphi = (i\sqrt{\lambda})\varphi^2$

$i\gamma^\mu\partial_\mu\varphi = 1/2\, \varphi(d\varphi/dx)\, i\sqrt{\lambda}/ik + 1/2\, \varphi\, (d\varphi/dt)\, \sqrt{\lambda}\, /{-\omega}$

$i\gamma^\mu\partial_\mu\varphi = 1/2\, \varphi\, r\, (d\varphi/dx)\, \sqrt{\lambda} - 1/2\varphi\, r(d\varphi/dt) i\sqrt{\lambda}/r\omega$

In the toroidal model, $r\omega = c$.

In natural units, c=1

$i\gamma^\mu\partial_\mu\varphi = \varphi\, r/2(d\varphi/dx)\sqrt{\lambda} - r\varphi\, /2\, (d\varphi/dt) i\sqrt{\lambda}$

$i\gamma^\mu\partial_\mu\varphi = r\, \varphi\, /2(d\varphi/dx)\sqrt{\lambda} - r\, \varphi\, /2\, (d\varphi/dt) i\sqrt{\lambda}$

Generalising to all dimensions,

$i\gamma^{\mu}\partial_{\mu}\varphi = \varphi\, r\sqrt{\lambda}/2\, [(d\varphi/dx)\, (d\varphi/dy) i\, (d\varphi/dz)\, -i\, (d\varphi/dt)]$

$i\gamma^{\mu}\partial_{\mu}\varphi = \varphi\, r\sqrt{\lambda}/2\, [\nabla\varphi\, -i(d\varphi/dt)]$

$i\gamma^{\mu}\partial_{\mu}\varphi = i\, \varphi\, r\sqrt{\lambda}/2\, [-i\nabla\varphi\, -(d\varphi/dt)]$

$\Box$ can be factorized as,

$\Box = [-i\nabla + (d/dt)]\, [-i\nabla\, -(d/dt)]$

Therefore, $i\gamma^{\mu}\partial_{\mu}\varphi = i\, \varphi\, r\sqrt{\lambda}/2\, [-i\nabla\varphi +/- (d\varphi/dt)]$

The above expressions are similar to quantum mechanical operators,

$\hat{P} = -i\hbar\nabla = -i\hbar(ik) = \hbar k$

$E = \hat{H} = i\hbar\partial t = (i\hbar)\omega = i\hbar\omega$

$i\hbar\partial t = i\hbar\omega = i\hbar\omega \sim iecA_{\mu}$ where A_{μ} is electromagnetic four potential

In natural units, $\partial t = eA_{\mu}$.

This resembles the gauge term.

It can be used as an axiom to make the toy model Lagrangian covariant. Next ansatz for toy model is: $D_{\mu} = [-i\nabla +/- (d/dt)]$

$D_{\mu} = d_{\mu} - (d/dt)$ or $D_{\mu} = d_{\mu} + (d/dt)$, according to the fermionic/antifermionic nature of field excitation.

Therefore, $i\gamma^{\mu}\partial_{\mu}\varphi = i\varphi r\sqrt{\lambda}/2 (d_{\mu}\varphi)$

$(d_{\mu}\varphi) = (-2i/\varphi r\sqrt{\lambda})\, i\gamma^{\mu}\partial_{\mu}\varphi$

From the on-shell scalar relation:

$(\nabla^2 - \partial_t^2) = m^2\varphi^2$, we see that the time and the spatial variation of φ form four natural components. These are precisely the four ingredients of the Dirac formalism.

The spinor may be defined by an ansatz as $\psi = i\gamma^{\mu}\partial_{\mu}\varphi\zeta = i\varphi r\sqrt{\lambda}/2 d_{\mu}(\varphi)\zeta$

$\zeta = 1/\sqrt{m}R$, where R is the dimensionless rotor in space time algebra[4]

Therefore, $\psi = (\rho e^{i\beta})^{1/2} R = i\gamma^{\mu}\partial_{\mu}\varphi\zeta$. This grounds the model in space time algebra.

This obeys Klein Gordon equation and Dirac equation.

Here ζ is a spinor seed with dimensions $[M^{-1/2}]$ and transform in the toroidal space alone.

In the toroidal model, this effective spinor emerges as the derivative of scalar field: the oscillatory scalar dynamics in four components encodes the spinor. The gamma matrices map the internal toroidal dynamics. In this model, the toroid is a scalar field oscillon which transforms (rotates) inside the spherical domain it lives and the spherical domain transforms in

Lorentz boost observing SO3 symmetry. ζ is an internal spinor seed which transforms internally. Therefore the toroidal structure is Lorentz Covariant geometrically.

$i\gamma^{\mu}\,\partial_{\mu}\psi= i\gamma\,\partial(i\gamma^{\mu}\partial_{\mu}\varphi\zeta)=-\,\square\,(\varphi)\zeta$

Therefore ψ obeys both Klein Gordon and Dirac eqpuations.

Therefore, $-\,\square\,(\varphi)\zeta=m^2(\varphi)\zeta$ and it obeys $\square\,(\varphi)+m^2(\varphi)=0$

The below equations are analogous in the toy model:

$i\gamma^{\mu}\partial_{\mu}\varphi\zeta= i\ \varphi\ r\sqrt{\lambda}/2(d\mu(\varphi)\zeta$

Therefore, $i\gamma^{\mu} = \mathbf{r}^{a}\ T^{a}\ i\sqrt{\lambda}\ \varphi\ /2$

$\mathbf{r}^{a}\ T^{a}$ is explained in the next section.

3. Relationship between the gauge symmetries and the model

The Lagrangian can be written as, $\mathscr{L}=\overline{\psi}\ (\mathbf{r}^{a}\ T^{a}\ i\sqrt{\lambda}\ \varphi\ /2\ D_{\mu}- m)\ \psi$

$\mathbf{r}^{a}\ T^{a}$ is introduced to make the toroidal geometry, Lorentz invariant. $\mathbf{r}^{a}$ can be regarded as the vectors along the radius of toroid. It is an octet. The radius vector inside a toroid can align in eight different orthogonal directions without repetition. Four directions are in the axial plane of toroid. Four directions are in the orthogonal planes. The octet is transformed by T^{a}, which are SU3 generators. The T^3 and T^8 , diagonal generators which cause rotation of radius vector in the plane of toroid. All other generators rotate radius vector to an orthogonal plane. The toroid can exist in three orthogonal planes. $i\sqrt{\lambda}\varphi/2$ is a scaling constant for a particular fermion which may be replaced by N.

The Lagrangian can be written as:

$\mathscr{L}=\overline{\psi}[N\mathbf{r}^{a}T^{a}D_{\mu}- m]\psi$

$\psi=\begin{pmatrix}\psi\ xy\\ \psi\ xz\\ \psi\ yz\end{pmatrix}$. The toroid can lie in XY, XZ or YZ plane at a point in time.

The Lagrangian observes an effective SU(3) symmetry.

$\psi\rightarrow U\psi$ where $U\in SU(3)$ and $UU^{\dagger}=I$

$\bar{\psi}\rightarrow U^{\dagger}\bar{\psi}$

Fermions, in this framework, are toroidal excitations of the scalar field. The sphere of radius (r), within which the toroid resides, can be interpreted as the local domain of the scalar field associated with a single fermion. Within this spherical region, transformations in various spatial directions take place, encoding the internal degrees of freedom. During boost, the toroid can choose one of the three spatial planes and propagate in one of the corresponding direction axial to the plane. There are eight directions or degrees of freedom the toroid can have a Lorentz boost. The Lorentz invariance occurs due to internal rotation or configuration change of the toroid, aligned with the boost direction. It is shown to cause the specific findings in Stern Gerlach experiment. It is described in below section. Therefore, the toroid observes SO3 symmetry with respect to the spherical domain where it lives and an effective SU3 symmetry of the toroid during boost allows for it. Toroid chooses the alignment in the direction of the boost. The effective SU3 symmetry is present in the toroidal geometry. The SO3 symmetry allows for Lorentz invariance based on geometry. Therefore $\mathbf{r}^a T^a$, effectively makes the toroid Lorentz invariant at the level of observables, though internally it is not Lorentz invariant.

The scalar field is expressed as:

$\varphi = Ae^{ikx-wt} = Ae^{i2\pi}e^{ikx-wt} = \varphi$

This expression indicates a global gauge symmetry due to the invariance of φ under global phase shift of 2π.

A global U(1)symmetry acts as:

$\psi \rightarrow e^{i\alpha}\psi, \ \bar{\psi} \rightarrow e^{-i\alpha}\bar{\psi}$

Therefore the phases cancel out in the lagrangian,

$\mathscr{L} = \bar{\psi}\, [\mathbf{r}^a T^a\, i\sqrt{\lambda}\, \varphi /2\, D_\mu - m]\psi$

The Lagrangian is invariant under global U(1), since φ is neutral scalar field and all other terms are scalar.

According to the toroidal model, the translational motion occurs in the XY plane, while the helical motion is inclined at 45° to both the XY and YZ planes. The upward and downward translational movement in XY plane contributes to the angular momentum, with its vector directed along the positive and negative Z-axis respectively. The motion of the particle isin a modified toroid. The periodic motion of the fermion along a helical trajectory on a toroid of radius r introduces an effective local gauge-like abelian symmetry, associated with the internal phase variation during propagation. This local structure reflects the dynamics of a point-like fluctuation (the fermion) within the neutral toroidal scalar field. The gauge term is derived in further section.

SU(2) symmetry is defined by using Pauli's matrices as generators. Through the below description, it is shown how the toroidal model is related to Pauli's matrices. In quantum mechanics, the fermion spin is given by, $S_i = \hbar/2\sigma_i$

$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

The determinant of each matrix is equal to -1.

From the geometrical model linking fermion spin to toroidal oscillation, it is learnt that the normalization of eigenspinors is due to the 45 degree helix on the toroid and the spin components make 45 degrees to x and y axis and the normalization of Pauli's matrices with $1/\sqrt{2}$ could be due to the geometry. Both the components are orthogonal to each other and orthogonal to the Z component Spin vector.

Therefore vector sum of magnitude of components

$|\mathbf{S_{xy}}| = \hbar/2 \sqrt{(S_x)^2+(S_y)^2}$

$=\hbar/2 \sqrt{(-1)^2+(-1)^2}$

$=\sqrt{2}\hbar/2$

$|\mathbf{S}| = \hbar/2 \sqrt{(S_{xy})^2+(S_z)^2}$

$|\mathbf{S}| = \hbar/2 \sqrt{(\sqrt{2})^2+(-1)^2}$

$|\mathbf{S}| = \sqrt{3}\hbar/2$.

The angular momentum obtained above is equal to the spin angular momentum of electron in hydrogen atom and it is at projection angle 54.7° to Z axis. It can be either at 35.3° to +Y axis or -Y axis [Spin-down and Spin-up configurations].

The Stern-Gerlach (SG) experiment used electrons in silver atoms to measure spin by passing them through a non-homogeneous magnetic field. The measured spin values appeared randomly distributed. Notably, even when spin-up electrons were separated and sent through another Stern-Gerlach apparatus in the same direction, they still exhibited equal probabilities of spin-up and spin-down. This inherent uncertainty posed a fundamental challenge to precisely quantifying both momentum and position.

In the proposed model, the bound electron adopts a modified toroidal configuration. This configuration naturally allows for two discrete spin states—spin-up and spin-down—and results in the electron behaving as a magnetic dipole. As a dipole, the electron experiences a torque in the presence of an external magnetic field, causing it to rotate and align with the field direction. Raedt et al. proposed an event-by-event simulation of the Stern–Gerlach (SG) experiment with neutrons [5]. Their model requires an initial alignment of the magnetic moment based on quantum mechanical assumptions but otherwise follows Newtonian dynamics. The modified toroidal model proposed here offers a tangible, geometric explanation for the initial magnetic moment alignment, providing an alternative to the probabilistic approach of quantum mechanics. Similarly, Mostafaeipour et al. conducted a finite-element simulation of atomic trajectories in the SG experiment [6]. Their model accurately reproduces the SG magnetic field and the resulting particle distribution, consistent with historical data. However, it also assumes predefined quantum spin states (e.g., for silver atoms), while the trajectory analysis is conducted using classical mechanics. The modified toroidal model can offer insight into the quantization of spin states, suggesting that spin is intrinsically aligned at an angle of 35.3° with respect to the +/- Y -axis due to the geometry of the torus. This naturally restricts the electron to two possible orientations, thereby offering a physical justification for spin quantization and its discrete nature. Unlike standard quantum mechanics, which invokes inherent probabilistic behaviour, this model explains spin orientation based on structural constraints. In the inhomogeneous magnetic field of the SG apparatus, which varies along the Z-direction, the magnetic dipole experiences a force given by: $F = \mu\ (dB/dz)$, where μ is the magnetic moment. If spin-up electrons are sent again through an SG device aligned along the Z-axis, they remain in the spin-up state, as no torque is applied. An atom in the $Sz = +½$ eigenstate remains in that state in a uniform Z-directed magnetic field and follows a predictable trajectory, reaching the detector at: $Zd=(\gamma t^2)/(4m)$, consistent with conventional SG analysis[7]. This indicates that a semi-classical treatment is valid for most of the silver atom's trajectory. However, problems arise at the entry point and when a perpendicular magnetic field is introduced. In these cases, classical analysis cannot provide a complete explanation. The modified toroidal configuration offers a potential physical hypothesis to account for these anomalies. When a magnetic field is applied in a direction perpendicular to the initial orientation, it exerts a torque: $\tau = \mu Bx \sin\theta$, where Bx is the external magnetic field along the X-axis, and θ is the angle between the magnetic moment and the field. Since the electron can be at any position within the toroidal structure when the field is applied, the torque disrupts the configuration irreversibly—this disruption can be interpreted as the collapse of the wave function. Following this disruption, the electron adopts a new toroidal configuration. The model assumes equal probability for the electron to transition into either spin-up or spin-down state $P(S^+) = P(S^-) = 0.5$. Thus, this model offers a new hypothesis for wave function collapse—one that is geometrically derived which requires further testing.

It can be defined that $\psi=\begin{pmatrix}\psi R\\ \psi L\end{pmatrix}$. In the toroidal model, right handed chirality results in the 45 degree projection of the spin to be contained towards the centre of toroidal structure. The left handed chirality results in the 45 degree projection of spin to be oriented outwards and it promote interaction with other particles. Therefore, ψ_L is a doublet, ψ_R and φ are singlets with respect to SU2 symmetry. Therefore weak interaction could be electromagnetic interaction which increases with contact time of particles. This means that the higher the speed of propagation of the oscillation along the toroid, higher the contact time and stronger the weak force interaction.

The Lagrangian observes SU(2) symmetry.

$\psi_L \rightarrow U\psi_L$ where $U \in SU(2)$ and $UU^\dagger = I$

$\bar{\psi}_L \rightarrow U^\dagger \bar{\psi}_L$

The Lagrangian has an inherent global U(1) symmetry, SU(2) symmetry and SU(3) symmetry. This suggests that the effective spinor may emerge from a more fundamental scalar field.[8,9].

4. **Mass Generation in the Model**

The toroidal model postulates that a minimal input of energy initiates zitterbewegung, possibly regulated by a Yukawa coupling.

$\mathscr{L} = \bar{\psi}[N\mathbf{r}^a T^a D_\mu - y\varphi]\psi$

This gives a Lorentz and gauge invariant Lagrangian with mass generation by Yukawa coupling.

The Euler Lagrange equation of the lagrangian, yield solution:

$[N\mathbf{r}^a T^a D_\mu - y\varphi]\psi = 0$, an expression similar to Dirac equation.

In the geometric toroidal model, an initial small amount of energy added maintains a constant rest energy for the particle. The initial energy may be added to overcome vacuum potential from self-interaction of KG field, $V_0 = 1/4\lambda\varphi^4$.The Lagrangian may be written as:

$\mathscr{L} = 1/2\, \partial_\mu\varphi\, \partial^\mu\varphi - 1/2\, m^2\varphi^2 - 1/4\lambda\varphi^4$[10]

This form allows for oscillon like solutions (φ^4 theory)

The corresponding Klein Gordon equation may be written as:

$(\nabla^2 - \partial t^2 - m^2)\, \varphi - \lambda\varphi^3 = 0$

Solving the equation, using ansatz $\varphi = A\mathrm{Cos}\,(kx-wt)$, the real part of the equation $\varphi = Ae^{ikx-wt}$

$k^2 A\mathrm{Cos}\,(kx-wt) - \omega^2 A\mathrm{Cos}\,(kx-wt) + m^2 A\,\mathrm{Cos}\,(kx-wt) - \lambda A^3\mathrm{Cos}^3(kx-wt) = 0$

$\cos^3(kx-wt) = 1/4\cos 3(kx-wt)) + \frac{3}{4}\cos(kx-wt)$

In case of small amplitude, the higher harmonic can be neglected.

$k^2 - \omega^2 + m^2 + \frac{3}{4}\lambda A^2 = 0$

$\omega^2 = k^2 + m^2 + \frac{3}{4}\lambda A^2$

Effective mass term, $m_{eff}{}^2 = m^2 + \frac{3}{4}\lambda A^2$

Therefore effective mass increases with amplitude. When the amplitude of oscillation is negligibly small, the equation becomes Klein-Gordon dispersion equation. Small amplitudes allow for particle like oscillon solution. Large amplitudes result in radiative process through third harmonic.

Applying toroid parameters, A=r/2 and k=1/r, the equation becomes

$\omega^2 = 1/r^2 + m^2 + 3/16\ \lambda r^2$

Therefore for small amplitudes, $1/r^2$ becomes dominant and do not allow for near zero solutions which requires infinite energy and thereby maintains stability of toroid structure. The larger the radius of toroid are , higher the energy needed in the system due to λr^2 term, which rules out possibility of very large toroids.

The Lagrangian is negative, with the kinetic term is balanced by the mass term and the self-interacting vacuum potential has an expectation value similar to Higg's field. But, as suggested by the toroidal model, the potential could be of much lesser magnitude.

In the toroidal model, $m = h/c\lambda_c = h/2\pi rc = \hbar/rc$,where λ_c is Compton wave length, m is rest mass, r is radius of toroid and c is speed of light. Therefore, the smaller the radius of the toroid, the greater the rest mass of the fermion. This framework could explain the varying rest masses of different fermions, such as neutrinos, electrons, quarks.

Since the kinetic term and mass term cancels each other exactly ($k^2=m^2$) as the Lagrangian which gives rise to effective spinor is on-shell and the self interaction term is very small, *Lc* is nearly zero and it gives the equation.

$Lc = -1/4\lambda\varphi^4$

The oscillon on the surface of compactified torus can be analogous to a spinor.

5. Gravitoelectromagnetism Derivation from an Ansatz and its Potential for Further Exploration in Quantum Gravity

It is defined that there is relation between scalar field and spinor: $\psi = ir\sqrt{\lambda}/2\sqrt{m}(d\mu(\varphi)R$

$\bar{\psi} = ir\ \varphi\sqrt{\lambda}/2\sqrt{m}\ (d\mu(\varphi)\bar{R}$

$\bar{\psi}\psi = (i\ \varphi\ r\sqrt{\lambda}/2\sqrt{m}\ d\mu(\varphi)\ \bar{R})(i\ \varphi\ r\sqrt{\lambda}/2\sqrt{m}\ (d\mu(\varphi)R)$

$\bar{\psi}\psi = (-\varphi^2 r^2/4m)\lambda\partial_\mu\varphi\partial^\mu\varphi\ \bar{R}R$

$\partial_\mu\varphi\partial^\mu\varphi = m^2\varphi^2$ in the model

$\bar{\psi}\psi = (-\varphi^2 r^2/4m)\lambda m^2\varphi^2\ \bar{R}R$

$\bar{R}R = 1$,then $\bar{\psi}\psi = (-r^2/4m)\lambda m^2\varphi^4$

If $a = r^2(-1/4\lambda\varphi^4)$, $a = r^2V_0 = r^2 1/4\lambda\varphi^4$, where V_0 is negative scalar potential.

$\bar{\psi}\psi = am = mr^2V_0$

$V = -m/r^1 = \bar{\psi}\psi/ar^1$

Therefore gravitational potential could be a function of fermionic density and which in turn is a function of mass and radius of fermion and scalar potential. It may be postulated that within a SO(3) invariant local spherical domain of radius r, the effective spinor field is related to gravitational potential at its surface by the relation: $V=-m/r^1=-\overline{\psi}\psi/ar^1$ .This postulate reflects an intrinsic geometric coupling of fermionic density and gravitational field.

Using the ansatz, $V=-\overline{\psi}\psi/ar^1$, if extrapolated for other components of Dirac probability current, $A_g=1/ar^1\ (\psi\gamma^0\gamma^i\psi\dagger)$, where i=1,2,3. The expression $\psi\gamma^0\gamma^i\psi\dagger$ represent fermion current and Ag is the gravitational magnetic potential.

The gravitomagnetic four potential may be defined as $A_{\mu g} =(V/c, A_g)$

The gravitoelectricfield in natural units is given by,

$E_g=- \nabla V - dA_g/dt$

The gravitomagnetic field is defined by

$B_g=\nabla \times A_g$

The unified Lagrangian of fields except gravitational field may be written as:

$L_{unified}= 1/2\ \partial_\mu\varphi\ \partial^\mu\varphi-1/2\ m^2\varphi^2-1/4\lambda\varphi^4-\overline{\psi}[N\mathbf{r}^{\mathbf{a}}T^aD_\mu-\ y\varphi]\psi-1/4F^b{}_{\mu\nu}F^{\mu\nu b}$

Where $L_K=-1/4F^b{}_{\mu\nu}aF^{\mu\nu b}$, is similar to the Yang-Mills Lagrangian density but include electromagnetic kinetic term also[11].

The stress energy tensor of the unified Lagrangian may be written as[12]:

$$T^\mu{}_\nu = \frac{\delta\mathcal{L}}{\delta(\partial_\mu\phi)}\partial_\nu\phi - \delta^\mu_\nu\mathcal{L}$$

This can be connected to general relativity,

$G_{\mu\nu} + \Lambda g_{\mu\nu} = (8\pi G/c^4)T_{\mu\nu}$ [13]

$\Lambda g_{\mu\nu}$ may represent the vacuum state of scalar field. In this model, the scalar field does not contribute to gravitation in its vacuum state but acquires gravitational effects when a fermionic mass current arises from it. Energy and momentum arising from the fermionic current along with fermionic density contributes to gravity. Modelling fermions as localized, toroidal excitations of a scalar field may potentially offer a novel mechanism for addressing the cosmological constant problem by effectively decoupling vacuum energy from gravity[14].

The Vacuum expectation value of Higgs field is 246 GeV. The Vacuum energy contribution from Higgs field is $\sim 10^8$ GeV^4. This is much larger than the predicted value of dark energy, which is $\sim 10^{-47}$ GeV^4. The toroidal model, as described in my article “Fermion Spin Linked to Zitterbewegung” (July Edition IJQF) , shows geometrically that zitterbewegung can be linked to rest energy and an initial infinitesimally small energy/boost is only required to set the particle or the field excitation (as in current article) into zitterbewegung /toroidal oscillation and doing so contribute to rest energy/ mass. By subsequent analysis in the current article, it is shown that the negative potential of Scalar field in Vacuum has to be negative and infinitesimally small such that it obeys scalar field lagrangian. It can be related to the vacuum energy contribution by dark energy of $10^{-47}GeV^4$. This potentially offers an alternative to Higgs mechanism. It has been proposed that the fermionic density and energy related to fermions contribute to gravity by observing SO(3) symmetry. It gives an opportunity to effectively decouple the vacuum energy of scalar field from the energy and matter contributing to gravity. This can explain the repulsive effect of dark energy. The model suggests that the dark energy could be scalar field energy in vacuum. This hypothesis needs further exploration. Self-interacting scalar fields have been extensively investigated as candidates for dark energy, while scalar field-based self-interacting particles have also been proposed in the context of self-interacting dark matter (SIDM) models.[15,16]. The negative Lagrangian density of the scalar field makes it a suitable choice for dark energy model. Therefore this approach can give a new direction to exploration of quantum gravity and cosmology.

6. Conclusion

It can be concluded that the Dirac spinor may be derived from a geometric toroidal model. This approach reproduces key spin features, connects chirality to geometry, and aligns with quantum field theory. It offers a novel framework for spinor dynamics with implications for foundational quantum theory. Fermion can be considered as localised excitation of a scalar field which follows a toroidal geometry. This toy model can potentially give an unified lagrangian density and can potentially solve the cosmological constant problem. This hypothesis needs further exploration.

References

1. Paul. A.M. Dirac. The Quantum theory of the electron Proc. R. Soc. A 117,610 (1928)
2. Suresh J. Fermion Spin Linked to Zitterbewegung: A Foundational Approach. International Journal of Quantum Foundations 11 (2025) 263–274
3. Naber, Gregory L.. Fermionic and supersymmetric harmonic oscillators. *Quantum Mechanics: An Introduction to the Physical Background and Mathematical Structure*, Berlin, Boston: De Gruyter, 2021, pp. 391-454. https://doi.org/10.1515/9783110751949-010

4. Hestenes, D. The zitterbewegung interpretation of quantum mechanics. *Found Phys* **20**, 1213–1232 (1990). https://doi.org/10.1007/BF01889466
5. De Raedt, H., Jin, F., &Michielsen, K. (2022). Classical, Quantum and Event-by-Event Simulation of a Stern–Gerlach Experiment with Neutrons. Entropy, 24(8), 1143. https://doi.org/10.3390/e24081143
6. FarazMostafaeipour et al 2025 Phys. Scr. 100 045410 DOI 10.1088/1402-4896/adbf6c
7. Wennerström, H., &Westlund, P.-O. (2017). A Quantum Description of the Stern–Gerlach Experiment. Entropy, 19(5), 186. https://doi.org/10.3390/e19050186
8. Barceló, C., Carballo-Rubio, R., Di Filippo, F. et al. From physical symmetries to emergent gauge symmetries. J. High Energ. Phys. 2016, 84 (2016).
9. J. C. Taylor: Gauge Theories in the Twentieth Century. World Scientific, Singapore (2001).
10. Denton, Peter B. Techniques for solving static Klein-Gordon equation with self-interaction λΦ4 and arbitrary spherical source terms. United States: N. p., 2024. Web. Doi:10.1016/j.physletb.2024.138860.
11. C. N. Yang and R. L. Mills, Phys. Rev. 96, 191 (1954). https://doi.org/10.1103/PhysRev.96.191
12. Tong, D. (2006). Lectures on Quantum Field Theory, University of Cambridge".
13. Einstein, A. (1916). The Foundation of the General Theory of Relativity. Annalen der Physik, 49(2), 769-822.
14. Weinberg, S. (1989) The Cosmological Constant Problem. Review of Modern Physics, 61, 1. https://doi.org/10.1103/RevModPhys.61.1
15. Han, Chengcheng&Xie, Ke-Pan & Yang, Jin Min & Zhang, Mengchao. (2024). Self-interacting dark matter implied by nano-Hertz gravitational waves. Physical Review D. 109. 10.1103/PhysRevD.109.115025.
16. Chakrabarti, Soumya&Lahiri, Amitabha. (2022). Scalar–fermion interaction as the driver of cosmic acceleration. Physics of the Dark Universe. 37. 101121. 10.1016/j.dark.2022.101121.