

*Original Paper*

# A Modified Toroidal Model of Proton In Nuclear Magnetic Resonance

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**Abstract:** In quantum mechanics, the zitterbewegung is a rapid oscillatory movement of quantum particle in single plane. It is a motion which can give idea about the sub structure of quantum particles. It is believed to be the reason for magnetic moment of the quantum particle. In 2023, a new toroidal model of proton has been proposed by Kovacs et al. In this article, a modification of the model has been proposed with the concept of ontic wave function and using the phenomenon of nuclear magnetic resonance. The analysis of zitterbewegung and the ontic wave function has shown that, using zitterbewegung and the concept of Larmor precession, a model for generation of radiant energy during nuclear magnetic resonance can be derived.

**Keywords:** ontic; quantum; wave function; precession; zitterbewegung

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In this article, the concept of zitterbewegung is considered as the reason for magnetic moment of proton. It has been described as the reason for magnetic moment of quantum particles like electron previously also. It is a back-and-forth rapid circulatory motion of quantum particle in a plane orthogonal to its propagation[1,2]. The maximum radius or amplitude of the zitterbewegung is found as equal to reduced Compton wavelength.

Therefore,  $r=\lambda/2\pi$  where  $r$  is the maximum radius of zitterbewegung.[3]. A toroid model has been described for proton by Kovacs et al[4]. It is described that the zitterbewegung has a helical pattern with uniform diameter which precess and forms a toroid in external magnetic field. Here in this article, a variable diameter (increasing and decreasing) of zitterbewegung has been considered in the presence of radiofrequency (Radiofrequency is used to tilt the magnetic moment orthogonally in nuclear magnetic resonance). The radiofrequency has a varying electric and magnetic field. This can explain nuclear magnetic resonance more accurately and can describe the generation of electromagnetic wave.

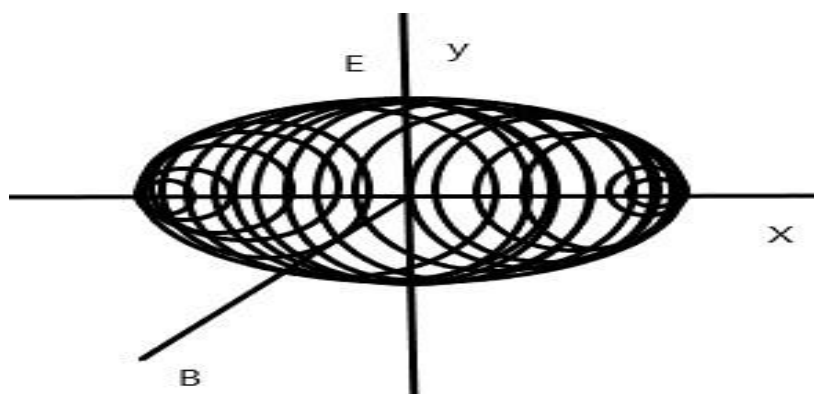


Fig 1. The zitterbewegung of quantum particle (increasing and decreasing diameter)

The relationship exists between the externally applied orthogonal magnetic field and Larmor precession frequency is described as:  $\omega = \gamma B$  where  $\omega$  represents the frequency of the Larmor precession;  $B$  is the external magnetic field intensity and  $\gamma$  represents the gyro-magnetic ratio of the quantum particle [5]. Here the angular frequency of precession should be equal to the angular frequency of the wave for generation of an electromagnetic wave.

In nuclear magnetic resonance, external magnetic field is applied which orient the magnetic moment to its direction. The radiofrequency pulse given orthogonally tilt the magnetic moment of the proton orthogonally and give it a precession which generates the electromagnetic wave once it goes back to the original alignment. The Larmor precession frequency will be the same as the frequency of radiofrequency pulse and the generated electromagnetic wave will also have the same frequency. This is a scientifically proven fact with experiments [6].

The equation for Larmor frequency is  $\omega = \gamma B$ .  $B$  is the magnetic field of radiofrequency pulse creating precession. Then  $E = cB$  is the electric field of the radiofrequency pulse. The Larmor frequency is equal to the angular frequency of the electromagnetic wave. Therefore the equation can be written as  $ck = \gamma B$ ,  $\gamma = gq/2m$  where  $q$  is the charge of proton and  $m$  is the mass. Therefore,  $ck = gBq/2m$ . Here  $k = 2\pi/\lambda = 1/r$ . We know that  $r$  is the maximum radius of zitterbewegung.

$$\text{Hence, } c/r = gEq/2mc$$

$$\text{Rearranging the equation, } mc^2/2g = qEr$$

$qEr$  in general means the work done by displacing a charge  $q$  over a distance  $r$  in an electric field  $E$

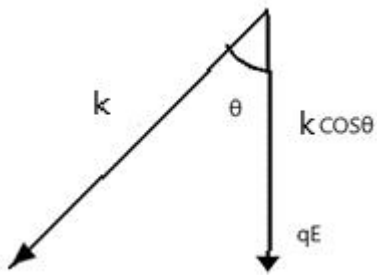


Fig 2. Work done by electric field of radiofrequency by flipping proton

The work done by the radiofrequency pulse to move proton to orthogonal plane is given by  $W = Fk \cos \theta$ . If the radiofrequency flips the zitterbewegung to orthogonal plane in a toroidal model of proton,  $r = k \cos \theta$ . The work done,  $W = qEr$  is equal to  $W = 1/2g (mc^2)$ .  $E = 1/2g (mc^2)$  is the energy required to flip the zitterbewegung to orthogonal plane. This energy is released as the radiant energy including the radiofrequency waves generated by Larmor precession. This same principle applies for electron magnetic resonance. There microwaves are utilised instead of radiowaves.

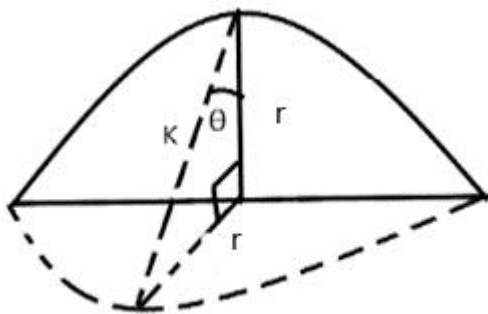


Fig 3. Flipping of plane of zitterbewegung to orthogonal plane by external magnetic field

The zitterbewegung of a charged particle in the way depicted in figure1 can create orthogonal magnetic field, which will be varying in magnitude. It can in turn create a varying orthogonal electric field. This magnetic field increases to the maximum value when the particle makes the largest circle. If the frequency of Larmor precession equals that of two cycles of zitterbewegung, the magnetic moment will reverse its direction in orthogonal axis after one cycle of zitterbewegung. The magnitude of magnetic moment due to zitterbewegung changes in direction but continues to increase and decrease in a sinusoidal wave pattern. It means that there is a generation of magnetic field and electric field as sine waves due to this process. Therefore it is possible to explain the generation of electromagnetic wave using the concept of Zitterbewegung and Larmor precession.

The potential application of such a concept is that it can explain the source of heat energy other than the mechanisms like conductive loop and antenna effect in nuclear magnetic resonance. It is a known fact that both these phenomena are not able to explain completely the tissue heat generation[7]. The model described above gives the concept of release of radiant energy during relaxation in magnetic resonance and it can explain why there is higher chance of burn injury with higher electric field of radiofrequency pulse. It also explains the increased chance of burns with the higher static magnetic field as it necessitates higher energy required to flip the zitterbewegung to orthogonal plane because the amplitude of zitterbewegung is increased by the external static magnetic field[8].

Next part of the particle is about linking the zitterbewegung in nuclear magnetic resonance to wave function. Wave function is used extensively in quantum physics to represent quantum particles in a probabilistic aspect. Here, a physical meaning of wave

function has been examined. It helps in supporting the derivation of modified toroidal model of proton. This model can explain accurately the radiant energy generation in nuclear magnetic resonance including the radiofrequency. A geometrical analysis has been done to understand the physical meaning of the wave function [9,10].

$$\Psi(x,t)=e^{i(kx-wt)} = \text{Cos}(kx-wt) + i \text{Sin}(kx-wt).$$

$$e^{i(kx-wt)} = e^{i \pi 2 (x/\lambda - t/T)}$$

$$e^{\pi i} = -1, \text{ according to Euler's equation. } (e^{\pi i} + 1 = 0)$$

$$e^{\pi 2 i} = 1$$

So by complex analysis, when  $kx-wt=0$ ,  $x/\lambda - t/T=0$  and  $e^{i(kx-wt)}=1$

In wave dynamics, when  $kx - wt = 0$ ,  $y=A\text{Cos}(kx-wt)$  or  $y=A\text{Sin}(kx-wt)$  represents a nondispersive wave like an electromagnetic wave in vacuum.

$$\text{Therefore, } \text{Cos}(kx-wt) + i \text{Sin}(kx-wt) = 1$$

$$\text{It can be rewritten as } 1 - i \text{Sin}(kx-wt) = \text{Cos}(kx-wt)$$

After taking the derivative with respect to  $dx$ ,

$$-ik \text{Cos}(kx-wt) = -k \text{Sin}(kx-wt)$$

$$\text{Therefore, } -\text{Cos}(kx-wt) = i \text{Sin}(kx-wt).$$

When square of the equation is taken, an equation with no imaginary numbers is obtained.  $\text{Cos}^2(kx-wt) = -\text{Sin}^2(kx-wt)$ .

The concept of standing wave for quantum particle is one of the basic consideration in quantum physics.[11,12,13]. In that view point, the meaning of wave function may be that the waveform  $i\text{Sin}(kx-wt)$  is an imaginary wave equal and opposite to a real  $\text{Cos}(kx-wt)$  wave and the superposition of these wave forms may result in closed curves like a standing wave. One closed curve of the standing wave is taken as a quantum in this article.

In Cartesian Co-ordinates, if  $x$  represents the displacement of free particle in the X direction and  $A$  represents the amplitude of oscillation in Y direction, it can be assumed as zitterbewegung of quantum particle in two dimensions. Zitterbewegung was first predicted by Schrodinger for Dirac electron[14].

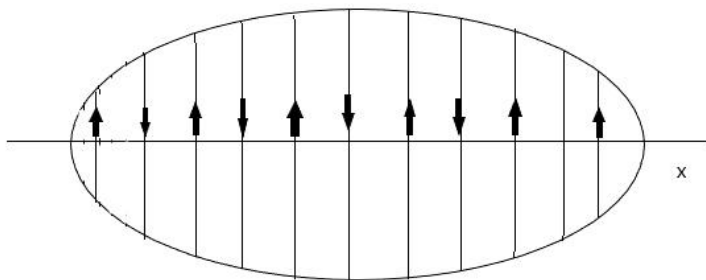


Fig4. Oscillating quantum particle forming an outline of a quantum of standing wave

$H = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \omega^2 A^2 \sin^2 \omega t$ . This equation can be visualised as an oscillating quantum particle within an outline of standing wave as in Fig 4.

But in this article, the zitterbewegung is considered to be a back-and-forth rapid circulatory motion of free quantum particle in XY plane rather than harmonic oscillation. Zitterbewegung of quantum particles has been considered as rapid circulatory motion previously[15]. In this article, it is examined using classical equations and geometry.

The hamiltonian can be written as  $H = P^2/2m + \frac{1}{2} m\omega^2 r^2$ . This is similar to the harmonic oscillator equation described earlier. The equation can be rewritten as,  $H = \frac{1}{2} m v^2 + \frac{1}{2} m v r \omega$ , where  $v$  is the linear velocity of the particle. In expansion,  $H = \frac{1}{2} m x^2 / t^2 + \frac{1}{2} m 2 \pi r x / t^2$

Taking derivative with respect to  $x$ ,  $dH/dx = (1/2m) t^2 d/dx(x^2 + 2 \pi r x)$

The total energy is constant for a quantum particle in harmonic motion.

$$E = H = \text{Constant (time independent hamiltonian)}$$

Applying chain rule,  $dH/dt = (dH/dx) (dx/dt) = 0$ . Therefore  $dH/dx = 0$  ( $dx/dt$  is a non zero term for a particle in motion)

Therefore, the derivative become,  $2x + 2 \pi r + 2 \pi x dr/dx = 0$

Solving the equation,

$$dr/dx = -1/\pi - r/x$$

$$r = -x(1/\pi + dr/dx)$$

$$|r| = x(1/\pi + dr/dx)$$



$$|r| = x/\pi + x \, dr/dx$$

This equation means that the quantum particle travels in helical motion with changing radius. Using the equation, a graph on X-Y coordinates can be plotted.  $x$  is the displacement in the equation. It ranges from 0 to  $\lambda$ .  $r$  is the changing radius with respect to the displacement. A graph (Fig.5) can be plotted in Cartesian co-ordinates by taking  $Y = \pm r$ .  $Y$  is similar to the amplitude of oscillation ( $A$ ) in the case of harmonic oscillator (Fig 4). A closed curve like a standing wave can be plotted. The graph has been drawn using boundary conditions, from  $X = -\lambda/2$  to  $X = +\lambda/2$  and assuming when position of the particle is in the negative  $X$  axis, the value of  $r$  increases and  $dr/dx$  is positive. When the position of particle is in the positive direction,  $r$  decreases and  $dr/dx$  is negative. It is obvious that the amplitude of oscillation should increase and decrease symmetrically for the particle to travel as wave as in Fig5.  $R$  is the radius when the position co-ordinate is  $X=0$  and displacement  $x = \lambda/2$ .

In this model, quantum particle forms a standing wave like structure with maximum radius  $R$  and wavelength  $\lambda$ .

$$\text{When } x = \lambda, \quad |r| = \lambda (1/\pi + dr/dx)$$

$$\text{So, } dr/dx = 0 - R - R/0 / \lambda = -2R / \lambda.$$

$$r/x = 0/\lambda = 0.$$

$$\text{Therefore- } 2R / \lambda = -1/\pi.$$

Therefore,  $\lambda = 2\pi R$ . This is equal to the predicted radius for zitterbewegung.[16]

The equation can be written as,  $|r| = x(1/\pi + dr/dx)$ .

Here  $|r| = x/\pi + xdr/dx$ . The graph can be drawn as the following.

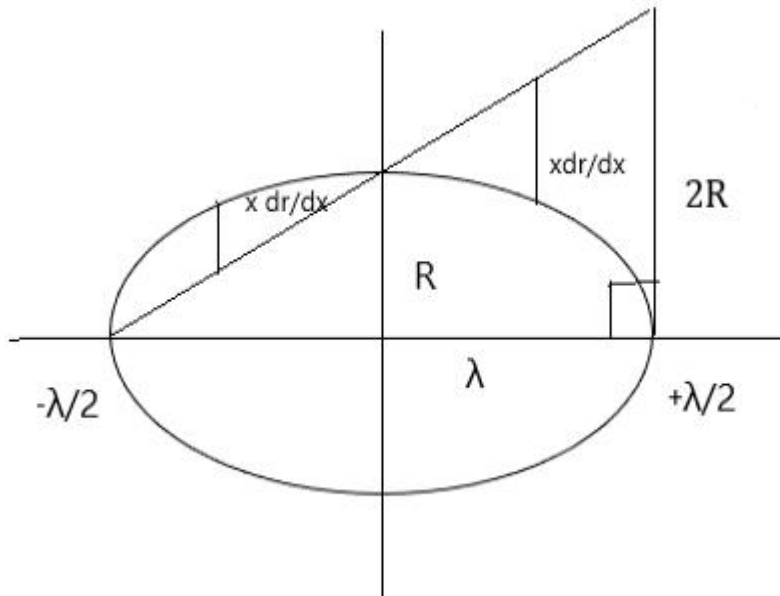


Fig 5. Outline formed by the quantum particle in one quantum

The equation can be written as  $r = r^1 + x dr/dx$ , (where  $r^1 = x/\pi$  and  $dr/dx = d\phi$  is the slope of the graph). The relation between the zitterbewegung and wave function can be proven using geometry (Fig3)

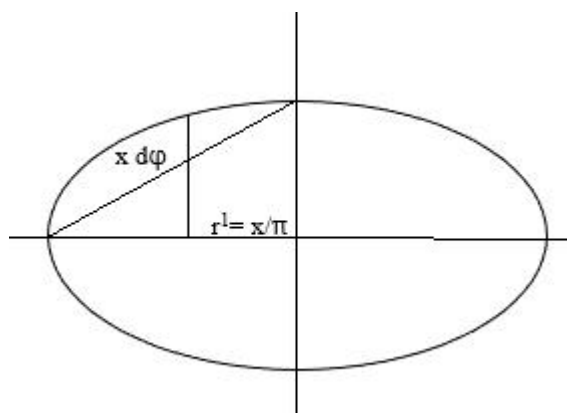


Fig 6. Relationship of one quantum with Cos wave

The slope of the graph,  $dr/dx = d\phi$ . It can be seen that the same right-angled triangle as in the graph can be drawn inside initial quarter of the cos wave  $y=1/2\cos x$ . The slope of the above graph and the Cos curve may be different. But all the other variables are the same for both the graphs. So, the difference between this graph and the Cos curve,  $y=1/2\cos x$  is the variable  $d\phi$ . In the graph drawn above,  $d\phi$  ( $dr/dx = d\phi$ ) represents the slope. If the difference of  $d\phi$  in the curves is zero, the graph is exactly the initial quarter of the Cos curve.

The equation of the graph can be written as  $|r| = x(1/\pi + dr/dx)$

Therefore when  $x = \lambda$ ,  $r=0$

$$d\phi = 1/\pi = 0.31$$

When  $r=R$ ,  $d\phi = 0$

So, the value of  $d\phi$  ranges from 0 to 0.3. The slopes of the half Cos curve ranges from 0 to 0.5. The difference of  $d\phi$  between the curves ranges from 0 to 0.19. Therefore, it is geometrically appropriate to approximate the difference of  $d\phi$  at each point of both the curves to zero. So one cycle of zitterbewegung can be approximated to non-dispersive wave function.

Therefore, the one quarter of the quantum is equivalent to the initial quarter part of  $y=1/2\cos x$ . In other words, the quantum may be considered as a superimposition of initial halves of two such Cos waves ( $y=1/2\cos x$ ), considering symmetry about Y axis. The upper half of quantum can be represented as part of the Cos wave,  $y=R \cos kx$  which is shown to

be equivalent to  $y=1/2 \text{ Cos } kx$ . Here,  $k$  is the wavenumber, which is equal to  $k = 2\pi / \lambda^1$ . It has to be noted that  $\lambda^1 = 2 \lambda$  where  $\lambda^1$  is the wavelength of Cos wave.

The standing wave is formed by superimposition of the Cos waves in opposite direction. The standing wave can be mathematically represented by the superposition of Cos waves as,  $\Psi(x) = R(\text{Cos}kx + \text{Cos}kx)$ . Both the waves have same frequency and amplitude. It has been found in the earlier part of this article that  $-\text{Cos}(kx - \omega t) = i\text{Sin}(kx - \omega t)$ . Hence,  $\Psi(x) = R(\text{Cos}kx + i\text{Sin}kx)$ . This is equivalent to  $\Psi(x) = R e^{ikx}$ . Therefore the wave function can be derived from the zitterbewegung using minor approximation.

The application of the above geometrical approximation is that a modified toroidal model can be derived from the concepts of wave function and zitterbewegung. The wave function represents the outline of zitterbewegung. The minor difference in the slopes may be the result of zitterbewegung occurring in three dimensional plane rather than two dimensional plane. The 360 degree precession of zitterbewegung gives rise to a modified toroidal shape and these concepts put together can explain the generation of electromagnetic wave in nuclear magnetic resonance. It means that the anapole magnetic moment changes its magnitude in a sinusoidal fashion and changes its direction continuously in the toroid. At the 180 degree precession, it completely reverses the direction. Therefore a modified toroidal model of proton can be proposed in nuclear magnetic resonance (NMR) and it may act as a mold for generation of radiofrequency wave.

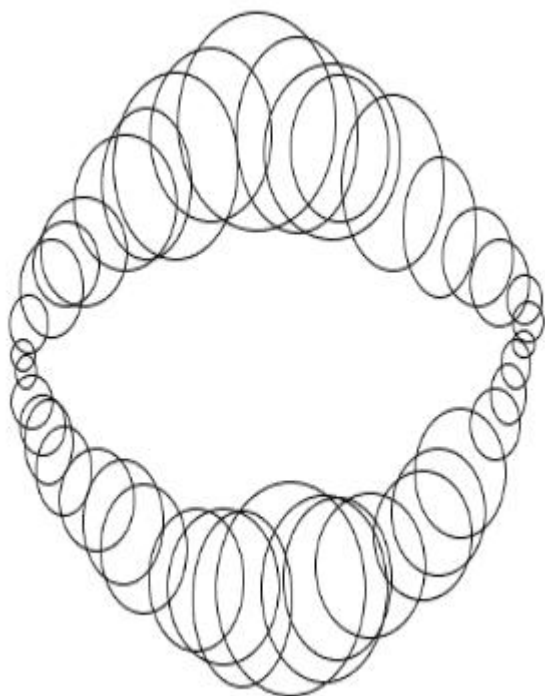


Fig 7. The Modified toroidal model of Proton in NMR

In this article, an equation for zitterbewegung was derived using hamiltonian. It was plotted in XY Cartesian plane using appropriate boundary conditions. The graph was shown to be similar to the non-dispersive wave function geometrically using minor approximation. The zitterbewegung and wavefunction represent the propagation of proton in single cartesian plane. The concepts of zitterbewegung and Larmor precession allow for derivation of a modified toroidal model for proton. The practical application of such a model is an alternate interpretation of nuclear magnetic resonance and an alternate explanation for generation of radiofrequency photons and the associated radiant energy. This concept can explain the unknown cause of burns during MRI imaging other than the usual mechanisms.

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