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Original Paper

Wigner's Friend Scenario, Born's Rule and an Alternative Formulation of Pilot Wave Theory

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Abstract: In this paper, we analyze the thought experiment of "Wigner's friend" and point out that new understanding should be made to Born's rule and measurement process: Born's rule is no longer seen as a rule based on the history of the quantum system's, and the measurement results are no longer directly related to the state of the measured object before the measurement. Inspired by Everett III and H. Zurek's views, we believe that Born's rule reflects the coordination between states of different parts in quantum entanglement systems, so it has nothing to do with the history of particles themselves but rather with the historical records. A new formulation of pilot wave theory, objective relative state formulation, or ORSF is suggested. Under this interpretation, micro-particles can also be assigned definite states before being observed. Based on this formulation, Wigner's friend-like scenarios can be effectively explained. We also notice that our universe can be totally retrocausal by the new formulation. The new interpretation brings new perspectives to many quantum phenomena.

Keywords: Interpretation of quantum mechanics, Born's rule, Observer independent facts, Pilot wave, Wigner's friend, Relative state

1. Introduction

In 1961, Wigner proposed a thought experiment [1], commonly known as "Wigner's Friend" scenario. In this scenario, there are two observers, W and F. Observer F, known as Wigner's friend, performs a measurement on a microscopic state, while Wigner himself, referred to as observer W, conducts a joint measurement on both the system comprising F and the microscopic state. The author argued that according to the quantum theory, the two observers will "have different realities". In fact, it was already pointed out in Everett III's paper in 1957 [2] that the presence of an observer within the observed system can potentially give rise to inconsistent outcomes when applying quantum theory. In recent years, researchers have integrated the conventional Wigner's friend thought experiment with Bell experiments, giving rise to a range of scenarios known as 'extended Wigner's friend scenarios (EWFS). A series of "no-go theorems" has been proposed to demonstrate that different observers may face different facts [3–6].

It is pointed out that according to quantum theory, the following three assumptions cannot hold simultaneously[6]:

(1) Absoluteness of Observed Events, AOE

(2) Locality (L).

(3) No-Superdeterminism (NSD).

The validation of these no-go theorems is achieved through multi-particle Bell tests, and the experimental results are in complete agreement with the predictions of quantum mechanics [5,6].

D. Frauchiger and R. Renner, on the other hand, proposed a thought experiment that suggested quantum mechanics cannot describe the use of itself consistently. This subsequently sparked a series of intense discussions [7]. However, the claims put forth by D. Frauchiger and R. Renner based on their thought experiment have also been met with some counterarguments and rebuttals. Some authors have pointed out potential logical loops in their reasoning[8,9].

Different assumptions listed above are rejected by different interpretations. If AOE is rejected, it can lead to a subjective interpretation, meaning that different observers can have different facts[2,10,11].

In this paper, we first return to the original thought experiment of "Wigner's Friend". As pointed out in the literature [12], the "observer-independent facts" that are manifested in EWFS are actually already present in the original thought experiment. Through simple analysis, it can be shown that the subjective interpretation does not completely resolve the contradictions in these thought experiments. We deem that what truly needs to be redefined is Born's rule, then we reinterpretate Born's rule. In this new interpretation of Born's rule, these thought experiments can be understood in a self-consistent manner. Based on the new understanding of the Born's rule, we suggest an alternative formulation of pilot wave theory of quantum mechanics. The new theory will brings a new perspective to many quantum phenomena.

This paper is organized as follows. In the second section, the original thought experiment of Wigner's Friend is carefully analyzed, and we identify the true reason for the "paradox." In the third section, the Born's rule is reinterpreted. Then a new formulation of pilot wave theory, objective relative state formulation is proposed. The fourth part discuss the contradiction with special relativity. The fifth section includes discussions on the significance of the new formulation, as well as the relation with other interpretations of quantum mechanics. The sixth section concludes the paper.

2. Analysis of the original thought experiment

Firstly, we consider a scenario similar to the original setting of Wigner[1]. There is a system S, an observer F, and a super-observer W. The initial state of S is:

$$|\psi\rangle_0 = \frac{1}{\sqrt{2}}(|0\rangle_S - |1\rangle_S),$$

where $\{|0\rangle, |1\rangle\}$ forms an orthonormal basis. Without further specification, we will adhere to this convention. For convenience, we call a projective measurement on this basis a Z measurement. Then F performs a measurement and obtains the following possible outcomes:

$$|\psi\rangle_{00} = |0\rangle_S, P = \frac{1}{2},$$
$$|\psi\rangle_{01} = |1\rangle_S, P = \frac{1}{2}.$$

Thus, for F, the probability of the composite system, which consists of him and the system, will be:

$$|\psi\rangle_{10} = |0\rangle_S|0\rangle_F, P = \frac{1}{2},$$
$$|\psi\rangle_{11} = |1\rangle_S|0\rangle_F.P = \frac{1}{2}.$$

$|0\rangle_F$ and $|1\rangle_F$ represent the states of F when he observes $|0\rangle_S$ and $|0\rangle_S$ respectively. But for the superobserver W after the interaction between S and F, what happens is:

$$|0\rangle_S - |1\rangle_S \to |0\rangle_S|0\rangle_F - |1\rangle_S|1\rangle_F.$$

Subjective interpretation considers it as evidence that they face different realities. So far there is no obvious contradiction between the results of W and F. Then F make an measurement on the joint system of S and F with following basis:

$$|\phi-\rangle = \frac{1}{\sqrt{2}}(|0\rangle_S|0\rangle_F - |1\rangle_S|1\rangle_F),$$

$$|\phi+\rangle = \frac{1}{\sqrt{2}}(|0\rangle_S|0\rangle_F + |1\rangle_S|1\rangle_F).$$

For F the joint system is either in $|0\rangle_S|0\rangle_F$ or $|1\rangle_S|1\rangle_F$ with equal possibilities. So, according to standard quantum mechanics, the probability distribution of the outcomes is:

$$P(\phi+) = P(\phi-) = \frac{1}{2}.$$

But for W the probability distribution is:

$$P(\phi+) = 0, P(\phi-) = 1.$$

This is further considered by proponents of subjective interpretation as evidence for observer-dependent facts. However, we argue that this actually suggests that subjective interpretation does not have a particular advantage in explaining the original thought experiment. One of the primary motivations behind the subjective interpretations is to ensure that all observers observe results that are consistent with their own application of quantum mechanics. However, the predictions of quantum mechanics and actual experiments clearly demonstrate that F's predictions are incorrect while W's predictions are correct. In fact, F may even realize that he has made incorrect predictions by applying standard quantum mechanics [13].

One could argue that the fact F making incorrect predictions can be seen as an illusion for W, as F still perceives their own predictions as correct. However, this argument seems to go too far. The establishment of physical laws are based on the ability to correctly obtain experimental results by communicating with others. If we were unable to do that successfully, it would be uncertain whether each person possesses their private laws of physics.

In fact, many subjective interpretations solve this problem by denying the observer's use of quantum mechanics for themselves. This further implies that this might be a problem of applying quantum mechanics rather than a problem of different perspectives. Hence, when addressing the original Wigner's friend scenario, the advantages of subjective interpretation may not be as evident as they initially appear.

3. Reinterpretation of Born's rule and a new formulation of pilot wave theory

3.1. Reinterpretation of Born's rule

From last section we can see that Born's rule is the problematic part. Actually, we have two seemingly consistent but actually different understandings of Born's rule.

Our first understanding of it is that it is a rule about the relationship between known states and their future measurement results, while the second understanding is about the measurement outcomes of a subsystem which belongs to an entangled system.

We deem that only the second understanding is the right one. Born's rule is meaningful only the measured system is a subsystem of a larger entangled system. The states before and after measurements are not directly related. So we make the following statement:

Statement 1: Born's rule is a rule that reflects the internal correlation of

dife=ferent parts within an entangled system.

It has been recognized by H.Zurek that the prediction of measurement outcomes based on Born's rule might be independent of the pre-measurement state[14,15]:

"Probabilities described by Born's rule quantify ignorance of the observer O before he or she finds out the measurement outcome. Therefore, envariant probabilities admit ignorance interpretation-O is ignorant of the future outcome (rather than of an unknown pre-existing real state, as was the case classically)."

According to the quote, it can be inferred that Zurek regards Born's rule as subjective. Statement 1 is mundane when it comes to a subjective interpretation. However, in this article we will try to propose a new formulation of pilot wave theory, which is an objective interpretation.

Since we are considering an objective interpretation, it is clear that we need a description of the observational instrument (which can also be seen as a description of the observer). In our view, the completion of observation does not mean the collapse of the entangled system, but rather the expansion of the entangled system. This means that we have descriptions of both the subsystem itself and the entire entangled system as two different levels, so the choice of pilot wave theory becomes inevitable.

3.2. The Bohm-Bell theory

The pilot wave theory typically suggests that, in addition to the exact state of the system, the pilot wave or the universal wavefunction has an influence on the system's evolution[16]. In this subsection we will briefly review the Bell version pilot wave theory[17].

John Bell proposed some new pilot wave theories that abandons the concept of particle trajectories put forward by de Broglie and Bohm, and instead considers it as a probabilistic theory [18]. In his 1984 version, it formulates as below [16]:

The evolution of quantum states is governed by a time-dependent state vector $|\Psi\rangle$, which satisfies the time dependent Schrödinger equation. The pilot vector can be written as linear combination of its components in the viable subspace S_i. And the real state is some vector $|\psi_i\rangle$, which can be either dependent of independent of time t. The transition probabilities w_{ij} is given by the Bell's postulate[19] [20]:

$$w_{ij} = \begin{cases} \frac{2Re[(i\hbar)^{-1}\langle\psi_i|H|\psi_j\rangle]}{\langle\psi_i|\psi_i\rangle}, & \text{if this is } \geq 0, \\ 0, & \text{if this is negative,} \end{cases}$$

where H is the Hamiltonian. Born's rule can be derived from the above formulation [19].

Although this formulation can reproduce all the prediction of quantum theory, we deem that this is still have two following shortcomings. Firstly, its physical interpretation is not very clear. Secondly, it is not convenient when discussing entangled systems. Furthermore, it is not easy to see its relationship with the standard form of quantum mechanics. Sometimes the "real Hamiltonian" need to be carefully constructed to match the prediction of standard quantum mechanics[20]. The formulation above also implies that the whole system may change spontaneously even when some subsystem is not experiencing an interaction.

3.3. A new pilot wave formulation

Here we suggest an alternative formulation of the pilot wave theory. Then new formulation is inspired by the "relative state formulation". In the relative state formulation interpretation, some observer will only perceive the Universe that is consistent with his measurement outcomes[2] [21].

The new formulation inherits the concepts of universal wave function and relative state[2] form H. Everett. However, we aim to propose an objective interpretation, implying that we strive to harmonize the "Universes" of all observers as much as possible. We jettison the unnecessary notions such as observers in other branches. This is also the aim of Bell's work. So we would like to call it "objective relative state formulation", or ORSF.

This new formulation of pilot wave theory has actually been implicitly used. When we consider a pair of entangled photons, supporters of the pilot-wave theory sometimes say that the measurement result of one photon can affect the other. This is the simplest scenario of the new formulation. However, this new formulation has not been systematically summarized in previous researches. The above description is obviously seen as potentially violating Lorentz symmetry, a problem that we will discuss later in the article.

In order to build the new formulation, we first apply the inertial law of quantum states. The law of inertial is:

Inertial Law: An quantum state at rest tends to stay at rest, and an object in motion tends to stay in motion with the same speed and in the same direction, unless there is an interaction.

Before reaching the interaction law, we first propose the right Born's rule after reinterpretation. By applying the notion of "relative state", we deem that the new definition of Born's rule is as follows:

Definition 1: Given that the state of a subsystem of some entangled system is $|\alpha\rangle$, it is projected to the $|A\rangle$ state. It is also known that the rest part of the entangled system

is in state $|\theta\rangle$, and $|\tau\rangle$ is the relative state of $|\theta\rangle$, then the probability of obtaining $|A\rangle$ as the measurement result is $|\langle\tau|A\rangle|^2$.

It can be easily seen by the definition above one quantum state will not change unless there is an interaction. Traditionally, we consider measurement as a process of acquiring inherent information about a physical entity. However, in quantum theory, measuring the y-direction spin of an electron will inevitably change the state of the electron that was initially in the z-direction spin eigenstate. We must regard measurement as a process that can have a significant impact on the entity being measured.

We view the measurement process as a process in which a microscopic system and a measuring instrument undergo some kind of coupling. The microscopic system's state after this coupling may not necessarily be the state before. We should keep in mind that quantum theory is based on measurement outcomes rather than on the "true state" that we might expect. We have reason to believe that there is consistency between the state after the measurement of a microsystem and the reading of the instrument. In the following text, we will consider this as a characteristic of the measurement process and our subsequent discussions will be based on it.

Now we use an example to show how to correctly apply Born's rule with the new formulation. Notice here the entangled system involve all observers(or apparatus), so we will not concern about the reduced density matrix in the following demonstrations.

0 and 1 represent an orthonormal basis as before. First we consider a system with two parts, A and B:

$$|H\rangle = \frac{1}{\sqrt{3}}\left(|0\rangle_A|0\rangle_B + |0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B\right). \tag{1}$$

It can be seen that the probability of A being in the $|0\rangle$ state is two-thirds. Then we import the following $\{|\theta+\rangle, |\theta-\rangle\}$ basis:

$$|0\rangle = (\cos\theta|\theta+\rangle + \sin\theta|\theta-\rangle), |1\rangle = (\sin\theta|\theta+\rangle - \cos\theta|\theta-\rangle).$$

Now we rewrite Eq.(1) as:

$$\begin{aligned} |H\rangle = \frac{1}{\sqrt{3}}(&|0\rangle_A(\cos\theta|\theta+\rangle_B + \sin\theta|\theta-\rangle_B) \\ &+|0\rangle_A(\sin\theta|\theta+\rangle_B - \cos\theta|\theta-\rangle_B) \\ &+|1\rangle_A(\cos\theta|\theta+\rangle_B + \sin\theta|\theta-\rangle_B)), \end{aligned} \tag{2}$$

Here we assume that after the measurement the "pointer" of the apparatus and the state of the observed system will be consistent. Then we use an apparatus D to measure B with θ

basis, which leads to:

$$
\begin{aligned}
|H\rangle = \frac{1}{\sqrt{3}}(&|0\rangle_A(\cos\theta|\theta+\rangle_B|\theta+\rangle_D + \sin\theta|\theta-\rangle_B|\theta-\rangle_D)\\
&+|0\rangle_A(\sin\theta|\theta+\rangle_B|\theta+\rangle_D - \cos\theta|\theta-\rangle_B|\theta-\rangle_D)\\
&+|1\rangle_A(\cos\theta|\theta+\rangle_B|\theta+\rangle_D + \sin\theta|\theta-\rangle_B|\theta-\rangle_D)),
\end{aligned} \tag{3}
$$

and it can be rewritten as:

$$
\begin{aligned}
|H\rangle = \frac{1}{\sqrt{3}}(&|0\rangle_A(\cos\theta|\theta+\rangle_{BD} + \sin\theta|\theta-\rangle_{BD})\\
&+|0\rangle_A(\sin\theta|\theta+\rangle_{BD} - \cos\theta|\theta-\rangle_{BD})\\
&+|1\rangle_A(\cos\theta|\theta+\rangle_{BD} + \sin\theta|\theta-\rangle_{BD})).
\end{aligned} \tag{4}
$$

Notice that Eq.(2) and Eq.(4) represent the whole entangled system, or the "pilot wave" in Bohmian mechanics[20], not product of the exact states of subsystems. Here comes the key point. Assume that the exact state is:

$$|0\rangle_A \otimes |0\rangle_B,$$

and we're going to make a projection on the $|\theta+\rangle$ and $|\theta-\rangle$ basis. Traditionally, we will think that the result will be:

$$|0\rangle_A \otimes (\cos\theta|\theta+\rangle_{BD} + \sin\theta|\theta-\rangle_{BD}).$$

However, according to definition 1, the outcome is not necessarily required to be as such. The reinterpreted Born's rule only need to keep all the states of the subsystems in the whole system consistent after the measurement. The result has no relation with the precious state $|0\rangle_B$.

The probability of A being in the $|0\rangle$ state remains unchanged. In order to predict the result of BD, our calculation will based on the rest parts of the entangled system, i.e., subsystem A. Now $|0\rangle_A$ is the exact state of A. If we consider the state of $|0\rangle_A$ by Eq.(1), we will find that its "relative state" is:

$$\frac{1}{\sqrt{3}}((\sin\theta|\theta+\rangle_{BD} - \cos\theta|\theta-\rangle_{BD}) + (\cos\theta|\theta+\rangle_{BD} + \sin\theta|\theta-\rangle_{BD})).$$

Notice that it is not just one of the two parts: $(\sin\theta|\theta+\rangle_{BD} - \cos\theta|\theta-\rangle_{BD})$ or $(\sin\theta|\theta+\rangle_{BD} - \cos\theta|\theta-\rangle_{BD})$, because we need to put them together. We need to consider the whole entangled system. So the probability distribution is:

$$P(BD = \theta+, A = 0) = \frac{1}{3}(\cos\theta + \sin\theta)^2,$$

$$P(BD = \theta-, A = 0) = \frac{1}{3}(\cos\theta - \sin\theta)^2.$$

And the conditional probability is :

$$P(BD = \theta + |A = 0) = \frac{1}{2}(\cos\theta + \sin\theta)^2,$$

$$P(BD = \theta - |A = 0) = \frac{1}{2}(\cos\theta - \sin\theta)^2.$$

The factor one-half can be seen as coming from the two normalized relative states of $|0\rangle_A$:$\sin\theta|\theta+\rangle_{BD} + \cos\theta|\theta-\rangle_{BD})$ and $\sin\theta|\theta+\rangle_{BD} - \cos\theta|\theta-\rangle_{BD}$, instead of just one. Obviously we have:

$$P(BD = \theta\pm, A = 0) = P(A = 0)P(BD = \theta \pm |A = 0).$$

This implies that a joint probability distribution do exist. It also can be seen in the above procedure local tomography will not be violated. Please note once again that equation Eq.(1) serves as a universal description for all components of the system, or the shared facts. And it does not belong exclusively to some particular observer's knowledge.

One question is how to understand the tremendous success of the conventional usage of Born's rule. This answer is simple: the prediction is not based on the history of some system, it is based on its historical records which are also quantum states. In most cases, the records of quantum systems are in "classical" states. In this scenario, the usage of Born's rule based on the history of the particles, does not lead to any problems. In the literature[22] it has been pointed out by A. Sudbery:

"If a system is linked to a memory which keeps a permanent records of a set B of a basis states of the system, then the probabilities which will be observed in the memory are the same as those which would be calculated in Copenhagen calculation that the system(without the memory)undergoes collapse at each time state onto a state of the basis B."

However the Wigner's friends scenario is a special case. For convenience the normalization factor is omitted. It can seen that the structure of the whole entangled system for S and F is:

$$|\psi\rangle_0 = |0\rangle_S|0\rangle_F - |1\rangle_S|1\rangle_F. \tag{5}$$

After the measurement of W, the new system is:

$$|\psi\rangle_0 = (|0\rangle_S|0\rangle_F - |1\rangle_S|1\rangle_F)|\phi-\rangle_W = |\phi-\rangle_{SF}|\phi-\rangle_W. \tag{6}$$

Since all the subsystems take part in the measurement, they all will be updated. There is no subsystem left that can be used to predict future result. So we need go to the "upstream" of the universal wavefunction. Usually it will have the following form:

$$|\Psi\rangle = |\psi\rangle_{0R}|\psi\rangle_0 + |\psi\rangle_{1R}|\psi\rangle_1 + ...$$

$|\psi\rangle_{iR}$s are the historical records of $|\psi\rangle_i$s. The rest parts of the system is $|\psi\rangle_{0R}$, whose relative state is $|\psi\rangle_0$, then we make an projection on the $\{|\phi+\rangle_{SF}|\phi+\rangle_W$, $|\phi-\rangle_{SF}|\phi-\rangle_W\}$basis. Then we have:

$$P(|\phi-\rangle_{SF}|\phi-\rangle_W) = 1,$$

$$P(|\phi+\rangle_{SF}|\phi+\rangle_W) = 0.$$

So the result of F and W are totally independent:

$$P(\phi \pm |0) = P(\phi\pm)P(|0\rangle_S), P(\phi \pm |1) = P(\phi\pm)P(|1\rangle_S).$$

To solve the contradiction in Wigner's friend scenario, modifying the conditional possibility is a reasonable choice[23] [24]. We have shown that for the following micro system $|\psi\rangle_0$:

$$|\psi\rangle_0 = \frac{1}{\sqrt{2}}(|0\rangle_S - |1\rangle_S),$$

we can obtain the following conditional probabilities:

$$P(\phi + |0) = P(\phi + |1) = 0.$$

The result obtained by Wigner through measurement is represented by $\phi+$ and $\phi-$ and the result obtained by F through measurement is represented by 0 and 1. For a different $|\psi\rangle_0$,

$$|\psi\rangle_0 = \frac{1}{\sqrt{2}}(|0\rangle_S + |1\rangle_S),$$

we will find that:

$$P(\phi - |0) = P(\phi - |1) = 0.$$

This is obviously a contradiction. However, the contradiction is from that we implicitly assume the conditional probability mentioned above is independent of $|\psi\rangle_0$, and is independent of measurement settings of Wigner.

If we assume that future choices cannot influence current outcomes, we need to adjust the conditional probability to the form of $P(\phi - |0, |\psi\rangle_0)$, which means that the conditional probability will depend on the state of the microsystem before F performs the measurement.

We can find many acceptable conditional probabilities, but most of them lack clear physical meanings. While ORSF presents a clear physical picture: subsystems that do not interact will not change and remain in a definite state. For subsystems that do interact, the probability distribution of the interaction's outcomes logically depends on the internal coherence of the entire system after the interaction has ended. This coherence is referenced to the universal wavefunction.

3.4. Universal wave function

When the universal function of the pilot wave is used, this means that we need to consider the entire history. We not only need to consider the history that has occurred, but also the history that could have occurred but did not. This is precisely what unitary evolution requires. In the worst cases, we need to consider from the Big Bang. However, in most cases, things are not that bad. Now let's take a look at the following example:

$$|\psi\rangle_{t0} = |A_0\rangle_X |B_0\rangle_Y - |C_0\rangle_X |D_0\rangle_Y. \tag{7}$$

In the equation, X and Y represent two different systems. We assume that the states B_0 and D_0 are "classical" states. and when studying the Y system we only consider it in one of the classical states. If A_0 and C_0 are orthogonal, the exact state of Y system, namely either B_0 or D_0, can be studied independently regardless of the universal wavefunction. If A_0 and C_0 are not orthogonal at the beginning, after a sufficient amount of time, the X system will undergo decoherence and evolve into a new state.:

$$|\psi\rangle_{t1} = |A_1\rangle_{X'} |B_0\rangle_Y - |C_1\rangle_{X'} |D_0\rangle_Y. \tag{8}$$

Due to decoherence involving other systems, X' is larger than X. Usually the different states of X' are orthogonal.

The procedure above clearly shows how we obtain an "entangled system", or a "effective pilot wave" from the pilot wave of the whole universe, which is the only one real pilot wave.

In the new formulation, the entanglement structure should contain all subsystems involved in the universal wavefunction. For a definite time, every entity has a definite state. Probability prediction is based on the entanglement structure and the state all subsystems except for the system will be updating in the interaction. There might be a concern that if the measurement result of a particle's position is independent of its previous position, it could lead to the "jumping" of the particle's position. However, the thought experiment of EPR demonstrates that the inherent properties of entangled systems determine that the outcome of such "jumping" is constrained by the entanglement itself. Similarly, various conservation laws in nature would also keep the measurement results of different physical quantities of particles within a limited range.

3.5. General interaction

The new formulation can be directly extended from the measurement process to general interactions. Consider that we have two entangled system:

$$|\psi\rangle_A = \sum_i |AR_i\rangle_{AR} |A_i\rangle_A,$$

$$|\psi\rangle_B = \sum_j |BR_j\rangle_{BR}|B_j\rangle_B.$$

For convenience we take all states to be othorgonal. AR and BR can be seen as historical records. After some interaction between A and B, the composite system will be:

$$|\psi\rangle_{AB} = \sum_i \sum_j |AR_i\rangle_{AR}|BR_j\rangle_{BR}\hat{U}(|A_i\rangle_A|B_j\rangle_B), \tag{9}$$

where $\hat{U}$ is the evolution operator. Notice again that the equations above are all universal wavefunctions, or pilot waves, not the exact states of the systems. Obviously, when the records are just macroscopic states, there is nothing new we can learn from the new interpretation comparing with the standard quantum theory.

4. On the contradiction with special relativity

4.1. Causal order

If we only consider measurements with time-like intervals, everything is ok. However, when we consider space-like intervals, contradictions will arise.

First we consider the an entangled electron spin system AB:

$$|\psi\rangle_0 = \frac{1}{\sqrt{2}}(|0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B). \tag{10}$$

Now we use apparatus C and D to measure A and B in different directions. It can be seen that this is essentially what happens in a Bell measurement with respect to an electron singlet[25]. Although they lead to the same joint probability distribution, different measurement orders can have different probability distributions for some intermediate states.

Now we consider two space-like separated measurements, performed by two apparatus C and D. They're measuring A with $\{|0\rangle, |1\rangle\}$ basis and B with $\{|+\rangle, |-\rangle\}$ basis. The $\{|+\rangle, |-\rangle\}$ basis is defined by the following relation:

$$|0\rangle = \frac{\sqrt{2}}{2}|+\rangle + \frac{\sqrt{2}}{2}|-\rangle, |1\rangle = \frac{\sqrt{2}}{2}|+\rangle - \frac{\sqrt{2}}{2}|-\rangle.$$

If we consider the measurement on A performed first, the state will be:

$$|\psi\rangle_0 = \frac{1}{\sqrt{2}}(|0\rangle_{AC}|1\rangle_B - |1\rangle_{AC}|0\rangle_B). \tag{11}$$

Then we make a measurement on B, and the universal wave function will be:

$$|\psi\rangle_0 = \frac{1}{\sqrt{2}}(|0\rangle_{AC}\left(\frac{\sqrt{2}}{2}|+\rangle_{BD} - \frac{\sqrt{2}}{2}|-\rangle_{BD}\right) - |1\rangle_{AC}\left(\frac{\sqrt{2}}{2}|+\rangle_{BD} + \frac{\sqrt{2}}{2}|-\rangle_{BD}\right). \tag{12}$$

If the exact state before measurement is $|0\rangle_A|1\rangle_B$ and the measurement order is described Eq.(11) and Eq.(12), there is only intermediate state of $|0\rangle_A$ and there will be no $|1\rangle_A$ state as intermediate state or final outcomes. However, if we consider the measurement performed on B happens first, the universal wave function will be:

$$|\psi\rangle_0 = \frac{1}{\sqrt{2}}(|0\rangle_A \left(\frac{\sqrt{2}}{2}|+\rangle_{BD} - \frac{\sqrt{2}}{2}|-\rangle_{BD}\right) - |1\rangle_A \left(\frac{\sqrt{2}}{2}|+\rangle_{BD} + \frac{\sqrt{2}}{2}|-\rangle_{BD}\right). \quad (13)$$

Then it's followed by a measurement performed on A. After measurement on B, the state of B is either $|+\rangle_B$ or $|-\rangle_B$, neither of whose relative state is $|0\rangle_A$. So the probability we obtain $|1\rangle_A$ is not zero. The contradiction between realistic quantum theory and Lorentz symmetry has already been noticed in literature [26].

But this problem is not as serious as it seems to be. The easiest way to solve this problem is to assume there exist a special time order that doesn't violate special relativity for every time-like event pairs. We can even appoint a definite inertial reference frame which determines the causal structure of all events. There won't be any observable effects. Any records of previous states which can be used to compare with current states will influence the measurement outcomes. For example, If there is a record of A, which we denote as AR, the state will be:

$$|\psi\rangle_0 = \frac{1}{\sqrt{2}}(|0\rangle_{AR}|0\rangle_A|1\rangle_B - |1\rangle_{AR}|1\rangle_A|0\rangle_B). \quad (14)$$

Therefore the Eq.(13) will become:

$$|\psi\rangle_0 = \frac{1}{\sqrt{2}}(|0\rangle_{AR}|0\rangle_A \left(\frac{\sqrt{2}}{2}|+\rangle_{BD} - \frac{\sqrt{2}}{2}|-\rangle_{BD}\right) - |1\rangle_{AR}|1\rangle_A \left(\frac{\sqrt{2}}{2}|+\rangle_{BD} + \frac{\sqrt{2}}{2}|-\rangle_{BD}\right). \quad (15)$$

Any measurements performed on A will only obtain the result of $|0\rangle_A$.

We also have many other ways to solve this problem. For two or more different order choices, we can consider that both are equally likely to occur. But these two situations would not be considered to happen simultaneously since we're considering an objective interpretation. Although it appears to be retrocausal, we should remember that quantum correlation is not the kind of causality in special relativity. The "future measurement event" predicted by A or B is outside its light cone.

Another choice is to deem that neither time order will occur. So we make another assumption here: The results of two space-like measurements is independent of both the states of the two measured systems before the measurement. It means that the results of BD will not depend on neither A or B. AC will not either. But the result of AC and BD will be correlated. Obviously it can be generalized to three or more measurements with space-like intervals.

Perhaps we can even use new ways to describe particle spin in relativity, such as considering the predictions of current quantum theory is valid only when measurement devices are almost at rest with respect to the particles, and then modifying existing theories to make the predicted probabilities consistent across different reference frames. It has already been known that boost will influence the pattern of entanglement[33]. Considering that the superposition states of particles may be related to some internal degrees of freedom which is independent of space and time, for example electric charge, this could imply a "larger unified theory," such as supersymmetry.

Nevertheless, we regard this as an open question. Although there is currently no completely satisfactory answer, there are some admissible answers.

This contradiction seems to be consistent with the expectations of subjective interpretation. But rather than being an appearance of observer-dependent facts, it can be seen as a result of frame-dependent facts. Symmetry is indeed another interesting issue, and it can potentially lead to wrong conclusions when inferring the entanglement of macroscopic systems based on the results of an electron singlet for their different symmetry.

Regardless of which viewpoint is adopted, it will not yield observable different results for current experiments, which is different from the situation stated in the literature[27]. The problems of EWFS then can be solved. For the three assumptions mentioned in the introduction, the locality is violated. The results can be generalized to larger systems.

Since the interpretation of quantum mechanics is based on personal preference, one can still accept the new definition of Born's rule and reject ORSF. If absolute reality is rejected, we can still use new Born's rule to study how classical reality emerge from quantum world.

4.2. Retrocausality

In the previous subsection, we only consider the causal order that will not violate the special relativistic causal structure. Now we can consider a more interesting case, what will happen the current events can be influenced by future events.

We can still focus on the following system:

$$|\psi\rangle_0 = \frac{1}{\sqrt{2}}(|0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B). \tag{16}$$

Now, let's consider a sequence of measurements: the first performed on system A using apparatus C, the second performed on system B using apparatus D, and finally, a measurement performed on the composite system AC using apparatus E.

Obviously, if the outcomes of C is determined by the future outcomes from E, the joint probability distribution of the outcomes from C and D will violate the prediction from standard quantum theory. Based on this violation, surely we can predict the future. We will know that there will be a future measurement on the composite system AC.

But this foresight will not happen for all practice purpose. Once an outcome is obtained by an apparatus, a joint measurement is required that includes the apparatus, the system, and all the environmental particles that have interacted with them. This presents from a different perspective why a large number of copies in quantum Darwinism form a reality.

This demonstrates that a quantum correlation should not necessarily be interpreted as a causal structure that follows the principles of special relativistic causality. It is possible that we possess a greater degree of freedom than we expect when it comes to selecting the causal structure within spacetime. This freedom has already been noticed in the standard Bohmian mechanics [28] [29], in which the particles has exact trajectories, while we are considering a probabilistic pilot wave theory.

It seems that it still doesn't exclude the situation where we can witness this retrocausality. If after the measurement the system and the apparatus undergo a very strong interaction that leads to measurement outcomes that are completely indistinguishable, the non-standard quantum correlation will be observed. The previous research has already observed that the modified Born's rule might be used to predict future probabilities [12], but further discussions were not conducted.

However, the complex human nervous system may prevent the above scenario from occurring. The process of human information acquisition involves a large amount of interaction and accompanying "historical records," which ultimately leads to the effect of Born's rule conforming to the standard quantum theory's description of it. Note that this process is accompanied by decoherence, but the two are not fully equivalent. The above discussion can also be seen as a related understanding of the view that "consciousness causes the wave function to collapse."

4.3. The manufactured measurement outcomes

One may also argue that we cannot assign a definite state to a subsystem of a singlet, because there are infinite many different ways to rewrite the wavefunction of the singlet. However, that is not the main issue we are currently concerned about. Having many options means that any one of them is feasible; it's a matter of degree of freedom. We are going from no choice to infinite choices. It can be shown that there is no contradiction when selecting different basis states for A and B.

In fact, we can find that the exact states of particle A and B of a singlet before measurement even can be $|0\rangle_A$ and $(|0\rangle + |1\rangle)_B$. By the new formulation, the measurement outcomes still can reproduce the predictions of standard quantum mechanics. We believe that the strong correlation observed in the measurement results may actually be an artifact created by the measuring instrument itself.

We propose an interesting conjecture that departs from standard quantum mechanics, suggesting that the correlation of measurement results actually retains a small amount of information about the states of the two particles before measurement. Please note that this conjecture is distinct from the claims made in other parts of this paper. As an example, for the state:

$$|\alpha\rangle = |0\rangle_A |0\rangle_B + |1\rangle_A |1\rangle_B,$$

we may speculate the existence of a relationship in the following form:

$${}_A\langle A_0|B_0\rangle_B = {}_A\langle A_1|B_1\rangle_B,$$

or something like:

$$f({}_A\langle A_0|B_0\rangle_B) = g({}_A\langle A_1|B_1\rangle_B),$$

where A_0 and B_0 are states of A and B before and A_1 and B_1 are the composite systems of the particle and the apparatus after the measurement. Theoretically, this can be observed if the conjecture is true.

5. The significance of the new Born rule and ORSF

In classical physics, measurements generally accurately reflect the properties of the object being measured prior to the measurement. This characteristic has been partially carried over to quantum physics. When measuring the eigenstates corresponding to certain observables with the appropriate basis, a unique and determinate result can still be obtained. However, the other aspect of quantum physics now deserves more attention: quantum measurements can significantly alter the state of the measured system. For instance, a state that is close but not equal to $|0\rangle$ can completely turn into $|1\rangle$, which is a state orthogonal to $|0\rangle$, after a measurement. The quantum Zeno effect further demonstrates the profound impact of measurement on quantum systems[30].

In the framework of ORSF, the measurement process is considered as the interaction between an observer and an existing entangled system, resulting in the formation of a new entangled system. During this process, both the observer and the directly measured subsystem will undergo changes, based on the structure of the universal wavefunction and the other subsystems except for the system undergoing the change. In the reference [31], it is also pointed out through a thought experiment that it may not be appropriate to predict the measurement outcomes based on current states. The conclusion of this article is similar, but we deem that in order to predict the measurement result of system A, one must invoke other parts entangled with A and the universal wavefunction. The results of Born's rule actually reflect the correlation between different parts of an entangled system. Every time a new measurement takes place, Born's rule strives to ensure that the subsystem undergoing changes in the newly formed system is as coordinated as possible with the other parts.

According to the new formulation of pilot wave theory, the measurement is no longer assumed to be based on the previous state of the measured object. A series of discussions based on quantum theory that denies the existence of definite results before measurement cannot prove the non-existence of a definite quantum state before measurement [32].

Similarly, in a Bell experiment, the two particles can also have definite spin directions before measurement.

The new interpretation, as a pilot wave theory, is not entirely the same as the traditional Bohmian mechanics, which also advocates a single-world interpretation [8]. Bohmian mechanics provides completely deterministic predictions for measurement outcomes, whereas the new interpretation still retains the probabilistic nature of measurement results in quantum theory. It is also different from the form proposed by J.Bell and applied by A.Sudbery[17] [20].

Indeed, it can be observed that the new interpretation also shares some similarities with the Existential Interpretation proposed by H. Zurek, which is also based on entanglement [15]. H. Zurek has pointed out that the emergence of classical objectivity comes from the proliferation of copies of the same information [15], and the new interpretation is in complete agreement with this viewpoint. However, there are interesting differences in their attitudes towards the physical world. The Existential Interpretation still uses Everett's "branching" concept and considers only the shared observed reality as the "objective reality". On the other hand, the new interpretation proposes the possibility that even microscopic particles can have definite states, which can be considered objective.

It appears that our results are similar to those in the literature [23], but they are not. The formulas in the literature still use the traditional state updating rule implicitly, which means that for a state A and an apparatus C, the measurement process can always be described as:

$$|0\rangle_A|R\rangle_C \to |0\rangle_A|0\rangle_C,$$

where $|R\rangle_C$ denotes the state of the apparatus before the measurement. However, this is not always true in the new interpretation. As mentioned in the above sections, the state will change according to the relative states of other parts in the whole entangled system.

Although in most cases, the old and new Born rules will yield identical conclusions, there can still be differences in certain special circumstances. Since in section 4 our discussion is based on the special relativity, there might be some observable differences in some non-trivial spacetime structures.

Clearly, the objective relative state interpretation of quantum mechanics, which is a quantum interpretation, has parts that cannot be verified experimentally. Supporters of other quantum interpretations may not support the pilot wave theory, or may hold viewpoints that are relational or perspectival about quantum reality. Nevertheless, the objective relative state interpretation may still contribute to deepening their understanding of subjective quantum interpretations. Understanding the "records of history" in the new Born rule makes it clearer to understand the difference between stable and unstable facts in relational quantum mechanics. The new Born rule also allows observers to apply quantum Bayesianism to themselves.

6. Conclusion

In this paper, we discuss the scenario of "Wigner's friend" and point out its fundamental contradiction is related to Born's rule. We combine Everett's relative state theory with H. Zurek's viewpoint of the Born's rule based on quantum entanglement and then propose a correct understanding and usage of the Born's rule to resolve the contradictions arising from the usage of quantum mechanics in scenarios like "Wigner's friend". Thus, reinterpreting the measurement process and Born's rule may be a truly effective approach. We suggest a new formulation of the pilot wave theory, objective relative state formulation. In the new formulation, each microscopic particle can be considered to have a definite state before being measured. The new form, compared to the traditional formulation, exhibits a clearer and more concise logical structure, and it is closely related to the standard form of quantum mechanics.

In traditional physical theories, measurement outcomes are directly related to the object being measured. However, based on the new interpretation, we clarify that the observation result is not directly related to the previous state of the observed object. We also point out that the Born's rule only makes sense within entangled systems. To make the new Born rule universal, we need a pilot wave theory, which is the objective relative state formulation. Its logical structure is more clear and straightforward and it has a closer connection with standard quantum mechanics, comparing with other pilot wave theories.

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