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Does Bohmian Mechanics Explain the Prototype of Quantum Measurement: Stern-Gerlach?

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Abstract: A recent reassessment of the Stern-Gerlach experiment of one hundred years ago demonstrates that the wavefunction of the transiting atom does not develop continuously through that magnet. That understanding can correct the Bohm-deBroglie description which claims a unique, continuous, and deterministic wavefunction evolution.

Keywords: Stern-Gerlach experiment, Bohmian Mechanics, quantum measurement, quantum wavefunction reduction, quantum energy exchange

1. Introduction

The recent understanding of the quantum measurement process within the Stern-Gerlach (S-G) experiment [1] can lead to illuminating insights into the foundations of quantum mechanics. The Bohm-deBroglie (B-deB) interpretation of quantum mechanics [2] also called Bohmian Mechanics, which claims to be entirely deterministic [3], and has received a great deal of current interest [4], can be tested by our new knowledge of the prototypical S-G measurement process.

2. The Stern-Gerlach device

The Schrodinger equation which prescribes the continuous time evolution of a quantum wavefunction is valid only for a closed physical system. Some of the most astute physicists who helped develop quantum mechanics have made that clear. According to von Neumann, "…the time dependent Schrodinger differential equation….describes how the system changes continuously and causally in the course of time, if its total energy is known." [5]. Eugene Wigner reiterated, "In quantum mechanics, as in classical physics, we postulate the existence of *isolated systems.* In both theories, if a complete description of an isolated system is given at one time, a complete description for any other time is uniquely determined as long as the system remains isolated – i.e. is not influenced by any other system. In this sense, both systems are deterministic." [6].

A few years ago, David Weinland, the Nobel laureate, and his colleagues reported macroscopic quantum jumps seen as intermittent fluorescence of trapped atomic ions [7]. The time required for the energy transition, they found, is smaller than can be measured with the best instruments available. Those experimentalists emphasized that the process is as nearly instantaneous as can be determined today; not a continuous one. If a quantum of energy, such as a photon, could be transferred as contiguous, partial pieces over some tiny time interval, it would not be a discrete quantum.

But, the accepted explanation of atomic wavefunction development within a Stern-Gerlach magnet, found in our quantum mechanics textbooks [8 – 10], describes continuous development of a spin-direction superposition throughout the magnet. There must, however, be energy quanta from the magnetic field transferred to a neutral, spin one-half atom, like silver, which kick the atom in a direction transverse to the incident beam. Observation shows that the atom gains that kinetic energy, either one way, or the opposite, from within the magnet, where the magnetic field is contained, not later when the atom reaches a downstream detector. So, evolution of the atom's wavefunction cannot be continuous throughout the S-G magnet.

Absorption of the first magnetic field quantum by the atom is a discontinuous process which immediately reduces the atomic wavefunction superposition to a single spin-direction eigenfunction. Subsequent transfers of innumerable field quanta continue to kick the atom in that same direction as it transits the magnet.

Moreover, consider the potential provided to the atom by a S-G magnet. The magnetic field is stable in time, so the potential, too, has no time dependence. Schrodinger's equation for the atomic wavefunction yields a separable solution [11]. That wavefunction has two factors, one dependent only on time, the other a function of position only. Each factor must equal the same constant, E, the total atomic energy. So, continuous Schrodinger evolution of that wavefunction exists only so long as the total energy is constant. That energy, however, iteratively increases with each quantum transfer, and the atom's wavefunction travels just one path, not a quantum superposition, through the magnet.

Long ago, David Bohm, himself, writing in his textbook, suggested that momentum transfer between the S-G magnet and the transiting atom would measure the direction of the spin of the atom. He wrote, "Thus, it would be possible in principle to measure the spin by measuring the momentum transmitted to the particle by the magnetic field." [12] Such measurement would, of course, immediately terminate a spin-direction superposition.

By contrast, interference phenomena, as a single photon through two narrow slits, or a photon moving through a Mach-Zehnder device, demonstrate a continuous quantum superposition of that object's position, without energy interchange, until detection. I have recently suggested a very simple and practical experiment to confirm wavefunction reduction, via energy exchange, in the S-G device [13].

Though it is widely recognized by quantum researchers that Schrodinger's equation is only valid for closed systems, most do not seem to realize that any discrete, quantized energy exchange must discontinuously disrupt Schrodinger evolution. Of the dozen, and more, current interpretations of the quantum measurement process, the majority do not predict a discontinuous wavefunction reduction at measurement.

Some, however, do; resulting from disparate events. Bohr and his adherents supposed that registration of the observation by a 'macroscopic' device would immediately collapse the quantum wavefunction [14]. Von Neumann and Wigner believed human conscious awareness of an empirical result would reduce the quantum wavefunction to a single eigenvalue [15]. More recently, the theory of Girardi, Rimini, and Weber [16] predicts a random, spontaneous wavefunction jump in both position and time at measurement. And the transactional interpretation of Cramer and Kastner [17], which treats the complex conjugate of the wavefunction as a wave traveling backward in time, also supposes immediate wavefunction collapse. One of the theories that does not predict a discontinuous wavefunction collapse is Bohmian mechanics.

3. Bohmian mechanics

We may compare the explanation of the Stern-Gerlach experiment, given in the Bohm-deBroglie interpretation, with the realistic understanding of immediate measurement inside the magnet. Bohmian Mechanics describes physical objects, including ones with mass, like silver atoms, as point particles moving on unique, continuous trajectories determined by the quantum wavefunction and an auxiliary 'guiding wave equation'. One of the prominent advocates for the B-deB theory, Sheldon Goldstein, writes that "This deterministic theory of particles in motion accounts for all the phenomena of nonrelativistic quantum mechanics, from interference effects to spectral lines to spin." [18].

During the advent of the quantum theory, it was suggested that expression of the quantum wavefunction in polar form, $\Psi(\mathbf{r},t) = R(\mathbf{r},t)e^{i\frac{S(\mathbf{r},t)}{\hbar}}$, may lead to productive insights into that physics [19]. Here, as Born told us [20], $R \equiv \Psi^*\Psi$ is the real probability density for observing an object. The polar form can be inserted into Schrodinger's equation, then real and imaginary parts equated. Two equations result,

$$\frac{\partial R}{\partial t} = -\frac{1}{2m}[R\nabla^2 S + 2(\nabla R)\bullet(\nabla S)], \text{ and,} \tag{1}$$

$$\frac{\partial S}{\partial t} = -\frac{1}{2m}(\nabla S)^2 + \frac{\hbar^2}{2m}\frac{(\nabla R)^2}{R} - V, \tag{2}$$

where m is the object's mass, and V is the real potential it experiences. Because it enters equation (2) the same way as the classical potential, Bohm named a quantum potential, $V_Q \equiv -\frac{\hbar^2}{2m}\frac{(\nabla R)^2}{R}$. The quantum potential can be calculated from Schrodinger's equation using the real potential.

A way to calculate the trajectory of a physical object while its wavefunction evolves continuously is to follow the flow of particle probability using Schrodinger's equation. Since $P = \int \Psi^*\Psi d^3r$ is the probability of observing the object within the infinitesimal volume element specified, the change of probability there is,

$$\frac{\partial P}{\partial t} = \int [\Psi^*\frac{\partial \Psi}{\partial t} + \Psi\frac{\partial \Psi^*}{\partial t}]d^3r. \tag{3}$$

Substituting from Schrodinger's equation, $i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi$, and rearranging, yields,

$$\frac{\partial P}{\partial t} = \frac{i\hbar}{2m}\int\frac{\partial}{\partial r}(\Psi^*\frac{\partial}{\partial r}\Psi - \Psi\frac{\partial}{\partial r}\Psi^*)d^3r. \quad (4)$$

The integrand is a perfect differential, thus

$$\frac{dP}{dt} = \frac{i\hbar}{2m}\left[\Psi^*\frac{\partial}{\partial r}\Psi - \Psi\frac{\partial}{\partial r}\Psi^*\right], \quad (5)$$

We recognize that the change in the probability must equal the difference in the probability density current, **j,** across the infinitesimal volume, times the infinitesimal area orthogonal to **j**. So we take,

$$\mathbf{j}(\mathbf{r},t) \equiv \frac{i\hbar}{2m}\left[\Psi^*\vec{\nabla}\Psi - \Psi\vec{\nabla}\Psi^*\right]. \quad (6)$$

Notice that, $(\Psi^*\vec{\nabla}\Psi)^* = \Psi\vec{\nabla}\Psi^*$. And, in general, for any complex number, z = x + iy, $z - z^* =$ 2iy = 2i Im(z). The probability current is then,

$$\mathbf{j}(\mathbf{r},t) = -\frac{\hbar}{m}\mathrm{Im}(\Psi^*\vec{\nabla}\Psi). \quad (7)$$

For point particles, as required by Bohmian mechanics, the direction of the velocity, **v**, of a particle along its trajectory is specified by the direction of **j**. Because $\mathbf{j} = \mathbf{v}\Psi^*\Psi$,

$$\mathbf{v}(\mathbf{r},t) = -\frac{\hbar}{m}\frac{\mathrm{Im}(\Psi^*\vec{\nabla}\Psi)}{\Psi^*\Psi}. \quad (8)$$

Bohm-deBroglie practitioners suggest that point particles moving on continuous trajectories will follow such flow lines from an initial position to an exact location downstream, demonstrating a deterministic quantum theory [21]. Equation (8) is often called the 'guiding wave equation' though three of the most knowledgeable Bohmian mechanics, Dürr, Goldstein, and Zangi, tell us that we discover it by "guessing" [22]. They say that a guiding wave equation, and the quantum wavefunction, from which it is determined, are sufficient to define the trajectories of Bohmian particles.

In 1986 Dewdney, Holland, and Kyprianidis calculated such flow lines for the particles (neutral, spin one-half atoms) traversing a Stern-Gerlach magnet. They said that such atoms have "well-defined and continuous trajectories and spin vectors", determined by the causal interpretation of Bohmian Mechanics [23]. That conclusion is mistaken.

When a silver atom first encounters the magnetic field, **B**, supplied by the S-G magnet, its magnetic moment, **μ**, will either line up parallel, or anti-parallel (space quantization), to **B** [24]. It is a quintessential uncertainty in the quantum theory, never resolved, whether **μ** will point one way or the other. Probability, only, is available. As Bohm suggested in his textbook, a momentum kick to the atom, in one direction, or the other, does determine the atom's spin direction; it is recorded by the correlated, opposite momentum kick to the magnet. Energy has been transferred. Schrodinger's equation, which is used to determine the Bohmian guiding wave equation, and trajectories, is discontinuous when energy is transferred.

Once spin direction is measured within the S-G magnet, subsequent field quanta continue to kick the atom to one side, or the other, of its initial direction. As Stern and Gerlach found, there are two traces on the downstream detector. But which particular atom arrived at one trace, or the other, is not determined by quantum mechanics, or the Bohm-deBroglie theory.

4. Conclusion

An ontology of Bohmian mechanics would include point particles and their quantum wavefunction. Those two elements are inadequate for a deterministic theory when the wavefunction is discontinuous, as in the Stern-Gerlach experiment. Bohmian mechanics has been able to predict the interference patterns for a double slit, or such interferometers as the Mach-Zehnder, because there is no energy exchanged.

I believe the prototypical S-G experiment can help us solve other intriguing problems shrouding quantum mechanics. Stern and Gerlach took care, even one hundred years ago, to prevent the environment in their laboratory at Frankfurt University from interfering with the spin measurement performed with their magnet. They used a vacuum apparatus and shielding to do so. But the decoherence theory of quantum measurement [25] well-regarded by many researchers, tells us it is not the magnet itself, but rather, extraneous physical elements interacting with a silver atom, which have determined spin direction. Careful consideration, I believe, demonstrates deficiencies in the decoherence theory.

In 2001 John Wheeler and Max Tegmark surveyed the prevailing appreciation, and misapprehensions, of quantum theory [26]. Lack of an equation depicting when and how a quantum superposition collapses was thought a fundamental failing. A more realistic explanation of the S-G experiment, now, indicates quantum energy exchange as its mechanism.

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