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Original Paper

On the Problem of Quantum Measurement in Quantum Mechanics

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Abstract: Using quantum foundations we give a new demonstration of a relation, existing in spin measurements, that engages a new subalgebra with a new commutation relation that exists during the measurement of a spin coordinate. Since the given projector is unique, we do not find a contradiction with the von Neumann postulate that an unique Hermitean operator is associated with the given observable.

Keywords: Quantum wave function collapse, Clifford algebra, quantum measurement, quantum foundations.

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1. Introduction

The problem of quantum measurement is the most difficult problem of physics: we do not understand as the collapse of wave function happens and we have an inconsistent theory that must adopt this problem as an additive external postulate to the theory. In 2009 and 2010 we gave a proof of von Neumann postulates in two separate papers [1,2] by using the Clifford algebra and in 2001 [3] we gave demonstration that the non commutation relation of quantum mechanics is the central problem for such missing demonstration of wave function collapse in the case of a system having three anti-commuting observables as the spin. We used a two states quantum spin system S, and thus considering the particular case of three anticommuting elements and the measurement of, we gave demonstration that, during the wave collapse, we have two existing set of spin operators and a transformation of Pauli standard commutation relation of the spin to new commutation relations occuring during the interaction of the S system with the macroscopic measurement system M. By this formulation we suggested a new method in attempting to solve the problem of wave function collapse: the concept of observable, in use in standard quantum mechanics, is resolved in an abstract entity to which is connected a linear hermitean operator that signs mathematically the operation that we must perform on the wave function in order to obtain the potential and possible values of the observable. It does not commute with a number of other operators characterizing the system and the non commuting rules have a fundamental role in quantum mechanics .They have a logic that must be analyzed in each phase of the non measuring and the measuring processes. When we consider the dynamics of wave function collapse we must account that the observed observable becomes a number, with proper unity of measurements ,during the measurement, thus the linear hermitean operator to which is connected before the measurement, disappears and in its place it appears a new operator that maintain the non commutativity with the other operators to which the old and disappeared operator was connected.

The general problem is as it follows:

Let us admit that we have a wave function $\Psi$and the corresponding density function ρ

$$\Psi = \sum_n c_n u_n \text{ and the density function} \quad (1)$$

$$\rho = |\Psi >< \Psi| = \sum_{nm} c_n c_m^+ |u_n >< u_m| \quad (2)$$

Doing the measurement of a dynamical quantity , as the energy of which the u_n are the eigenfunctions , we obtain the different eigenvalues $E_1, E_2,$............ with probability $|c_n|^2$, but when the result of the measurement has been obtained ,say E_i, the system is necessarily in the state u_i. At the end of the measurement we have a mixture of Ψ that is $u_1, u_2, ...u_k,$....... with the probabilities $|c_1|^2$,$|c_2|^2$,...... and $|c_k|^2$.

In ninety years since its beginnings, quantum mechanics has had great functional and theoretical success leaving little reason to doubt its intrinsic validity. Nevertheless, we cannot ignore that some questions concerning the foundations of this theory remained unsolved, and a historic debates among scientists arose and deeply influenced the early development of the theory. The solution of the quantum measurement would be of relevant significance because it would provide us with a self-consistent formulation of the theory, which presently depends on the von Neumann postulates that have been added from the outside to the body of the theory. Starting with 2009 [1,2,3] our tentative approach was to use the Clifford algebra with the aim to construct a bare bone skeleton of quantum mechanics but giving collapse. We will deepen here some basic features but remaining fully in the aim of the foundations of quantum mechanics and thus without recurring to the Clifford algebra.

2. Theory Elaboration

Let us consider the measurement of e_3 spin z-component. We have three operators e_1,e_2,e_3 and that satisfy the relation

$$e_1e_2=ie_3,\ e_2e_3=ie_1,\ e_3e_1=ie_2 \quad (3)$$

We showed that, during the measurement, we non more can assume as existing the classical non commutation observables that in the case of spin are the following

$$e_1e_2=ie_3,\ e_2e_3=ie_1,\ e_3e_1=ie_2 \quad (4)$$

but the previous ones must be substituted by the following operator relation

$$e_1e_2=i\ ,\ i= e_1e_2 \quad (5)$$

with $e_3$that becomes a classical physical observable that may assume the value ±1 during the measurement.

The aim of the present letter is to show that the operator equation

$$e_1e_2=i\ ,\ i= e_1e_2 \quad (6)$$

actually holds during the e_3 measurement. Doing this we apply only the foundations of quantum mechanics.

Consider a quantum system S on which we intend to perform the e_3 measurement. Let x_1,x_2,x_3 be three real numbers and let

$$a = \sqrt{x_1^2 + x_2^2 + x_3^2} \qquad (7)$$

Consider the quantity represented by

$$x_1e_1+x_2e_2+x_3e_3 \qquad (8)$$

Its square is represented by

$$(x_1e_1+x_2e_2+x_3e_3)^2 =(x_1e_1+x_2e_2+x_3e_3)^2 1=(a^2)1 \qquad (9)$$

Its square can have only the value a^2. Therefore the quantity itself can have only the values (-a) and (a). Its mean value must be between -a and a. Thus we have

$$-a\leq\langle x_1e_1+x_2e_2+x_3e_3\rangle\leq a \qquad (10)$$

Generally we have that

$$\langle x_1e_1+x_2e_2+x_3e_3\rangle=x_1\langle e_1\rangle+x_2\langle e_2\rangle+x_3\langle e_3\rangle \qquad (11)$$

Therefore we have also

$$-a \leq x_1\langle e_1\rangle+x_2\langle e_2\rangle+x_3\langle e_3\rangle\leq a \qquad (12)$$

This relation holds for any real number and in particular for

$$x_1=\langle e_1\rangle,\ x_2=\langle e_2\rangle, x_3=\langle e_3\rangle \qquad (13)$$

therefore

$$x_1\, x_1 + x_2\, x_2 + x_3\, x_3 \leq a \qquad (14)$$

On the other hand we have $a^2 \leq a$ and $a \leq 1$ and this implies that $a^2\leq 1$.
We obtain that

$$\langle e_1\rangle^2+\langle e_2\rangle^2+\langle e_3\rangle^2\leq 1 \qquad (15)$$

This is a general relation of quantum mechanics that involves the three basic operators of the spin. Admit that we are measuring e_3. We will obtain either or 1 or -1 and $\langle e_1\rangle^2+\langle e_2\rangle^2 =0$. This result implies that

$$\langle e_1\rangle^2= - \langle e_2\rangle^2 \qquad (16)$$

and consequently

$$\langle e_1\rangle = i\langle e_2\rangle \qquad (17)$$

which solution is $\langle e_1\rangle =\langle e_2\rangle = 0$ and it implies that

$$e_1= i\, e_2 . \qquad (18)$$

as established in (5).

3. Final Considerations

This is the proof of our demonstration that is based on the previous demonstration that is due to Thomas F. Jordan who published his proof in a celebrated book entitled "Quantum Mechanics in a simple Matrix form "in 1985 by editor Wiley-Interscience Publication. Apparently we could encounter a contradiction with the von Neumann 's assumption that says that to each observable a unique linear hermitean operator is associated [5]but it must be noted that our model of wave function collapse, introduced in [1] and [2], considers two subalgebras one prior and the other after the collapse that are resumed in the $A(S_i)$ and $N_{i=\pm 1}$ of such previous papers]1,2] and thus this von Neumann's assumption relates these two algebras separately and in each of this two algebras the given projector is unique.

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