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Original Paper

What Is It That 'Waves' in Wave Mechanics?

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Abstract: What is it that 'waves' in wave mechanics? It is thought that the waves represented by the wave function are not waves *of* anything. But this sits uneasily with the seeming physical significance of the phase relations between the linearly superposed elements of the wave function. For example, motion in quantum mechanics is described by the interference of superposed wave functions belonging to different energies, controlled by the phase relations between them. Quantum field theory, too, remains rooted in the 'harmonic paradigm' of waves and wave packets. So some think there is more to be said. The present article seeks to contribute to the ongoing debate about the ontology and meaning of the wave function by offering a new perspective on what it is that 'waves' in quantum mechanics. It postulates an underlying periodic physical process that all spin-half particles are taken to undergo in the 'shadow' of the Heisenberg uncertainty principle. It shows how the underlying process can account for the waves and wave packets of the quantum-mechanical formalism (including in the spin-one case)—the mathematical formalism 'modelling' the underlying actual physical process. The new perspective seems to provide insight into other aspects of quantum mechanics as well, including its linear superposition principle, the Schrödinger *Zitterbewegung*—and, rather unexpectedly, into the quantum field-theoretical problem of why a finite particle mass and charge is always observed despite the potentially infinite field energy surrounding a particle.

Keywords: quantum theory, wave function, complex phase, renormalization, zitterbewegung

1. Introduction

In 1987 Bell wrote,

> What is it that 'waves' in wave mechanics?… In the case of the waves of wave mechanics we have no idea what is waving… and do not ask the question. What we do have is a mathematical recipe for the propagation of the waves, and the rule that the probability of an electron being seen at a particular place when looked for there… is related to the intensity there of the wave motion [1].

Modern quantum field theory (QFT), too, as Zee complains, "remains rooted in this harmonic paradigm… We have not been able to get away from the basic notions of oscillations and wave

packets" [38]. Although great improvements in experimental techniques since the early days of quantum theory have clarified some of the related issues [22], the meaning of the wave function and its ontological status remain open questions and the subject of on-going investigation, e.g. [6], [15], [16], [18], [25].

This article proposes a novel answer to the question of what it is that actually waves in wave mechanics, or quantum mechanics (QM).

The proposal is put forward as a heuristic. It is based on (i) an extension of an early idea of Dirac's and (ii) the dropping of a commonly held assumption. The dropped assumption is the irrelevance of Dirac's 1930 hole picture of positrons to modern quantum theory and the interpretation of QM. The proposal takes the modern picture of positrons (in which the negative energy solutions of Dirac's 1928 equation of a free electron in an electromagnetic field refer physically to positive energy positrons) and Dirac's hole picture of positrons (in which positrons are holes in a negative energy sea of electrons), as equivalent and alternative descriptions of the same underlying reality. The aim is to show that the proposed heuristic can recover not only the main results of QM (at least in principle), but also that it can explain much that presently remains ill-understood in those results.

Of course nobody worries anymore about issues to do with Dirac's early picture of positrons as holes in a negative (–ve) energy sea of electrons, since the –ve energy solutions of Dirac's equation correspond, via charge conjugation, to the positive (+ve) energy solutions of a similar *positron* equation, and quantum theory contains no physically real –ve energy states. Moreover, the Dirac theory is not quite right, e.g. it is thought it cannot be applied to massive bosons, and it seems wrong to assume that the infinite number of electrons in the vacuum 'sea' don't interact. However, as David Bohm once remarked to P.C.W. Davies, progress in physics is usually made by dropping assumptions. It is shown how the above-mentioned objections to Dirac's original theory don't apply to the present picture.

Since the proposal is most easily understood in the context of an early insight of Dirac's, connected with his hole theory, this article begins (after some preliminary setting up) by describing that insight. Next, it describes the article's core idea—the central notion underpinning all its arguments. It is the postulation of a fundamental underlying periodic physical process ('basic process') that every fundamental spin-half particle, such as an electron, is taken to undergo in the 'shadow' of the Heisenberg uncertainty principle (HUP). It is shown how the process is mathematically modelled by the wave function\state vector in the relativistic QM formalism—the proposed process being the underlying reality and the wave function its model.

It is argued that the process is behind the harmonic paradigm of QM. It generates the probability amplitudes of QM and is the underlying physical basis of its linear superposition principle, explaining quantum systems' ability to form wave packets and to self-interfere when unobserved. It creates and annihilates virtual quanta, identified with the 'cloud' of virtual photons and other quanta dressing the electron\atom in quantum electrodynamics (QED). Massive bosons and composite systems are brought into the picture. It is argued that the proposed process is the underlying physical generator of the Schrödinger *Zitterbewegung*. Possible objections are noted and responded to. In the last two sections the proposal is tied in more closely to the mathematical formalism of QED, including QED's successful but uninterpreted 'subtraction formalism' or renormalization to remove its (QED's) besetting infinities. It is shown how the proposed process offers a physical explanation of just *why* the subtraction formalism works.

First, some setting up. The linear superposition principle has a central role in the Schrödinger (or time) evolution of the wave function\state vector. In that evolution, the phase

factor plays a key role. It is at the heart of the dynamical prescription for a QM system. Since many of the arguments of this article turn on the role and interpretation of the phase factor, it is appropriate to begin with a quick 'refresh' of just how the QM prescription for the time evolution of a quantum state actually works, and of the crucial role of phase in it.

2. The phase factor

Below is an arbitrary state vector in an orthonormal basis at the time t_0 in the Schrödinger representation. We select an orthonormal basis for simplicity, which is always possible to do. It is a linear superposition of 'waves' (or wave functions):

$$\left|\varphi(t_0)\right\rangle = \alpha_1\left|\psi_1(t_0)\right\rangle + \alpha_2\left|\psi_2(t_0)\right\rangle + ... + \alpha_n\left|\psi_n(t_0)\right\rangle. \tag{1}$$

Suppose that the superposed state vectors are energy eigenvectors, for simplicity. The Schrödinger evolution of an energy eigenvector is given by a complex function of time, known as the *phase factor*. Every true state vector\wave function carries it. That factor is

$$e^{-iEt/\hbar}, \tag{2}$$

where E is the given energy eigenvalue and $\hbar$ is Planck's constant.

(2) is an operator which continuously rotates a vector to which it is applied in the complex plane. In an energy basis, it means that energy eigenvectors are undergoing a ceaseless rotation, or 'waving', the frequency of which is proportional to the corresponding energy E. Such a rotation is in fact the dynamical prescription for a QM system [14].

The Schrödinger evolution of our basis in (1) is then given by the equations

$$\left|\psi_j(t)\right\rangle = e^{-iE_j t/\hbar}\left|\psi_j(t_0)\right\rangle, \tag{3}$$

for j = 1, 2, … n, where E_j is the energy eigenvalue corresponding to the eigenvector $\left|\psi_j(t)\right\rangle$.

When we substitute these expressions into our linear superposition, we have the formula for the Schrödinger evolution of the state vector.

So the Schrödinger or time evolution of the state vector is given in a very remarkable way—by a superposition of unobservable simple oscillations or wavings of 'something we know not what', i.e. through the terms involving the interference of the contributions of different oscillating stationary states. The oscillations are unobservable because the process is complex—all the time-dependence is in the exponential, or the phase factor. And it is generally agreed that the state vector doesn't correspond to any physical quantity or process. Probabilities and probability densities are then obtained by turning to another empirically obtained rule, Born's rule, which says to take the modulus squared of the complex amplitudes of this 'something we know not what'. Upon a measurement, the state vector reduces to one of its constituent eigenvectors in accordance with Born's rule.

The prescription works perfectly. The phase factor determines completely the Schrödinger evolution of every state vector in the Hilbert space of QM. All the interference effects of QM depend on it, as does the spread of expectation values for states described by identical state vectors. As one investigator put it, "…this mysterious complex oscillation… controls everything, but itself eludes scrutiny" [35].

But is that remark correct? Surely what are waving are the *fields* of QFT? So the rotation of the phase factor simply describes the waving of the (potentially mutually interfering) modes of the quantum-theoretical fields, or field quanta. That may be so, but interpretationally that's not much help. That's because the quantum-theoretical fields themselves remain uninterpreted and intrinsically probabilistic, in a way that, for example, the familiar classical electromagnetic fields are not.

Because of the probabilistic nature of QFT, it retains the interpretative problems of non-relativistic QM in one form or another. There is also the worry that we've put too much into our quantum-theoretical picture of the vacuum, as exemplified by its estimate (on the basis of seemingly reasonable assumptions) of the energy density of the vacuum, at variance with the observational limit by up to 120 orders of magnitude, in units of Planck mass [27].

In view of the seemingly intractable nature of QM's interpretative problems, it may be productive to look at the algorithm that is QM from a different starting point—a different set of assumptions—than is customary in current quantum theory, including field theory, and see where that might lead. We begin by describing Dirac's early idea.

3. Dirac's prescient idea

In 1928\1929 Dirac was faced by two (now entirely historical) problems, the first well known and the second perhaps not so well known, both arising from his 1928 relativistic equation of a free electron in an electromagnetic field. Here is a recap of the first. In non-relativistic mechanics, the energy E of a particle is given as a function of its velocity v or its momentum p by $E = \frac{1}{2}mv^2 = p^2/2m$, which corresponds to an always positive E. Dirac's equation replaces these by the energy-momentum relation $E^2 = c^2p^2 + m^2c^4$. Because the relation has two roots, namely $E = \pm c\left(m^2c^2 + p^2\right)^{1/2}$, relativistic mechanics permits in principle two sets of energy level distributions of matter: those with energy $\geq mc^2$, and those with energy $\leq -mc^2$, where m is the mass. A 'forbidden' zone of width $2mc^2$, energies for which the Dirac equation has no solutions, separates the lowest positive energy states from the highest negative energy states.

It is a well-established principle that physical systems tend to seek states of lowest energy. Moreover, in QM we have dynamical variables able to jump discontinuously from one value to another. Thus, if the negative energy states exist and there is an unlimited number of them, electrons ought to be unstable in a vacuum because there seems to be nothing preventing them from jumping from states of positive mass to states of ever increasing negative mass, such jumping being accompanied by a catastrophic emission of energy without limit. That was known as the 'negative-energy catastrophe'.

As is well known, Dirac's solution to the problem was to assume that nearly all the states of –ve energy were already occupied by electrons, one electron per each state in accordance with the Pauli exclusion principle. The few vacant states or holes in the –ve energy 'electron sea' were particles of +ve energy and charge, now known as positrons. In the hole theory, a perfect vacuum is simply a state in which all the –ve energy states are filled and all +ve energy states are empty.

Having solved that problem to his satisfaction, Dirac was soon busy trying to solve a new problem with his theory (the second historical problem referred to above) created by his solution to the first problem. The problem arose in the context of the Klein-Nishina formula on Compton scattering, based on the Dirac equation. It turned out that nearly all the electron

scattering comes from the system's jumping into brief-lived *intermediate* states with –ve energy [9]. (Intermediate state electrons are electrons in transition from an initial to a final state, the intermediate transitional state being a superposition of the initial and final states.) Yet, given the exclusion principle and Dirac's own theory, it was absolutely forbidden for the electron (of +ve energy) to jump into states of –ve energy, no matter how briefly, since the postulated –ve energy electron sea is filled or almost filled; a conundrum. Soon Dirac had an answer.

He described his new idea to Bohr (as recounted by Pais [28]):

> On my new theory ... there is ... a new kind of double transition now taking place in which first one of the negative-energy electrons jumps to the proper final state with emission (or absorption) of a photon, and secondly the original positive-energy electron jumps down and fills up the hole, with absorption (emission) of a photon.

Dirac published the idea in 1930, explaining that the new kind of process makes up for the more direct excluded processes because, "the matrix elements that determine the transition probabilities are just the same in the two cases, though they come into play in the reverse order" [9]. The process which he described is unobservable even in principle and is now called virtual electron-positron formation and annihilation. (Virtual processes are well established in relativistic physics, e.g. [17], [23], [31], [37], though there is disagreement about their ontic status.)

We are now in a position to describe the present proposal. What follows constitutes a single extended argument.

4. A heuristic proposal

The proposal's core idea is to start with Dirac's 1929\1930 double transition process (the process Dirac described to Bohr in the context of Compton scattering and the hole picture), in which a –ve energy sea electron jumps into a +ve energy state and a +ve energy electron jumps into the hole it left in the–ve energy sea, with an emission & absorption of radiation (virtual pair-creation & annihilation). Dirac's double transition process is then extended in an original and far-reaching way: the process is postulated to be *periodic* and occurring all the time quite independently of scattering or other interactions—though it can result in them.

The idea is to treat the periodic double transition process as a *fundamental underlying process* that every electron, and every other spin-half particle, is ceaselessly undergoing in the 'shadow' of HUP. The process is taken as *physically real*, though intrinsically unobservable, and in that sense 'virtual'. The process's frequency and period are readily derived from the uncertainty principle, being $\omega = 2E/\hbar$ and $\tau = \hbar/2E$ respectively, where E is the electron's mass energy. (We focus in this article on the electron for simplicity, though the process is taken to apply to all fundamental spin-half particles. Photon 'waving' is brought into the picture later, as is the idea's application to composite systems.)

Even though, the proposed process results in pair-creation & annihilation, similar to that occurring in the quantum vacuum owing to Heisenberg uncertainty, the process is always associated with *individual electrons*, never with the fields of QED, which are replaced by the Dirac seas. Moreover, the process is strictly periodic rather than stochastic, of frequency $2E/\hbar$ and period $\hbar/2E$.

The proposed process is obviously very rapid. Its frequency for a low-energy electron works out at $\sim 10^{21}$ per second ($\geq 1.56\times10^{21}\mathrm{s}^{-1}$) and its period (the inverse of the frequency) at

$\sim 10^{-21}$ second ($\leq 6.41\times10^{-22}$s). The frequency & period are a simple consequence of HUP and the periodicity postulate. Roughly speaking, HUP allows a fermion of electron mass energy to disappear from the world for the above period ($\sim 10^{-21}$s) provided that it is replaced within the period. According to the present proposal, it *must* so disappear—and keep on doing so. The process is unobservable save by its effects, falling, as it does, within the uncertainties of the Heisenberg time-energy relation $\Delta t \cdot \Delta E \approx \hbar/2$. Owing to these limits, it is necessarily quantized and indivisible, as is each of the attendant 'flashes' of virtual radiant energy or photons.

The main objection to the Dirac hole picture is not the infinite energy and charge densities, requiring renormalization—their equivalents exist in all serious approaches to a realistic theory of particle physics, as Penrose points out [31]—but that the Dirac picture cannot be applied to massive bosons (force-carrying integer-spin particles). However, this objection doesn't apply to the present proposal, as will become evident. There are also other objections to the Dirac theory of the electron, connected with its failure to account for the interaction of an electron with the vacuum (e.g. Lamb shift, vacuum polarization). However, as we shall find, the present proposal allows for the electron's interaction with the vacuum (or rather, with its equivalent of the QED vacuum).

It will be argued that the proposed double transition process is the underlying physical basis of QM's mathematical de Broglie waves\matter waves\ψ waves, the evolution and behaviour of which are described in a precise way in QM. We shall find that the process extends to composite systems and particles in all possible states of energy without limit. For now though, to 'fix' the idea, we focus on a low-energy electron whose total mass energy E is of the order of its rest mass energy.

Take a single occurrence of the proposed periodic double transition by such a low-energy electron during the period Δt. The electron jumps into the –ve of its +ve energy state—into a hole in a –ve energy electron sea—filling the hole and emitting a pair of virtual gamma photons. As Dirac showed, physically the hole is a positron—an 'electron' of opposite charge, momentum and spin to the real electron—and therefore the downward jump represents pair annihilation (electron-positron annihilation).

How come the hole, since it is stipulated that there is no real positron in the vicinity? As already mentioned, the hole is left by the *simultaneous upward jump* of a –ve energy sea electron into a +ve energy state with an absorption of a pair of virtual gamma photons, replacing the electron that jumped down. So each jump needs the inverse jump for the possibility of its own occurrence.

Since the upward jump results in the creation of an electron and the creation of a hole (or positron), that jump represents pair-creation. So there is simultaneous pair annihilation and creation within the period $\hbar/2E$ (in the shadow of HUP). The positron member of the pair always remains virtual, annihilated as soon as it is created, whereas the electron member always remains real, its annihilation being only 'virtual', i.e. unobservable, concealed by HUP. The process repeats periodically.

The simultaneous (inverse) jumps may be schematically depicted as in Fig. 1.

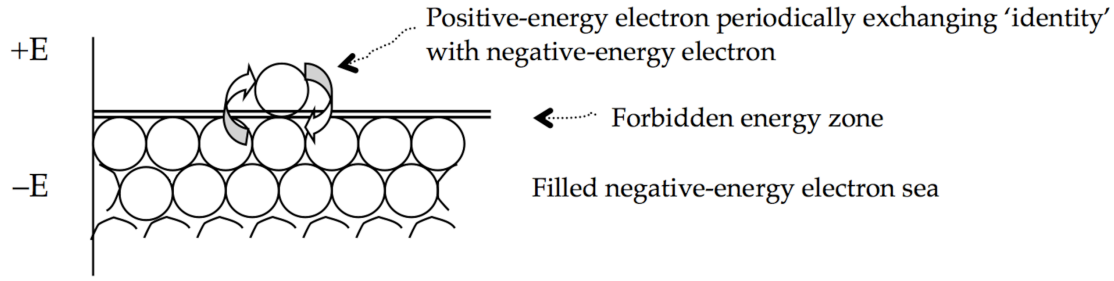

Fig. 1 An electron exchanging 'identity' with a –ve energy electron in the filled –ve energy electron sea, the exchange repeating $\sim 10^{21}$ times per second, its frequency derived from HUP

One might be tempted to think of the coupled jumps as occurring in a certain order, namely that one of the jumps occurs *first*, *followed* by the other. But since the process is masked by HUP, the order is arbitrary.[1] It is best to think of both as taking place all at once.

In this first approximation and in the absence of an interaction, the proposed process repeats without end with the frequency $(E-[-E])/\hbar \geq 1.6\times10^{21}\mathrm{s}^{-1}$ and period $\hbar/2E$. Consequently the electron behaves as a simple harmonic oscillator, periodically 'flashing' in the shadow of HUP with virtual radiant energy of the above frequency. The process is similar to the negative-energy catastrophe that seemed in 1930 to be implied by the –ve energy solutions of the Dirac equation, save that, each time, the catastrophe is offset within the Heisenberg limit by its inverse: a *positive*-energy 'catastrophe'.

In an energy basis, the diagonal elements of the system's density matrix will be constant, as is necessary, because the total probability that the state is occupied by one or the other of the electrons will be constant. The reduced probability that one electron occupies a state is balanced by the increased probability that the other electron occupies that state, and the off-diagonal elements (the cross\interference terms, which rotate) will be in accord with quantum theory, yielding the usual *Zitterbewegung* (*Zb*) or 'trembling' motion (Sect. 7); the loss in oscillation amplitude as one electron leaves the state is balanced by the gain in amplitude as the other electron arrives. Save for its periodicity and universality, this is just the process Dirac described to Bohr.

To 'catch' the electron in the act of transition, time would have to be defined as sharply as $\Delta t\left(\approx\hbar/2E\right) \leq 10^{-21}\mathrm{s}$ (the transition period), where Δt is the remaining uncertainty in the time. But then the ensuing in-principle uncertainty in the system's energy, $\Delta E\left(\approx\hbar/\Delta t\right)$, would be as large as\larger than double the electron rest energy E_R itself, i.e. $\Delta E/2E_R \geq 1$, foiling the attempt. So there is no *empirical* bar to the proposed periodic virtual process. Similarly, the created positron always remains directly unobservable, existing only for the period $\hbar/2E$, concealed by HUP.

A single occurrence of the proposed periodic double transition is mathematically equivalent to the familiar QED process (connected with the electrical polarization of the quantum-theoretical vacuum) where a vacuum fluctuation creates a virtual electron-positron pair in the vicinity of a physically real electron (cf. [21]). The real electron and the positron member of the virtual pair mutually annihilate, leaving the virtual electron member of the vacuum pair, now

[1] This is to do with the fact that when "two or more states are superposed, the order in which they occur is unimportant, so the superposition process is symmetric between the states that are superposed" [10].

without its positron partner, as the new real electron in a slightly different location, no more than half an electron Compton wavelength from that of the original. (This is why the position can't be defined to better than this in relativistic QM [8]). In the present proposal, there has been no spatial transportation of the original electron, as is shown in Sect. 6. Thus the proposal is consistent with field-theoretical models of the *Zb* [7].

The double transition described above is equivalent to the symbolic representation of a term in second-order perturbation theory. There are infinitely many terms in the perturbative expansion of the S (scattering) matrix, and *all* need to be taken into account. The more complete picture is given later in this article, in Sections 7-9, including the proposed solution to the infinities problem. Here we focus on a simple illustration to lay the groundwork for the more complete picture.

It has been argued above that, *physically*, the virtual positron of the electron-positron pair is actually a consequence of the periodic jumping up of a –ve energy sea electron and the resultant hole in the sea. What about mathematically? *Mathematically*, in the QM formalism, the positron is the –ve frequency half of the solution of the Dirac equation for a free electron when the period of interest is sufficiently short for Heisenberg uncertainty to kick in fully, i.e. when $\Delta t \leq 10^{-21}$s. The most *general* solution of the Dirac equation for an electron consists of superpositions of +ve and –ve frequency solutions, reinterpretable physically as superpositions of electron and positron states, as in (4):

$$\Psi(\mathrm{x},t) = \sum a_{+} \exp\left\{(i/\hbar)\left[(\mathbf{p}\cdot\mathbf{x}) - c\left(p^2 + m^2c^2\right)^{1/2} t\right]\right\} + \sum a_{-} \exp\left\{(i/\hbar)\left[(\mathbf{p}\cdot\mathbf{x}) + c\left(p^2 + m^2c^2\right)^{1/2} t\right]\right\}. \quad (4)$$

The summed solutions are de Broglie waves possessing well-defined values **p** and E of the momentum and energy, but with an ambiguity in sign. The summations extend over all possible values of **p**, and $a_{\pm}$ are the +ve and –ve energy state amplitudes (4-component spinors), with $E = \pm c\left(p^2 + m^2c^2\right)^{1/2}$, where p is the absolute value of the momentum, the latter expression giving the relation between the energy and the momentum [20].

For low energies and long durations we can pick either the +ve energy part of the solution or the –ve energy part, and discard the other. But when $\Delta t \approx \hbar/\Delta E \approx 10^{-21}$s (the period we are concerned with), that is no longer possible, and the solution represents both at once. But how is one to understand a single particle state as a superposition (a sum) of electrons and positrons? The usual response is to go to a multi-particle model and QFT, more specifically, QED. In QED, the Dirac equation for a free electron is treated as an evolution equation in a Hilbert space. The Hilbert space contains states which are inescapably superpositions of +ve and –ve energy states; physically, superpositions of electron and positron states [36]. But the nature of the postulated fields remains mysterious, and field theory breaks down at high energies.

This article proposes an alternative, but related, idea, in which (i) the electron\positron fields (+ve & –ve frequency fields) of QED are replaced by their equivalent Dirac's 'seas',[2] and (ii) the summed solutions of the above general solution (the uninterpreted de Broglie plane waves, of +ve and –ve frequency) mathematically model the electron's proposed underlying and physically real periodic double transition process, in a way to be described. It

[2] The real electron itself may be represented as a matching hole in a positive energy sea of advanced or backward-in-time electrons, equivalent to a hole in a –ve energy sea of retarded or forward-in-time positrons, the latter hole analogous (save for the sign change) to a hole in the –ve energy sea of retarded electrons that corresponds to a virtual positron\advanced –ve energy electron.

is an entirely *new idea*.

We shall find that the proposed physical process readily 'maps' to the standard relativistic formalism of QM. For example, the process's frequency ($\geq 1.6\times10^{21}\mathrm{s}^{-1}$), derived directly and simply from HUP, is identical to the frequency of oscillation of the cross terms with complex exponentials in the probability distribution function between the +ve and –ve energy solutions of the Dirac equation for a free electron when we superpose all our plane wave solutions to form wave packets. The cross terms oscillate (rotate) rapidly in time with the angular frequencies $\omega = 2mc^2/\hbar \approx 2\times10^{21}\mathrm{s}^{-1}$, where mc^2 is the rest mass energy [3], [33].

The present picture takes the rotation of the cross terms in the relativistic formalism as modelling the proposed *actually occurring* underlying periodic double transition of the same frequency—an 'internal' rotation—and the consequent periodic pair-annihilation & creation in the shadow of HUP. Details bringing in the phase are in Sections 6 & 7. (Spin-1 systems are brought into the picture in Sect. 5.)

The two pictures, the presently proposed one and QED, are like the two sides of the same coin, save that the proposed double transition process is periodic (though observationally it 'looks' stochastic), in contrast to the QED one, which is taken to be intrinsically stochastic. Both the QED process and the presently proposed process involve pair creation and pair annihilation in an essential way. In the QED picture, virtual creation and annihilation of *new* matter occurs all the time in the field-theoretical vacuum, with important consequences. In the present, alternative, picture in which the vacuum is replaced by the Dirac seas, a similar process is ceaselessly occurring in respect of already *existing* matter: the virtual annihilation and (re)creation of existing matter occurs periodically and without cessation—with similar consequences.

So why bother with the present proposal if the two pictures are mathematically identical (in principle)? Because there is no agreement as to what QM\QED 'means'. The latter's algorithmic formalism lacks a generally accepted interpretation. The present picture proposes an interpretation of the formalism—an interpretation that appears to have considerable heuristic potential.

I shall refer in brief to the proposed underlying process as the 'basic process' or 'double transition process' or 'basic (double transition) process', and sometimes just as an 'oscillation', where 'oscillation' is shorthand for the basic process.

5. Virtual quanta generated by the underlying basic process; linear superpositions; massive bosons

As the electron undergoes the proposed basic process (the –ve & +ve energy 'catastrophe'), it 'flashes' with virtual radiant energy in the course of each up and down leg of the oscillation, emitting pairs of virtual photons on the way down the energy scale and simultaneously absorbing pairs of virtual photons on the way up.[3] Each such photon is of an energy\frequency equal to a difference in energy\frequency between two adjoining energy levels or eigenstates of energy covered by the electron in its oscillation. In the present idealized example (to be made more realistic presently), the emission and absorption is of a single pair of gamma photons, of frequency $\omega \approx 10^{21}\mathrm{s}^{-1}$.

[3] It might be objected that for the free Dirac equation there is no coupling to the photon. But there are no truly free electrons. The Dirac equation with its solutions in the present proposal is taken to be the equation of an interacting *electron*, i.e. a particle *with* electric charge, and the coupling constant of the interaction between electrons and photons is the electric charge. In QED, photon coupling involves fields with independent degrees of freedom. In the present proposal, the field equivalents are created by the proposed basic process.

In quantum theory, a physically realistic electron state is a wave packet—a linear superposition of wave functions (de Broglie plane waves, interpreted as probability amplitudes, or probability density amplitudes). When a quantum wave function contains a sum of contributions from many states, the system must be thought of as somehow covering all the states at once, and the superposition of wave functions representing the different possible states of the system itself represents a possible state of the system. For example, if $|\psi\rangle$ and $|\psi'\rangle$ represent possible states of a system, so does $\alpha|\psi\rangle+\beta|\psi'\rangle$, where α and β are arbitrary complex numbers. The superposition principle applies to a domain in which the values of observables haven't yet been fixed—Heisenberg's world of potentiality.

That is the domain of the virtually oscillating electron of the present proposal. Its state within the Heisenberg limit as it executes the proposed double transition process is expressible as a linear superposition of eigenstates of the packet.

In each round-turn of the electron's double transition, in an energy basis, the electron covers all the eigenstates of energy represented by the packet, emitting and absorbing the covered mass energy as virtual electromagnetic radiation\photons, with mutually interfering field modes. The emitted and absorbed virtual radiant energy is itself expressible as a linear superposition of oscillations by a collection of *m* harmonic oscillators (i.e. virtual electron oscillators), *m* being the number of transitions between energy eigenstates, including –ve energy ones—potentially infinite in number as the time is better and better defined. *Bound* electrons, too, can make quantum jumps to states of –ve energy with photon emission even when isolated [23]. So they, too, undergo the proposed oscillation, emitting and absorbing (mutually interfering) virtual photons expressible as superpositions.[4] Energy differences due to fine structure would need to be included in the superposition.

The present proposal identifies the emitted\absorbed virtual photons and other similarly generated by-products of the underlying double transition process with the cloud of virtual quanta 'dressing' the electron\atom in QED—the force-carrying bosons responsible for the electromagnetic exchange force. So, instead of the cloud being generated by Heisenberg uncertainty applied to the electron & radiation fields of QED, it is generated by the double transition process occurring in the 'Dirac' sea\seas, *concealed* by Heisenberg uncertainty.

There are of course no –ve energy states associated with photons, and they don't undergo the double transition process. Photons are simply created and annihilated in virtual form by the *electron's* double transitions. They don't exist independently of their sources. Occasionally a virtual photon transitions into a physically real photon (makes the transition from Heisenberg's potentiality to reality). The real photon's frequency is always the same as that of its source virtual photon, and thus related to the frequency of the underlying virtual electron oscillator that generated the virtual photon. In this way the proposal brings into the picture not only the 'waving' of spin-half particles and the resulting exchange force, but also the 'waving' of physically *real* photons and other bosons.

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[4] Since, in an energy basis, the electron exists virtually at every instant in a linear combination of all the eigenstates of energy represented by the basis vectors $|E_j\rangle$ of the system's H (with some expansion coefficient a_j of each), the electron may be modelled as undergoing the proposed double transition between *each* such eigenstate of energy and the eigenstate's negative, with a frequency proportional to the energy difference between the two. So all the time there is occurring a superposition of virtual electron oscillations between the $|E_j\rangle$ and their negatives $|E_{-j}\rangle$, at various frequencies. Since each oscillation generates a (pair of) virtual photon of the corresponding frequency, the process results in the existence at every instant of a superposition of such virtually co-existing, linearly combined mutually interfering photons, their oscillations expressible as a superposition of beats between the modes.

The above picture suggests that the virtual photons created by the basic process *mediate* physically real interactions, such as the localization of an electron by a measurement or the emission or absorption of a real photon by an atom. If so, the process might go some way towards a physical explanation of the *probability amplitude* nature of QM's ψ waves and the attendant measurement problem, something not explained by Born's rule—which is just that: a rule. As Gao states, an important aspect of the measurement problem is to explain the origin of the Born probabilities [15]. Here is how such an explanation could go.

The probability $P_{\mathbf{x}}$ per unit time of a measurement locating, say, an electron in some selected volume element or region of space $\mathbf{x}$, is proportional to the intensity $I_{\mathbf{x}}$ (and thus the energy density) of the virtual electromagnetic field ε_{X} in that region, generated by the double transition process (basic process) that the electrons in the volume element are taken to be ceaselessly undergoing. That is, $P_{\mathbf{x}}$ is proportional to the number $N_{\mathbf{x}}$ of virtual photons per unit volume element per unit time in that region, generated by the electrons' virtual oscillations up and down the energy scale. (The greater the intensity per mode, the greater the number of electrons in the region.) By the same token, the greater the intensity, the greater the probability of a resonance interaction with a real photon from a measuring instrument, probing the volume element, transforming one of the virtual photons created by the double transition process into a real photon, which interacts with the measuring instrument. But in orthodox QM that same probability is proportional to the modulus squared of ψ (Born's rule). So, given the present picture, $|\psi(\mathbf{x})|^2$ must be proportional to the number of virtual photons created per unit time by the oscillations in the region. Thus we have the relations $P_{\mathbf{x}} \propto I_{\mathbf{x}} \propto N_{\mathbf{x}} \propto |\psi(\mathbf{x})|^2$. When $I_{\mathbf{x}}$ is high in some region $\mathbf{x}$, meaning that the square of the virtual electric field strength ε_{X}—and thus the flux of virtual photons—is high there, the probability density $|\psi(\mathbf{x})|^2$ for locating an electron there, is correspondingly high. And when $I_{\mathbf{x}}$ is low and the virtual photon flux low, $|\psi(\mathbf{x})|^2$ is correspondingly low—in accordance with Born's rule. It would seem, at least in this particular case, that the amplitude of the *un-interpreted* mathematical ψ (probability) wave refers to, or piggybacks on—or *supervenes* on—the amplitude of the *interpreted* underlying virtual electromagnetic wave ε_{X} generated by the presently proposed physical process.

This is of course not yet a solution of the measurement problem of QM, since the details of how the "resonance interaction" mentioned above could work are not provided. So here the idea must be left as (at best) a promising conjecture.

How would the present picture apply to massive bosons since there is no exclusion principle to prevent such bosons undergoing the –ve energy catastrophe? The answer is that massive bosons are not fundamental particles. As soon as the boson begins to undergo the 'catastrophe', it decays into intermediate particles, which always seem to include at least one spin-half particle, e.g. in the case of pion decay, a mu-electron (or muon). The intermediate spin-half particle then itself undergoes the virtual double transition process, or basic process, dropping into the negative of its +ve energy states while simultaneously being replaced by a –ve energy muon jumping up into the negative of *its* –ve energy state, i.e. into a +ve energy state. This *intermediate* double transition process prevents the massive boson's immediate permanent or physical annihilation, since it entails not only the parent boson's rapid periodic annihilation but also its equally rapid periodic re-creation. For example, HUP allows the intermediate negatively charged muon (which also feels the weak charge), to undergo the above 'catastrophe' on average $\sim 10^{24}$ times per second, decaying virtually each time into an electron, a muon-neutrino and an electron-antineutrino (plus the mediating W^-). This means that, like the electron, the muon undergoes a

Zb, flickering back and forth between a right and left corkscrewing form (spin-flipping) at the above frequency. The underlying rapid virtual process continues until the boson's eventual, much slower and *non*-virtual physical decay into the above decay products or 'constituents'.

What about (fundamental) spin-half particles with no electric charge, such as neutrinos? It is conjectured that they undergo their own version of the proposed double transition process, emitting and absorbing gauge bosons associated with their particular 'charges'.

6. The role of phase in the proposed basic process, and an objection

Consider the general solution (4) of the Dirac equation for a free electron. It may be written more usefully for present purposes in the following form:

$$\Psi(\mathbf{x},t) = \sum a_{+}e^{i(\mathbf{p}\cdot\mathbf{x}/\hbar - Et/\hbar)} + \sum a_{-}e^{-i(\mathbf{p}\cdot\mathbf{x}/\hbar - Et/\hbar)}, \qquad (5)$$

where the notation makes it evident that for the physical particle associated with the hole (the positron), **p** has the meaning of the physical momentum of the particle (see e.g. [33]).

One group of solutions in (5) corresponds to +ve frequencies and one group to –ve frequencies, the latter group associated with negative energies. The former solutions propagate forward in time, the latter backward.[5] The former solutions are often called 'retarded' and the latter 'advanced'. In the associated probability distribution function there are cross terms connecting the +ve and –ve frequency states, giving rise to an oscillatory time dependence between the eigenstates belonging to each group of solutions, as mentioned in Sect. 4.

When the electron's momentum, and thus its energy, is well defined, the state may be approximated by a constituent solution of the general solution (5), i.e., by

$$\Psi(x,t) = a_{+}e^{i(p_x\cdot x/\hbar - Et/\hbar)} + a_{-}e^{-i(p_x\cdot x/\hbar - Et/\hbar)}, \qquad (6)$$

which may be written simply as:

$$\Psi(x,t) = a_{+}e^{i(k\cdot x - \omega t)} + a_{-}e^{-i(k\cdot x - \omega t)}. \qquad (7)$$

In QM, (6) & (7) represent the linear superposition of a retarded plane wave (a de Broglie wave) and its complex conjugate plane wave, the latter being a wave of –ve frequency. The former wave is an electron wave function of four-momentum p and the latter wave is an advanced –ve energy\frequency electron wave function of four-momentum $-p$, reinterpretable as a +ve energy\frequency positron wave function.

As described in Sect. 2, the superposed plane waves (or wave functions\state vectors) are each associated with an undetermined and uninterpreted phase factor, $e^{\pm iEt/\hbar}$, or $e^{\pm i\omega t}$, which continuously rotates an energy eigenvector to which it is applied through the angle ωt in a complex plane with the frequency $\omega = E/\hbar$.

According to the present heuristic, the rotations of the state vectors mathematically model the proposed underlying basic process (the periodic double transition process). That process is taken to be the underlying physical generator of the wave properties of spin-half systems in QM,

[5] Dirac interpreted the backward-in-time, or 'advanced', –ve frequency solutions of his time-symmetric equation as referring to the motion of a *hole* in a sea of –ve energy electrons. The hole, i.e. the absence of a –ve energy electron with four-momentum $-p_{+}$ and spin $-s_{+}$, is recorded as the presence of a +ve energy positron with four-momentum p_{+} and spin s_{+}. In Dirac's positron theory, the hole picture ensured the boundary condition of backward-in-time propagation for the –ve frequency solutions [3]. Feynman, in turn, showed that the hole (the positron) is equivalent to a –ve energy advanced electron [12].

such as their capacity for self-interference. That's to say, the proposed process is the underlying reality, and the state vectors and their rotations are its formal 'shadows'. The coupled rotations of the positive and negative frequency state vectors can be 'mapped' one-to-one with the proposed underlying basic process. Here's how.

Take the constituent solution (6)\(7) for simplicity. We've seen that the solution represents a superposed electron-positron *pair* during sufficiently short periods $\Delta t \approx \hbar/2E$ ($\leq 10^{-21}$s), even when there is no physically real positron in the electron's vicinity. How come? Well, the physically real electron and a –ve energy sea electron are ceaselessly undergoing a periodic exchange of 'identity' pursuant to the basic process (Fig. 1)—as the –ve energy sea electron jumps into a +ve energy state, the +ve energy electron jumps into the hole left in the sea. Therefore, during the period $\hbar/2E$, an electron and a hole (a positron) exist simultaneously. The hole—the positron term of (6)\(7)—is a –ve frequency solution of the Dirac equation for an electron, of frequency $-\omega$, where $-\omega$ is the negative of the +ve energy electron's frequency ω. The electron's jump into the hole constitutes pair annihilation. Each such jump and the filling of the hole is modelled in solution (6)\(7) by a $2i$ rotation of the state vector with the exponential $e^{i\omega t}$—one of the two summed state vectors, or plane waves (de Broglie waves), of the solution. Thus the $e^{i\omega t}$ vector may be thought of, in mathematical QM-speak, as an *annihilation operator* of the electron and the hole\positron, and a *creation operator* of a pair of resultant virtual photons.[6]

But each time the annihilation of the electron-positron 'pair' is short-lived owing to the simultaneous occurrence of the *inverse* process, namely the jump by a –ve energy sea electron into the +ve energy state, modelled in (6)\(7) by a $-2i$ rotation of the component state vector with the exponential $e^{-i\omega t}$. The latter rotation restores the system to its initial state. So the jump up constitutes pair creation. Thus the $e^{-i\omega t}$ vector may be thought of as a *creation operator* of the electron and the hole in the –ve energy 'sea' (the positron), and an *annihilation operator* of a pair of photons.

Here is an equivalent and alternative description of a single round turn of the above periodic double transition. Pursuant to a $2i$ rotation of the state vector with the exponential $e^{i\omega t}$, the real electron jumps into the –ve of its +ve energy state (as above)—into a hole in a –ve energy sea of electrons. So there is pair annihilation. Simultaneously, pursuant to a $-2i$ rotation of the state vector with the exponential $e^{-i\omega t}$, the virtual *positron* member of the 'pair' jumps into the –ve of *its* +ve energy state, i.e. into a hole in a –ve energy sea of *positrons*. The latter hole is physically an *electron*, of opposite momentum and spin to the positron. At first, one might think that this jump, too, represents pair annihilation (of the real electron and its virtual positron counterpart). But the physically real electron is *already* annihilated by its own downward jump and there never was a physically real positron to be annihilated. Therefore the *second*, simultaneous, annihilation of the pair resulting from the virtual positron's jump, is an *annihilation* of an annihilation—i.e. a creation of a state of something. That 'something' is the real electron and a hole in the –ve energy electron sea. So the virtual positron's downward jump equates to an *upward* jump of a –ve energy *electron* from the –ve energy electron sea into a +ve energy state, restoring the system to its initial state. Neither jump may be taken as really occurring 'first', since the two are simultaneous within HUP's envelope. The identical situation would exist if the virtual positron's downward jump were to be described as occurring first, followed by the electron's downward jump. Always, the one jump, whichever one, undoes the other.

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[6] This is consistent with the QED picture of an electron, in which an electron is only partially to be associated with the electron field alone; it is also partially to be associated with the photon field, because the two are in interaction [34].

Whichever the representation, what has *physically* happened is that the *original* real electron and hole have both been transformed by the electron's jump into the hole into a virtual electromagnetic wave, of frequency $\omega = 2E/\hbar \approx 10^{21}\text{s}^{-1}$ and period $\tau = \hbar/2E \approx 10^{-21}\text{s}$, where E is the electron's total energy. The 'wave' is decomposable into a pair of virtual photons.

The photons 'propagate' the distance $\Delta\mathbf{x} = c\Delta\tau = \hbar/2mc \approx 10^{-13}\text{m}$ before their absorption. The absorption is by the above-mentioned ex-negative energy sea virtual electron, now located in our +ve energy world at the distance $\sim \frac{1}{2}\lambda\!\!\!^{-}_{c}$ from the original real electron, say at **B** if the original electron was at **A**. The virtual electron has no choice but to absorb the wave\photons since the conditions for perfect *resonance* between the two exist—the incoming wave\photons are *in phase* with the electron's jumping. There is never any spatial transportation of the electron between **A** and **B**. Thus it avoids a possible quantum field-theoretical objection [21] that there can be no *Zb*, where *Zb* is understood as a bodily motion of electron mass. See also [7] for discussion.

The absorption (i) transforms the ex-negative energy sea electron into a physically real +ve energy electron and (ii) recreates the virtual hole in the –ve energy sea (the virtual positron). The process repeats with reversed direction and helicity (cf. [31]). It results in the impossibility of localization of an electron to better than $\Delta\mathbf{x} = \hbar/2mc \approx \frac{1}{2}\lambda\!\!\!^{-}_{c}$.[7]

So we now have an *interpretation* in terms of an underlying physical process of QM's mathematical creation and annihilation operators, falling naturally out of the proposed 'basic process'.

It might be objected that the preceding discussion of the role of the phase factor lacks physical content since the same form of solutions can be found for *classical* waves (no –ve energy sea) when written in Fourier form as linear superpositions, or for classical harmonic oscillators if one represents the position-momentum pair by a single complex number.

Such an objection would fail because, although classical waves *can* be represented in Fourier form as linear superpositions, they need not. That's in contrast to QM, where the linear superposition principle is fundamental—and ill-understood. Likewise, although classical harmonic oscillators *can* be represented by a single complex number (for mathematical convenience), they need not. In contrast, the psi waves of QM are *intrinsically* complex: the QM complexity is fundamental, and ill-understood. The intrinsic complexity strongly suggests a mystery remaining to be solved. The present heuristic suggests a way of tackling the mystery.

7. Modelling the basic process; more possible objections

It was shown in Sect. 6 how the underlying physically real (though unobservable) basic process can be 'mapped' one-to-one with the paired rotations of the state vectors of solutions (6)–(7). The basic process, and thus the rotations of the state vectors of solutions (6)–(7), may be modelled by a geometrical representation of Euler's analytical expression, or formula, $\frac{1}{2}(e^{i\omega t} + e^{-i\omega t})$ for a cosine function (Fig. 2). Take (7) for simplicity. There is a pair of

[7] The process is time-reversal invariant. In a time-reversed frame of reference, the roles of the real and virtual particles in the superposition (6)\(7) would be swapped over—the virtual positron being the real particle jumping down and our real electron merely a short-lived hole in a –ve energy sea of positrons. That's because (i) their 'identities' (in the sense of the signs of their charges), are defined by the sense of rotation of the state vectors, and (ii) the phase relations of the components of the superposition that make up a wave packet contain information that is temporally directed; consequently, "one cannot reverse time without reversing those phase angles" [32]. See also [11]. The two opposite reversals of handedness of phase offset each other. There would be no overall change.

superposed modulus vectors or phasors of length one—the coupled state vectors with the phase factors $e^{i\omega t}$ and $e^{-i\omega t}$ of (7)—with a common origin on a unit circle on a complex plane with constant length one, rotating anticlockwise and clockwise, with the frequencies ω and $-\omega$, respectively, where $\omega \approx 2mc^2/\hbar$ and m is the electron's relativistic mass. The paired rotation generates a pair of superposed complex circular helices of opposite handedness centred on the time axis, lying normal to the unit circle. (Different values for the frequency may be subbed in to model rotations covering an arbitrary range of energies.)

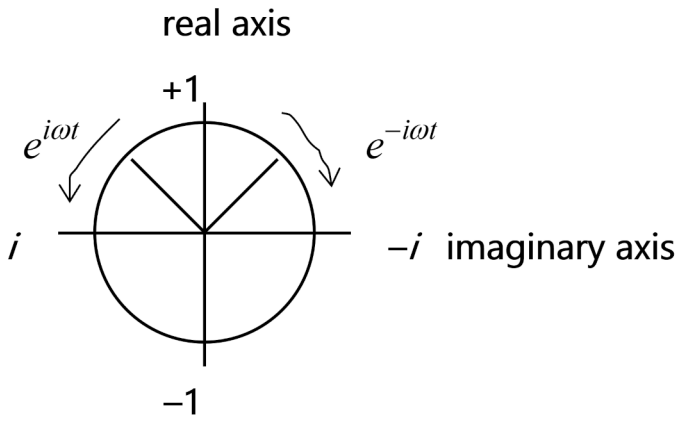

Fig. 2 A geometrical representation of Euler's analytical expression, or formula, $\frac{1}{2}(e^{i\omega t}+e^{-i\omega t})$ for a cosine function, modelling the proposed double transition process

The two rotating phasors add vectorially to give a periodic oscillation or waving that always lies completely along the *real* axis (call it the x-axis) of the unit circle, alternating between the –ve and +ve x-direction, the imaginary components cancelling out. Its frequency and period (i.e. $2mc^2/\hbar$ and $\hbar/2mc^2$), still in our idealized version, works out at $\sim 10^{21}\text{s}^{-1}$ and $\sim 10^{-21}\text{s}$ respectively. The oscillation's x-amplitude (real-axis diameter of the unit circle) works out at $\hbar/2mc$, i.e. half the electron Compton wavelength: $\frac{1}{2}\lambda\!\!\!^{-}_{c}$ ($\approx 10^{-13}\text{m}$).

On a spacetime diagram, the electron's oscillatory time dependence generated by the rotating modulus vectors of Fig. 2 is a zigzag, as depicted in Fig. 3, where the spatial distance **A** to **B** is the real-axis diameter of the unit circle.

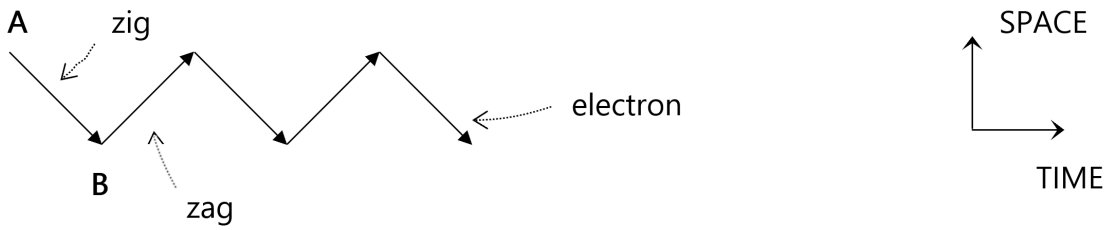

Fig. 3 An idealized representation of an electron zigzagging at the speed of light, constantly reversing its velocity and helicity at locations **A** and **B**

A rotation of $|\pi|$ on the complex plane, in radian measure, by each of the two paired vectors of Fig. 2 (total of $|2\pi|$) represents one round turn of the basic (double transition) process. It takes

the pair of vectors from +1 to −1 on the real axis and shifts the electron from **A** to **B** in Fig. 3.[8] Even though the system has rotated by $|2\pi|$, its physical state at **B** is not yet the same as its starting state at **A**. The spin eigenfunctions are now the negatives of the initial spin eigenfunctions, with peaks and troughs interchanged. A *further* rotation of $|\pi|$ by each vector is required to restore the system to its original state. The vectors cross over at −1 on the real axis and the rotations continue, now with reversed helicity, bringing the pair back to +1 and returning the electron to **A**. So a total rotation of $|4\pi|$ or *two* 360° rotations is required to bring the electron back to its original physical state, suggesting spin. The magnetic dipole moment generated by the coupled rotations is $e\hbar/2m_0c$, where e is the charge and m_0 the rest mass (as it happens, the same as the magnetic dipole moment usually ascribed to the electron's intrinsic spin).

In (relativistic) QM, just such a spacetime zigzag is the fundamental part of the electron's *Zb*, a speed-of-light oscillation about a mean position: "[T]he electron executes a complicated motion which is a superposition of an average motion with the expected velocity plus an oscillating motion with a frequency $2mc^2/\hbar$" [33]—the oscillatory component of the motion being the *Zb*.

In quantum theory, the *Zb* motion is a consequence of the coupled rotation of the superposed state vectors with the phase factors $e^{-i\omega t}$ and $e^{i\omega t}$ in the solutions of the Dirac equation and the associated cross terms, when a_- and a_+ (i.e. –ve and +ve energy state amplitudes) are both present—as they are in (7). See e.g. [33], [36]. The quantities $\hbar/2mc$, $\hbar/2mc^2$ and $2mc^2/\hbar$ (the amplitude, period and frequency of the above spacetime zigzag) are the generally accepted characteristic amplitude, period and frequency of the (fundamental part of the) electron's *Zb* [19], [23), [26].[9]

Since identical quantities also describe the amplitude, period and frequency of the speed-of-light spacetime zigzag entailed by the presently proposed basic process, it seems reasonable to equate the two speed-of-light oscillations\zigzags. If so, the proposed basic process, modelled by the complex phase factors of (5), (6), (7), may be taken as the underlying generator of the *Zb*. Such an association of *Zb* motion with phase shouldn't be surprising. After all, as Hestenes remarks [19], the *Zb* clearly arose from wave function oscillations in Schrödinger's free particle analysis, even though Dirac himself never considered a general connection of *Zb* to wave function phase and his analysis never went beyond the free particle case. However, as Hestenes also remarks, the *Zb* is nonetheless present as a rotating phase factor in the Dirac wave function [19]. The present article has suggested a *physical interpretation* of the (complex) wave function oscillations and their connection to phase. Nor should the (paired) circular helices of Fig. 2 of the present physical model be too surprising; other investigators too, e.g. Hestenes [19], Natarajan [25], Niehaus [26] have proposed circular helical models (and interpretations) of the electron *Zb*; however, space precludes further discussion of these.

A physically realistic electron's state is a packet. Therefore the *Zb*, and thus the electron's zigzag motion (Fig. 3) during any arbitrary period, needs to be thought of as constituted of a

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[8] Roughly speaking, the electron 'collides' with its virtual positron counterpart at **A** (drops into a hole in a –ve energy electron sea—pair annihilation). It reappears at **B**, where it has been recreated (together with its virtual positron counterpart—pair creation) owing to the inverse process. At **B**, it again collides with its virtual positron counterpart, and reappears at **A**, etc.

[9] The double transition frequency $2mc^2/\hbar$ is also the accepted value of the rotational frequency associated with the intrinsic *spin* of an electron in motion with the momentum **p** [24], though I don't argue here that the presently proposed process *is* spin. Dirac associated *Zb* circulation with spin, see [19], and classical models of the electron exist in which spin and *Zb* are associated [2], [26].

superposition of a very large number of such zigzags, even an infinity of them, each associated with a different period, frequency and direction, and the periodic emission/absorption of the related virtual quanta. Such a case is covered by the group of solutions of de Broglie waves in (5). Each element of the group is undergoing the postulated basic process, and may be modelled by the pair of appropriately rotating vectors of Fig. 2.

It may be objected that using Euler's formula for a cosine function to model the underlying origin of the *Zb* and 'waving' of de Broglie waves seems rather simplistic. As it happens, however, Euler's formula is also important in perturbation theory (Sect. 9) for treating of perturbations that are periodic in time. Roughly speaking, the emission or absorption of energy by an atom to which a periodic force is applied arises from the splitting up of $\cos \omega t$ in an expression for the Hamiltonian of the system ($\hat{H}^{(1)}(t) = \hat{H}_0^{(1)} \cos \omega t$) into two exponentials (Euler's formula), carrying out time integration on the result and squaring it. Whether the atom loses energy or gains energy is determined by the exponentials, where $e^{i\omega t}$ is responsible for the loss of energy and $e^{-i\omega t}$ for the gain [29], much as in the proposed underlying double transition process. No problem there then.

But isn't this still overly simplistic? What about the fact that the pair annihilation\creation associated with the emission\absorption of a pair of photons is in QED just a symbolic representation of a single term in second-order perturbation theory? There are *infinitely many* terms in the perturbative expansion of the S (scattering) matrix, as Feynman diagrams reveal, with each term contributing to the result. *All* need to be taken into account, in the manner of Feynman graphs. Similarly, as already mentioned, the zigzag of Fig. 3 is just one of an infinity of such superposed zigzags. Moreover, as Penrose points out [31], an *actual* electron (as opposed to an idealized free one) will also be continually undergoing interactions with other particles, such as photons, all of which also need to be included in the overall superposition—but which don't appear in (5). Dirac's theory of the electron is not general enough. Hence QED (and QFT).

Indeed so. And when all *that* is taken into account in QED, one gets divergent integrals and nonsensical answers (infinities) to perfectly reasonable questions concerning the electron's mass and charge, requiring as a fix an ill-understood 'renormalization' of the answers—the so-called subtraction formalism. What about the present picture? Unexpectedly, it seems to offer a possible solution to the infinities problem of quantum theory. So far the description of the proposed underlying 'basic process' has mainly focused on its main elements using the simplest case of a double transition covering a clearly defined range of energy and Δt. Now it needs to be brought into closer correspondence with the mathematical formalism of QM and physical reality, where the transition can cover any range of Δt and include any number and kinds of interactions with other particles, even an infinite number of them. QED's subtraction formalism also needs to be brought into the picture. These tasks are attempted in the next two sections.

8. An uncertainty in the energy all the way up and down the energy scale: the electron as a virtual 'synchrotron'

So far in this article, the proposed 'basic process' has been mainly characterized in terms of a periodic double transition of a *particular* frequency $2E/\hbar$, where E is of the order of the rest energy E_R of the electron, with a consequent uncertainty of $\sim 2E_R$ in the energy of the system during the period $\hbar/2E$. However, that's an idealization. The energy is in fact intrinsically uncertain *all the way* up and down the energy scale, without limit, as the time t is better and better defined (i.e. as the uncertainty Δt in the time is reduced), as briefly described in Sect. 5. The better the definition of the time, the greater the frequency of the proposed process. We

now extend the basic process to include an arbitrary range of energies, as flagged in Sections 4 & 5.

Owing to the extended process, the unobserved electron is to be taken as periodically transiting to arbitrarily deep states of –ve energy and radiating virtual photons of arbitrarily high energies, along with the offsetting inverse process. It's as if it were always in the presence of a perturbing potential, making virtual transitions towards *all possible states of energy, and radiating virtual photons of all possible energies*. Even in orthodox QM, as Bohm states [5], in the presence of a suitable perturbation "it is necessary to imagine that the system fluctuates simultaneously into all possible states, so that it covers all states simultaneously"—i.e. the state is a linear superposition.

The emitted and absorbed mutually interfering virtual photons generated by the underlying (extended) basic process may themselves be associated with other even more fleeting transients, such as virtual electron-positron pairs. Since there is effectively little or no limit to the accuracy with which the time t can be defined, an unobserved free electron should be thought of during a sufficiently short period as akin to a *synchrotron* of virtually unlimited potential, periodically creating and annihilating the gamut of the known sub-atomic particles and their antiparticles, along with their intermediate bosons, all co-existing virtually in a monster superposition, described by some idealized Feynman diagram.

For example, we saw in Sect. 5 that the uncertainty in the energy enables the electron to exist during sufficiently short periods as an electron, an electron anti-neutrino, a muon neutrino, and hence as a muon (mediated by a W) and its virtual positron counterpart as a positron, an electron neutrino, a muon antineutrino, and hence as an anti-muon (mediated by a W).

The smaller the Δt, the more numerous and energetic the fleeting transients during each double transition. As $\Delta t \to 0$ and $\Delta E \to \infty$, the radiated energy $E \to \infty$, as does the frequency $\omega = E/\hbar$ of the periodic virtual double transition, while its period $\tau \to 0$. (Notice that as $\tau \to 0$, the electron's effective 'diameter' tends to zero.[10]) Then we have something like the conditions of the big bang even associated with a single electron—occurring periodically and ceaselessly with an unimaginably great frequency, but always concealed by HUP. It should be emphasized that this has nothing to do with measurement. Unlike in a normal synchrotron, we don't create the transients from the energy *we* put into the system. It is something that's taken as happening (virtually) all the time completely independently of us.

All possible states of the system already co-exist during sufficiently short periods, virtually, even in the absence of observation or measurement, and every electron and every other spin-half particle is always oscillating or rotating (undergoing the double transition process) with the frequency $\omega = E/\hbar$ *and period* $\tau = \hbar/E$, *where* $E \to \infty$ *and* $\tau \to 0$.

The above 'monster' periodic transition can, in principle, be Fourier decomposed into a linear superposition of periodic double transitions or wavings (rotations) of longer periods and lesser frequencies or energies, undergone by an appropriate number of virtual particles, or 'oscillators', consistent with Bohr's frequency condition $\omega_{mn} = (E_m - E_n)/\hbar$. That means that the particle undergoing the double transition process with $\omega \to \infty$ and $\tau \to 0$ *may be taken, equivalently and alternatively, as waving or rotating with all the different frequencies and periods in the relevant Fourier decomposition of the principal oscillation*. That's because during each double transition, the electron covers all the eigenstates in the superposition, including those of atomic fine structure, meaning that there are as many simultaneously

[10] At $\omega = \infty$, $\tau = 0$, the electron would be a point particle, since its radiated quanta would only have time to propagate a zero distance from their point of emission before their reabsorption.

existing oscillating virtual electrons\complex oscillators (together with their by-products) as there are terms in the superposition of states. Hence the infinitely many terms in the perturbative expansion of the S matrix, and the infinity of superposed *Zb* zigzags in a physically realistic version of Fig. 3.

We may focus on any one of these component or sub-wavings, such as the one occurring with the frequency $\omega \approx 10^{21} \mathrm{s}^{-1}$ (associated with the electron's rest mass energy), as we did earlier in this article. That's because we can take any energy as our zero of energy [13], and the presently proposed basic underlying field-generating oscillation is ever-present—a background *potentia*—equivalent to the fields of QED\QFT.

Whichever the selected energy\frequency, the ensemble of the resulting created and annihilated and mutually interfering transients constitutes an ever-present 'cloud', or field, of virtual quanta about the electron, similar to that of QED, representable in principle by a single wave function in a mathematical configuration space. Although virtual, the transients can be raised into real physical existence by providing the system with sufficient energy, as is indeed done every day in synchrotrons and linear accelerators. Precisely *how* this happens, i.e. why some particular member of the ensemble or linear combination of possibilities is selected by a measurement as the value actually observed, is known as the measurement problem. It was touched on in Sect. 5, but is beyond the scope of this article—as is how the proposed process might be extended to include the superposition of entangled systems. These are among the areas requiring further work.

Nonetheless, the proposed underlying physically real basic process has now been brought into closer correspondence with the mathematical formalism of QM and physical reality. The ensuing picture contains infinities, as does QED. Why, then, is a particle of *finite* mass and charge always observed despite the (potentially infinite) field energy surrounding it? Here is the present and perhaps surprising answer, falling out of the basic process in a natural way.

9. QED infinities & renormalization—the 'subtraction formalism'

The basic equations of QED were already known by the late 1920s. However, the main approach to the solutions of the equations by a method of successive approximations (perturbation theory) was halted by a seemingly insurmountable obstacle. Only the principal approximation gave results in agreement with experiment. The successive approximations gave divergent integrals and nonsensical answers. Consider an electron of energy mc^2. Even if 'at rest', it is always accompanied by its electromagnetic (or radiation) field. If the electron is treated as a point charge, the accompanying field energy is infinite. Because of the equivalence between mass and energy, the electron's mass, too, must be infinite. Similar arguments apply to the electron's charge. But those results are wrong: measurements give perfectly sensible finite results for the electron's mass and charge.

It was necessary somehow to 'cut off' the divergent integrals at high energies. But how, and what could be the physical justification for such a limitation?

The solution, known as renormalization, was based on the realization that not all the radiation corrections obtained in the higher-order approximations are observable. In particular, the *first* term in the perturbative expansion—the mass of a non-interacting, fictitious or 'bare' electron (i.e. an electron without a surrounding radiation field)—let us call it $m\_$—is non-physical and therefore not observable; similarly for the 'bare' charge (i.e. the charge in the absence of vacuum polarization). Even the radiation field itself is not directly observable, but only observable though its effects. However, in the second approximation, the sum

$m_o = m_- + \alpha\, m_1$ *is* an observable, representing the mass of the actual electron which interacts with the electromagnetic field. Here, m_o is the electron's *observable* or measured mass, m_- is the 'bare' mass, m_1 is the radiative correction to the mass given by a divergent integral, and α is the fine-structure constant (cf. [4], [30]).

This allowed for the idea that the two non-observables could offset each other. In particular, if we reduce the radius of the electron, the bare or 'undressed' electron mass could *decrease* in direct proportion to the increase in the physical electron's mass due to its increased field energy. The bare mass could even become negative, infinitely negative [30]. The two effects would cancel out, yielding the actual electron mass just as is observed; similarly for the electric charge:

$$m_- = m_o - \alpha\, m_1 \qquad e_- = e_o - \alpha e_1 . \tag{8}$$

The subtraction formalism works, at least for QED. But many regard it as a mathematical sleight-of-hand, along with the notion of a 'bare' electron. Nobody really knows why it works, except it is thought that unknown physics must come into play beyond the integration cut-off [38].

Here is the present heuristic's suggestion for what is *physically* happening and why the formalism works. Take an observable electron of mass m_o. As we've seen, the proposed periodic double transition\basic process has two simultaneously occurring 'legs' in the shadow of HUP. In what I'll call the 'first' leg, the electron jumps down into the –ve of its +ve energy state pursuant to the process. When it is *in* that state, having emitted away double its mass energy as virtual quanta (photons), think of it as the unobservable, non-interacting 'bare' electron of QED—not quite bare yet but certainly partially undressed—and now of *negative* mass m_- (potentially infinitely negative and fully 'bare' when $\Delta t \to 0$, as described in the preceding section). Think, too, of the mass energy\virtual quanta emitted pursuant to the jump down as QED's unobservable radiative correction m_1 to the partially undressed\bare mass m_-, where m_1 is the field energy divided by c^2 (also potentially infinite: as $\Delta t \to 0$, $m_1 \to \infty$). The two, m_- and m_1, necessarily always add up to the observable mass m_o, i.e. $m_o = m_- + m_1$ (equivalently, $m_- = m_o - m_1$). Similarly, $e_o = e_- + e_1$, where e is the charge. For simplicity we ignore here the fine-structure constant α.

In the 'second' leg, occurring simultaneously with the first, a –ve energy electron jumps up replacing the original electron, absorbing the emitted virtual quanta, i.e. the field energy m_1's contribution to the electron's partially undressed\bare mass m_- (as the case may be). The same relations apply to the –ve energy electron. Again, $m_o = m_- + m_1$. Thus, an electron of mass m_o is always observed, just as expected and a physical interpretation is given of both the field-theoretical bare mass and the radiative correction to the electron mass.

Above, we've made two important identifications: (i) we've equated the uninterpreted quantum-theoretical 'bare' or partially bare electron with the physically real electron when the latter has dropped into a –ve energy state pursuant to the proposed basic process; and (ii) we've equated the mass energy\'virtual' quanta *actually* emitted by the real electron pursuant to its drop, with the uninterpreted quantum-theoretical radiative correction m_1 to the theoretical mass m_-.

In this way, the proposed basic process\double transition process seems a natural fit with the mathematical subtraction formalism of QED. It seems to offer, at least in principle, a *physical* explanation of why m_o is always observed, rather than $m_o + m_1$, despite the potentially infinite electric field energy always surrounding the electron. Moreover, it does so without

detracting from the *reality* of the latter—the latter's ubiquity and capacity for interacting with matter or radiation. The explanation is a simple consequence of the proposed heuristic. In short, the proposal suggests an underlying *physical basis* for QED's successful but uninterpreted 'subtraction formalism' or renormalization to remove the infinities.

10. Conclusion

What is it that 'waves' in wave mechanics? This article has proposed an answer. The answer arises from an underlying fundamental physical process that every electron and every other spin-half particle is taken to undergo ceaselessly in the 'shadow' of the Heisenberg uncertainty principle. The process—a periodic double transition process between states of +ve and –ve energy—is an extension and generalization of an early idea of Dirac's. The postulated process is 'mapped' one-to-one with elements of the standard relativistic quantum-mechanical formalism. It is argued that this process is the underlying physical basis of the 'waving' of probability amplitudes in QM, and that it is this process that gives quantum particles their ability to be in a superposition of states—even an infinity of states—and to self-interfere when they are unobserved. It is further argued that it is the underlying generator of the Schrödinger *Zitterbewegung*.

The process offers a *physical* explanation, as contrasted with a purely mathematical one, of the role of QM's ubiquitous and mysterious complex phase factors, carried by every true wave function. The picture put forward takes the phase factors of ψ waves, representing spin-half systems, as mathematically modelling an *actually occurring* underlying periodic double transition process (a simultaneous –ve & +ve energy 'catastrophe') which all spin-half particles undergo.

In particular, it is argued that the proposed underlying process is (in effect) modelled in the formalism of standard QM by the +ve and –ve frequency rotations of the state vectors with the exponentials $e^{i\omega t}$ and $e^{-i\omega t}$ in the most general solution of the Dirac equation for an electron in an electromagnetic field. It is shown how the proposed double transitions also determine the details of the related 'waving' of spin-one ψ waves, such as photons, created by the transitions, even though the latter don't themselves undergo that process.

The proposed picture takes the double transition process as the underlying reality and the rotation of the state vectors as its mathematical model. In contrast, standard quantum theory is forced to put in by hand both (i) the wave or field aspect of matter, and (ii) the particle aspect of the quantum-theoretical fields, for the purpose of modelling the experimentally observed dualistic behaviour of matter. They cannot be derived and no physical explanation is on offer.

Composite systems are brought into the present picture. Possible objections are noted and responded to. It is shown how the double transition process seems to go some way to providing an underlying physical explanation of why the ψ wave amplitudes turn out to be *probability* amplitudes, which is something not explained by Born's rule, which is a postulate.

Finally, an account is given of how the proposed process seems to shed light on the QED problem of infinities—why a finite particle mass and charge is always observed despite the potentially infinite field energy surrounding a particle.

It is emphasized that this article is put forward as a heuristic—a new way of looking at some old things. It is by no means a definitive account of those aspects of QM it covers, and many gaps in the explanations need to be filled. For example, it hasn't touched on how the heuristic might be extended to include the superposition of entangled systems, nor does it specifically engage with the measurement problem. However, it does seem to this author that the proposal may provide a useful starting point for the required further work, and perhaps

even a point of vantage from which to examine anew the ontic state of the wave function.

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