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Original Paper

Hidden Variables Foundation of Matrix Mechanics

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Abstract: Bell inequalities violation is generally interpreted to rule out local and/or non-contextual hidden variables theories. Recently, an actual hidden variables model in matrix mechanics formulation was presented which is based on ontologically classical endogenous motion. All results of standard matrix mechanics are reproduced. A critical feature of the model is that it reproduces the mathematics of quantum observables including measurability, and hence quantum experiments. This feature can be a characteristic of a given class of hidden variables theories. There is then a direct conflict with the consensus interpretation of violation suggesting a need to reconsider Bell's theorem. It is found there is an additional assumption restricting the type of hidden variables theories inequalities represent by excluding those which reproduce quantum mathematics. Any theory which reproduces the mathematics of quantum observables is thereby not subject to Bell-type constraints.

Keywords: Quantum foundations, hidden variables theories, Bell's theorem, inequalities

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1. Introduction

Bell inequalities originate from Einstein-Podosky-Rosen (EPR) questioning the completeness of quantum mechanics (QM) and von Neumann's proof that the quantum description is complete [1]. The much considered inequalities, which have been rigorously tested, are central to exploring quantum foundations. Experiments verify the mathematical correctness of quantum mechanics and incorrectness (usually termed violation) of the inequalities. Violation is commonly interpreted to exclude, among other possibilities, local/non-contextual foundational theories.

An *actual* hidden variables (HV) model in matrix mechanics formulation was recently published [2]. Heisenberg's non-path postulate was replaced by an ontologically classical endogenous periodic motion describing transition paths. Locality and non-contextuality are both preserved. Obviously, this ontology directly contradicts the consensus. Mathematically, the proposed micro-paths replace transition amplitudes as elements in Born-Jordan infinite order matrices. All standard results are reproduced. The endogenous motion is found to average out over a cycle, and so, is unseen by the wave function. Nevertheless, the standard non-commutation for Schrodinger operators is reproduced. Hence the wave function is not affected.

The proposed foundational model is subject to the more stringent criteria of reproducing the mathematics of quantum observables, including measurability. As such the model is independent of Bell inequalities. For the purpose of this discussion however, it is not the model specifics which are of interest but rather general characteristics which can define a given type of hidden variables theories (HVT). The most critical feature is that the model is of a HVT-type which reproduces the mathematics of quantum observables including measurability. Any such HVT irrespective of its ontology must reproduce QM experiments including Bell-type.

While the HVT-type as defined has been identified by the proposed model it can be hypothesized independently. Also, its ontology is not defined.

While Bell's theorem has inspired many interpretations, this discussion is narrowly focused on the constraints violation imposes on the class of HVT identified, and in particular the model being proposed. Although the theorem is predominantly about testing assumptions, it is also a predictor of possible HVT. That means any *actual* HVT is a test of Bell's theorem. Meaning, it is the theorem whose veracity is under scrutiny.

The following discussion is also informed by the classical ontology of the HV quantities, and the wave function not being affected. A consequence of the latter is that completeness of QM and that of the wave function are de-coupled.

Revisiting Bell's theorem to accommodate the HVT of the type identified leads to contradictions which, to maintain logical consistency, requires an additional assumption of excluding theories which reproduce QM mathematics. Bell's theorem is thereby not relevant to such theories.

Broader issues associated with the inequalities will not be explored. Departure of quantum mechanics from classicality in relation to the proposed model has been previously discussed in the context of Mermin's EPR conundrum [2]. Familiarity with Bell's theorem is assumed [3].

2. Discussion

Despite ambiguity over its physical meaning, quantum mechanics is a mathematical calculus for predicting quantum experiments. Any HVT which reproduces the mathematics of QM will also predict quantum outcomes, including Bell-testing experiments. This reasoning was used by Bell in regard to Bohmian mechanics as an example of an actual non-local HVT which should not be possible according to von Neumann's proof. For Bell the issue was whether non-locality would be a feature of any HVT. EPR had assumed locality by virtue of it being the simplest explanation.

Bell's primary objective was to place the EPR paradox in an empirically testable context. Any conclusion on locality would then be evidence based and must be accepted as a fact-of-nature to be accommodated in any HVT. At issue is the degree to which this criterion is met by the inequalities.

Inequalities are a non-QM modelling which does not reproduce QM experiments. Therefore the inequalities are incorrect. Since their purpose is to test assumptions, incorrectness is not an immediate concern. Possible fatal assumptions are identified by inspection. Any conclusion is then conditional on there being no further unidentified assumptions. That the current list of candidate assumptions is complete is not established. This short coming challenges whether inequalities conclusions meet the stronger criterion.

Bell introduced a mathematical condition which in the physical experimental context can be an assumption of locality. Violation would then establish non-locality as an empirically supported *option*. However, to meet the stronger criterion requires additional assumptions.

Either locality is the only possible fatal assumption or non-local inequalities are not violated. Neither of which is established.

Other potential fatal assumptions have been identified notably non-contextuality, statistical independence, and recently, additivity of expectation values [4, 5, 6, 7]. This list is a selection only. Determining the merits of each assumption is not relevant to this discussion. Interestingly, Bell's own latter work with original EPR observables which shows *no disagreement* with QM has attracted very little attention [8]. There is also an interesting discussion that violation excludes *classicality,* which could encompass some if not all of the above assumptions [9]. This is not however a simplistic statement that quantum is not classical mechanics. Rather, it suggests inequalities are about the characteristics of QM. Speculations on the nature of HVT or metaphysical realities would then be suggestive.

Bell introduced a hidden variables parameter which is claimed to be completely general in representing "additional variables" even to the extent of possibilities not yet thought-of [3]. Because of its generality the parameter is not mathematically defined. That means the mathematics of hidden variables quantities, HVT and their relation with QM does not feature in Bell analysis. For the proposed model however these mathematics are defined. That creates additional conditions to be considered in Bell's theorem.

Via conditional probability definition a *statistical relation* is established between the parameter and the HVT it represents, and experimental outcomes. To be consistent with the locality condition the parameter must represent a local HVT. A non-local HVT would lead to the contradiction where within the same model locality is conditioned on non-locality. Violation would then result from flawed logic.

Because of violation, it is concluded that any local HVT cannot reproduce quantum experiments and so be rejected. However, for the type of theory reproducing quantum mathematics, and thereby QM experiments, there is again a logical contradiction where an experimentally correct HVT conditions incorrect inequalities. Logical consistency can be preserved by an additional assumption on the type of HVT which is relevant to the inequalities. Those reproducing QM mathematics cannot be included. Bell's theorem is then irrelevant to such theories.

An influential alternative to locality is contextuality leading to rejection of non-contextual HVT. The above reasoning also applies to assuming non-contextuality. Another possible fatal assumption, usually termed statistical independence, is that HV are independent of experimental settings. Various interpretations are given for its violation including contextuality and superdeterminism [4, 6]. The stated reasoning also applies to this assumption.

This basic reasoning applies generally: a HVT which reproduces QM mathematics and is thereby correct cannot, while maintaining logical consistency, facilitate conditions leading to incorrect inequalities.

A recent study, which also follows from an *actual* HVT, shows that Bell's theorem can also be based on an additional assumption of the additivity of expectation values [7]. The argument is in two parts. Firstly, it is shown that the standard Bell inequality is a tautology, which assumes its conclusion where the premise is also the upper bounds but expressed differently. Secondly, a correction term is added to the standard expression to accommodate non-commutation, leading to a modified inequality whose bounds are as predicted by QM and observed experimentally. As well as local realist, the HVT of the study is specifically contextual. That follows from hidden variables quantities of the initial *actual* HVT being non-commuting.

In terms of the HVT-type being considered the same conclusion of irrelevance applies to the first part since it is just the standard Bell-type inequality. For the extended inequality, which accommodates non-commutation and thereby the relevant quantum mathematics, there is no logical contradiction. Again, a HVT which reproduces quantum mathematics and so experiments, conditions an inequality which also reproduces experiments.

The HV quantities of the proposed model are ontologically classical so that local realism and non-contextuality are preserved. Since these quantities are incorporated as matrix elements in Born-Jordan infinite order matrices, non-commutation of quantum observables is preserved by the usual mathematics of matrix algebra. This is not however the only possible connection between hypothetical hidden variables and the mathematics of QM observables. Hidden variables observables can be directly non-commuting if the space and time physical quantities they represent are defined in a non-metric space subject to projective rather than Euclidean geometry [10]. The proposed model is an evolution from an original non-metric space model. Remaining in a Euclidian space was found to be the simpler explanation.

As briefly discussed, the proposed model and Bohmian mechanics do not *necessarily* contradict [2]. Since the proposed HV quantities do not impact on the wave function it remains as described by the Schrodinger equation [2]. This enables an alternative interpretation to non-locality for interdependence of particle positions in Bohmian mechanics; the features which most interested Bell. An ensemble of quantum particles moving along deterministic endogenous paths, and subject to conservation constraints, *may* be explained by synchronicity rather than non-locality. There is also commonality in that both approaches introduce position as a foundational quantity describing deterministic motion.

Any HVT reproducing quantum mathematics also includes a hypothetical General theory of the type where QM is a limiting case, in the manner of Newtonian mechanics in relation to Relativity as an example. For such a theory the above reasoning also applies since Bell-testing experiments are in the domain of QM as a limiting case. Bell's theorem is again irrelevant. By excluding significant classes of HVT the theorem is then restricted to only include theories which reproduce quantum experiments but not quantum mathematics. But are such theories possible?

Under the plausible expectation that a HVT which reproduces experiments would also, even if approximately, reproduce quantum mathematics, it is interesting to ask what theories are applicable to Bell's theorem. That raises the disturbing question whether inequalities are incorrect because they include only incorrect foundational theories.

3. Conclusion

An actual local, non-contextual hidden variables model in matrix mechanics formulation was recently published. A critical feature of the model is that it reproduces the mathematics of quantum mechanics, and hence, outcomes of quantum experiments. This feature can be hypothesised to be an aspect of any hidden variables theory. Any such theory, irrespective of its ontology, must reproduce quantum experiments including Bell-testing. That leads to a direct contradiction with the consensus conclusions on violation. On examination it is found that to maintain logical consistency inequalities must make an additional assumption excluding hidden variables theories which reproduce quantum mathematics. Hence, Bell constraints do not apply to such types of theories, including the proposed model.

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