Quantum Speculations **5** (2023) 1-19

Original Paper

Does the "Complex" Wave Function in Quantum Mechanics Represent Anything "Real" at all?

Natarajan T S

Department of Physics, Indian Institute of Technology, Tirupati - 517506, Andhra Pradesh, INDIA.
E-mail: tsniit@iittp.ac.in

Received: 26 April 2023 / Accepted: 15 June 2023 / Published: 24 June 2023

Abstract: "Quantum Reconstruction" attempts to rebuild the highly successful Quantum Mechanics (QM) from scratch to understand the "real" meaning of its mathematical structure. In addition, perhaps, we must re-look at the role the constant 'c' plays in physics. It would be shown here that this constant has a more crucial role at the foundations than what Relativity envisaged. It was Einstein, who postulated in his Special Theory of Relativity (SR), that the velocity of light is invariant for all inertial observers! This is counter-intuitive. Another mystery from QM is Schrodinger's "zitterbewegung" (ZB) phenomenon which is a mathematical extension of Dirac's free electron theory. By integrating these two concepts into physics at the foundational level we can rebuild a fairly consistent model which seems to unify SR and QM by giving a geometrical interpretation to the "complex wave-function" as representing a helical trajectory of particles like electrons. Helix being a geodesic on a cylinder accommodates "quantization of energy" and is a three-dimensional wave having all the properties that we are familiar with the 2D wave. Thus by postulating an internal structure to these fundamental particles consistent with ZB, many of the results of QM and SR which are at present purely based on intuitive mathematics, can be understood in a simple and "realistic" way.

Keywords: Quantum reconstruction, Special theory of Relativity, Quantum mechanics, zitterbewegung, Complex wave-function, helix, geodesic, Schrodinger's wave equations

---

1. Introduction

Quantum Mechanics (QM) has been in existence for almost a century now. But there is hardly any consensus among physicists across the world on what the quantum theory says about reality. In recent times some scientists are engaged in "quantum reconstruction" which is an attempt to rebuild the theory from scratch based on a few simple principles. In this context, it is interesting to observe, that the other "twin" of QM, namely, Einstein's theory of Special Relativity (SR) enjoys orders of magnitude higher levels of "acceptance" among physicists. This is despite equally weird results of SR such as time dilation, length contraction, mass increase with velocity, etc. It might be necessary perhaps to look at both these theories together and take into account a few of the possible inconsistencies in these while attempting to rebuild a new theory.

To start with, it might be useful to investigate whether the enigmatic fundamental constant 'c' the velocity of light in free space has any other role in physics other than setting an upper limit to the velocities that material particles can reach! It was Einstein who first recognized this constant and its role in his theory of Special Relativity (SR) [1]. He postulated that the velocity of light is invariant even when measured from an inertial frame moving uniformly with a very high velocity close to 'c' . It is counter-intuitive. **If we believe that the magnitude of this constant cannot be affected by the motion of the frame from which it is measured, then its converse that this constant 'c' on its part cannot affect the events happening in these frames must also be true!**

Einstein wrote this condition for the light beams in two inertial frames (S and S') in relative motion as

$$x^2 + y^2 + z^2 - c^2t^2 = 0 = x'^2 + y'^2 + z'^2 - c^2t'^2 \tag{1}$$

He went further and used this condition which, by his postulate, is **true only to light beams** as a necessary condition to derive the transformation equations between frames for any event! Thus it appears that the consequences derived from these Einstein-Lorentz transformation equations must be valid **only for light beams!** However, the results of SR such as time dilation, mass increase with velocity. etc., have been proved to be true time and again for even particles such as electrons, muons, etc., and the factor responsible for these effects is, $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, which depends on this very constant! Hence we must assume that this equation (1) must have much **wider applicability beyond light beams, to even fundamental particles such as electrons and muons!**

Dirac's relativistic wave equation developed to be consistent with equation (1) has some clues in this direction. When applied to free particles such as electrons (fermions), Dirac gets their instantaneous velocity to be $\pm$c [2]. This is a strange result and this confirms that the above equation (1) must be true not only to light beams but also to electrons, etc.

However, nobody was ready to recognize this! Again, Schrodinger extended the ideas of Dirac's theory and discovered a new phenomenon, namely, "zitterbewegung" (ZB) [3]. Several papers have been published till now to show that ZB is a real phenomenon where the electron has an intrinsic oscillatory motion which is circular with a radius equal to Compton wavelength and instantaneous velocity equal to 'c'[4][5][6][7] . It was also shown that it can account for the intrinsic "spin" angular momentum of electrons [8] [9]. **Thus It is interesting that, while physicists are ready to accept Dirac's equation they did not want to integrate this ZB motion which is also part of his theory into physics!**

In addition, there are a few more unanswered questions in SR. When one considers the strange but experimentally established result, namely, the mass increase with velocity, **one wonders, "What prevents the electron from getting accelerated by an external electric field to velocities beyond 'c'?"** This truly be due to some kind of a **constraint imposed by an internal structure** of these particles!

Einstein's profound equation $\mathrm{E} = \mathrm{mc}^2$ makes an inconspicuous entry into physics as a corollary of a simple exercise to obtain transformation equations between frames of reference in relative motion! This mass-energy equivalence is indeed at the very foundations of Physics! **Will it be possible to derive this equation independently from QM which is indeed the theory concerned about mass, energy, momentum, etc., of particles and their interactions?**

Another amazing result of SR is Time Dilation. One of the dramatic proofs of this result came from the study of decay times of muons in cosmic ray showers. It showed that muons live purely according to their 'proper time'. However, we must find out **which clock keeps time in muon's 'rest frame'?** Maybe, an internal periodic motion as evidenced by ZB might help in this respect.

All the above facts lead us to believe that the fundamental particles like electrons must have an internal structure and in addition, the constant 'c' must play an important role in that design.

No one can indeed replace the mathematical structure of existing theories as they have been impeccably precise to amazing levels of accuracy, be it SR or QM! However, it is worth investigating whether a new insight into the structure of these fantastic theories based on a new role for the constant 'c', can advance our understanding of the underlying reality without imposing major revisions in the mathematical framework. This is the motivation of the "quantum reconstruction" attempted here. Of course, it is a completely different approach compared to what one comes across as it takes on boldly both the theories QM and SR together[10]!

In the larger interest of Science and Physics, it would be singularly important to keep an open mind and go through the rest of the model presented here **however 'crazy' it might appear** in the wake of the voluminous data accumulated over the past century.

2. The Postulates

POSTULATE I Every microparticle (such as electrons) is endowed with two types of motion, namely, Internal Motion, and External Motion. The "intrinsic" internal motion is circular in a plane perpendicular to its possible external motion with a radius characteristic of the mass of the particle.

POSTULATE II The angular momentum and the radius of the internal motion are unaffected by the external motion and are therefore the same for all observers.

POSTULATE III The 'action' performed during one period of its internal motion is equal to $\frac{h}{2}$, while that due to its external motion is equal to h where h is the Planck's constant.

POSTULATE IV The magnitude of the instantaneous velocity tangential to the motion of the particle which is the resultant of its internal and external velocities is always equal in magnitude to 'c', the velocity of light in free space. This makes this constant 'c' truly universal.

POSTULATE V The internal periodic motion postulated is the "wrist-watch timer" of the fundamental particle to reckon the "proper time" in its rest frame. Thus fundamental particles carry their clock which also integrates effectively Space and Time into a continuum.

POSTULATE VI Physical laws are the same in all inertial frames.

3. Discussions

When the particle has no external motion $(v_{ex} = 0)$, the action, $S = \int p.ds$ corresponding to one revolution along its internal motion is

$$m_0 c 2\pi a_0 = h/2 \tag{2}$$

where $p = m_0 c$ is the instantaneous momentum, ds is an infinitesimal displacement along the orbit, m_0 and a_0 are the rest-mass and radius of internal motion respectively. Hence we get $a_0 = \frac{\hbar}{2m_0 c}$ where $\hbar = \frac{h}{2\pi}$. This value $a_0 \sim 1.93 \times 10^{-13}$ m corresponds to half the value of the Compton wavelength. The angular momentum of this intrinsic motion which is identified with the "spin" of the particle, is $m_0 c a_0 = \hbar/2$. Hence it becomes clear that there is no such thing as a "state of rest" for these particles and there is also energy associated with this internal motion which we can identify as its "zero point" or "rest" energy.

When the particle is found to move, say in the laboratory, due to some external force it does so along the direction perpendicular to the plane of its internal motion and hence it effectively moves along a cylindrical helix. Thus the velocity that we normally measure for the particles is just its external velocity along the axis of the helix. It should be recognized that the helix is a 3D wave. The word "wave" when uttered invariably generates in everybody's mind a stereotyped image of a 2-dimensional wave! The projection of the helical trajectory on any two mutually perpendicular planes having the axis of the helix as the line of intersection would be sine and cosine waves. The helical trajectory of fundamental particles ensures "directivity" as they spiral through space! The "pitch" of the helix, which is considered as the de Broglie wavelength is the distance traveled by the particle along the axis during one period of revolution around the axis. If the external momentum of the particle is 'p' along the z-axis and 'λ' is the wavelength of the helix then, the action corresponding to this external motion is $p.\lambda = h$. Thus we obtain an expression $\lambda = \frac{h}{p}$ which is de Broglie's expression. Further, we obtain the expression $p = \frac{h}{\lambda} = \frac{h}{2\pi}\frac{2\pi}{\lambda} = \hbar k$, where $\hbar$ is the reduced Planck's constant and k is the wave-vector. This becomes three dimensions

$$\mathbf{p} = \hbar\mathbf{k} \tag{3}$$

Thus the "wave nature" of fundamental particles can be understood through this helix!

It is worth digressing a bit here and observing amidst our surroundings, the ubiquitous nature of the helical path. It already exists among the largest astronomical objects such as planets, stars, etc., to the smallest micro-organisms! We need to recognize that all the planets around the sun, even the sun, and other objects do not describe closed orbits in the form of circles or ellipses, but their 'world line' are helices. [11]! Again, it is known that many microorganisms, such as bacteria, protists, and sperms, perform a kind of self-propelled swimming act in an aqueous environment along a helical trajectory to actively find food, escape predators, or produce off-springs to sustain their species [12][13].

4. Representation of the Helix

The parametric equations for a helix are $x = a_o cost; y = a_0 sint; z = t$ where a_0 is the radius of the cylindrical helix. Let us obtain a representation of the helix in terms of the internal and external motions. We will first consider the case of the particle which has only the internal motion in a frame S' at rest. Such a particle, is moving along a circle, say, in the xy- plane with a radius a_0 and an angular velocity ω, about an axis passing through its center and along the z-axis. The instantaneous position of the particle with respect to a coordinate system having the origin coinciding with the center of the circle is given by the tip of the radius vector rotating (in a counter-clockwise direction) which can be represented by

$$x = a_0 \cos\phi; \; y = a_0 \sin\phi; \; z = 0$$

where $\phi = \omega t$ is the angle swept by the tip in a time interval t seconds. The choice of the x and y axes is arbitrary due to the inherent cylindrical symmetry of the system. Hence, here we make use of the symbol "$\mathbf{i} = \sqrt{-1}$". We interpret this symbol $\mathbf{i}$ as a geometric operator in the well-known fashion, i.e., the operation of $\mathbf{i}$ on a vector (multiplying the vector by $\mathbf{i}$) rotates the vector by 90° in a counter-clockwise direction without changing its magnitude. Therefore, the position vector $\mathbf{r}_{xy}(t)$ in the xy-plane corresponding to the particle in this representation becomes,

$$\mathbf{r}_{xy}(t) = (a_0 \cos \omega t)\, \boldsymbol{\varepsilon}_x + (a_0 \sin \omega t)\, \boldsymbol{\varepsilon}_y + (0)\boldsymbol{\varepsilon}_z$$

where $\boldsymbol{\varepsilon}_x, \boldsymbol{\varepsilon}_y$, and $\boldsymbol{\varepsilon}_z$ are the unit vectors along x, y, and z axes and $\mathbf{r}_{xy}$ is the position vector on the xy plane. Since $\boldsymbol{\varepsilon}_y = i\boldsymbol{\varepsilon}_x$, and by Euler's identity

$$\mathbf{r}_{xy}(t) = a_0(\cos \omega t + i \sin \omega t)\boldsymbol{\varepsilon}_x + (0)\boldsymbol{\varepsilon}_z == a_0 e^{i\omega t} \boldsymbol{\varepsilon}_{\boldsymbol{x}} \tag{4}$$

If ω is clock-wise then,

$$\mathbf{r}_{xy}(t) = a_0(\cos \omega t - i \sin \omega t)\boldsymbol{\varepsilon}_x = a_0 e^{-i\omega t} \boldsymbol{\varepsilon}_{\boldsymbol{x}} \tag{5}$$

Any one of these two equations (4) & (5) represents a particle having no external motion and "at rest" but they differ in the sense of rotation along its internal motion. If we plot these each would describe a helix along the time axis.

If the same particle is observed from another frame S, with reference to which S$'$ is moving with an external velocity v along the positive z-axis, then to this observer the particle would be describing a helix with the z-axis as the axis of the helix. After a time t, this particle would have covered a distance $\mathrm{z} = v\mathrm{t}$ along the z-axis (assuming at $\mathrm{t} = 0$, the two origins coincide). Hence the instantaneous position vector from the origin in S would be represented by the pair of equations:

$$\mathbf{r}_{xy}(t) = a_0 \boldsymbol{\varepsilon}_x e^{i\omega t},\ \mathbf{r}_(z) = \boldsymbol{\varepsilon}_z v t \tag{6}$$

$\mathbf{r}_z$ is the distance covered during time t along the z-axis (axis of the helix). We can also represent the same motion equivalently as follows: Let the particle travel a distance of z units along the z-axis during a time interval t and 'λ' the pitch of the helix, and T the period of internal motion during which the particle sweeps an angle 2π in the xy-plane. The angle swept for distance z, $\phi = \frac{2\pi}{\lambda} z = kz$ where $k = \frac{2\pi}{\lambda}$ is the wave-vector. Hence

$$\mathbf{r}(z) = a_0 e^{ikz} \boldsymbol{\varepsilon}_x \tag{7}$$

Equation (4) represents the orientation of the position vector with respect to time at a given z coordinate, while (7) represents its orientation at a given time t as we go along the z-axis. Both these equations trace helixes with the corresponding axis, the former with time and the latter with the z-axis as its axis. Again, the orientation of the particle at any t is the same as that at an earlier time $t' = \left(t - \frac{z}{v}\right)$. Again, since $\omega = 2\pi\nu$ where ν is the frequency of internal motion and $v = \nu\lambda$, we can rewrite equation (4) replacing t' by t, $\mathbf{r}(z,t) = a_0 \boldsymbol{\varepsilon}_x e^{i\omega\left(t - \frac{z}{v}\right)}$. Since $\frac{\omega}{v} = k$ we can write,

$$\mathbf{r}(z,t) = a_0 \boldsymbol{\varepsilon}_x e^{i(\omega t - kz)} = a_0 \boldsymbol{\varepsilon}_x e^{-i(kz - \omega t)} \tag{8}$$

This represents the orientation of the instantaneous position of the particle explicitly with respect to both space and time coordinates. This is the well-known "plane wave" representation in physics. Figure1(a) shows the simulation of this "complex exponential" function where we see the evolution of the helix and also the projection of the helix on two mutually perpendicular planes having the axis of the helix as the line of intersection representing "Sine" and "Cosine" waves [14] [15]. Hence every particle in this model is automatically endowed with a wave aspect. Figure1(b) shows the position of the particle for two arbitrary values (z_1, t_1) and (z_2, t_2) at the tip of the radius vector drawn on xy-plane which is propagating continuously along the z-axis.

Figure 1. (a) 3D Simulation of complex wave function using the mathematical tool. The trajectory is a helix. Its projection on any two mutually perpendicular planes to the direction of propagation gives sine and cosine waves.
(b) Represents the position vector corresponding to two values (z_1, t_1) and $z_2, t_2)$ from the origin.

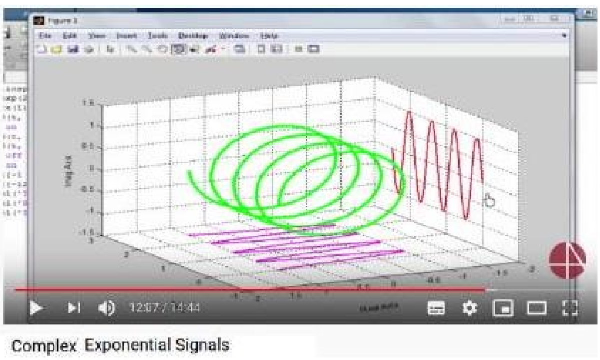
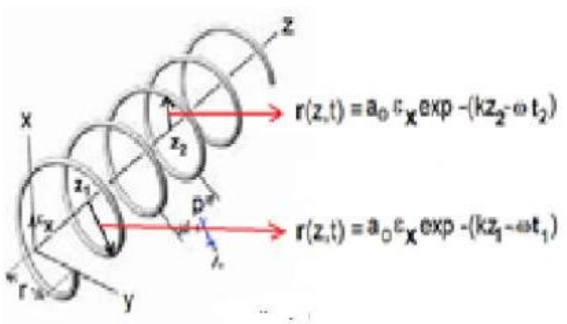


Figure 1(a)



Fibure 1(b)

There is nothing "complex" about this. This is an amazing result in the sense that if we attempt to represent the motion of a free particle electron (say), along a helix it leads to a "complex" representation as given in equation (8) above. This model has, therefore. given a new phenomenological interpretation of the complex wave function. This 3D wave represents a particle traveling along the positive z-axis and the expression $\mathbf{r}(z,t) = a_0\boldsymbol{\varepsilon}_\mathbf{x}e^{i(kz+\omega t)}$ represents a particle traveling along the negative z-axis. In QM, the wave-function $\Psi = Ae^{-i(kz-\omega t)}$ represents the probability amplitude according to the Copenhagen Interpretation. There are in general two distinct helices, the left-handed and the right-handed which are mirror images of each other and they cannot be made to coincide through simple geometric transformations! In our familiar wave equation the coefficient a_0 represents the magnitude of the "displacement" (amplitude) of the wave disturbance in 2-dimensions (xz-plane, say). But here the amplitude is constant (radius of the helix) in 3-dimensions. At this juncture, it will be interesting to observe the simulated complex wave function tracing helix in the Quantum Wave Function Visualization obtained by Eugene Khutoryansky in his brilliant series of Physics Simulation Videos [16].

5. Quantization of Energies

Helix is a geodesic on a cylinder! That means, we can apply the well known Euler-Lagrange equation from the Calculus of variations, to derive the equation for the "shortest path" between two points A and B, say, a distance L apart, parallel to the axis of an imaginary cylinder The result we obtain is helix [17]. The extremum paths accommodate not just one helix but a family of helices! We obtain helices that pass through point A and encircle the cylinder once, twice, thrice, etc., before reaching point B. See Figure 3. During a given unit of time, corresponding to the "fundamental" helix, the other helices must complete two, three, etc., revolutions around the imaginary cylinder! Thus the velocities (or momenta) of the particles corresponding to these helices would be increasing as they go round the cylinder multiple times during the same interval. Correspondingly their wavelengths (pitches) would decrease as required by the de Broglie relationship $\lambda \propto 1/p$ where p is the momentum. The external kinetic energies of these particles would also increase as we consider the higher modes.

Now let us consider a particle trapped between rigid walls at $\mathrm{z} = 0$ and $\mathrm{z} = \mathrm{L}$ as in the standard problem in QM corresponding to 1 D infinite potential well. Inside the well, the potential is zero and outside these limits the potential is infinity. The particle is having some kinetic energy initially and when it hits the walls at the two limits, its velocity component perpendicular to the walls gets reversed. In the case of helical waves, it means the "handedness" of the helix would change on reflection at the walls. The left-handed helix would become right-handed and *vice versa*. Hence the particle moves to and fro suffering reflections at the turning points on the walls. This is no different from the case of two 2D waves traveling back and forth leading to time-independent "standing waves" where the amplitude at any intermediate z value remains the same with time. Here too, we generate what are known as "nodes" and "antinodes". If we perform a simulation with a helix in "fundamental mode" traveling in opposite directions, we find at both the walls the particle is momentarily at rest (nodes), and in between at $z = L/2$ the instantaneous positions of the particle traveling in opposite directions are the farthest (equal to the diameter of the imaginary cylinder on which the helix is evolving). Let us typically assume that the particle is at the point $(a_o, 0, 0)$ when at $z = 0$, then in the fundamental mode it would be at $(-a_o, 0, L)$ at $z = L$. At the point $z = L/2$, the particle would be either at $(0, a_o, L/2)$ or at $(0, -a_o, L/2)$, that is along the Y-axis. That means, at "antinodes" the particles will be separated by $2a_o$ along the Y-axis. When we consider higher "modes" the nodes will be at $+a_0$ and $-a_o$ along X-axis alternately, while at antinodes they will be at a $2a_o$ distance apart along Y-axis. In the fundamental "mode" there is no node between 0 and L along Z-axis. However, for higher modes, we will see additional nodes appearing at $L/2$ for the second harmonic, and two more nodes at $L/4$ and $3L/4$ for the third harmonic, etc. See the simulation results in Figure 4. The pitches of the helices (de Broglie wavelength) will go as $L = n\frac{\lambda_n}{2}$ or $\lambda_n = \frac{2L}{n}, n = 1, 2, 3$, etc. The corresponding wave vectors (k_n) would go as $n\frac{\pi}{L}$. Hence the kinetic energy of the external motion for Newtonian velocities would go as

$$K_n = \left(\frac{(hk_n)^2}{2m}\right) = n^2\left(\frac{h^2}{8mL^2}\right) \tag{9}$$

This is exactly what we obtain when we solve the time-independent Schrodinger equation for this case.

The trajectory of the particle confined to the infinite potential well can be described by the time-independent part given in equation (7), namely, $\mathbf{r}_1 = a_0 e^{ikz}\boldsymbol{\varepsilon}_x$ in the forward direction. The reflected path will be given by $\mathbf{r}_2 = a_0 e^{-ikz}\boldsymbol{\varepsilon}_x$ and hence the vector displacement between the two points corresponding to the paths at any given z value would be given as

$\mathrm{r}_{1-2} = a_0 e^{ikz}\boldsymbol{\varepsilon}_x - a_0 e^{-ikz}\boldsymbol{\varepsilon}_x = a_0\left(e^{ikz} - e^{-ikz}\right)\boldsymbol{\varepsilon}_x = a_0 2i\,\mathrm{Sin}(kz)\boldsymbol{\varepsilon}_x = a_0 2\,\mathrm{Sin}(kz)\boldsymbol{\varepsilon}_y$ since $\mathrm{i}\boldsymbol{\varepsilon}_x = \boldsymbol{\varepsilon}_y$. If we apply the boundary condition corresponding to the "fundamental helix": At $\mathrm{z} = 0, \mathbf{r}_{1-2} = \mathbf{0}$; and at $\mathrm{z} = \mathrm{L}, \mathbf{r}_{1-2} = a_0 2\,\mathrm{Sin}(kL)\boldsymbol{\varepsilon}_y$, or since $k = \frac{\pi}{L}, \mathbf{r}_{1-2} = \mathbf{0}$; and finally at $\mathrm{z} = \mathrm{L}/2, \mathrm{r}_{1-2} = a_0 2\,\mathrm{Sin}\left(\frac{\pi}{L}\cdot\frac{L}{2}\right)\boldsymbol{\varepsilon}_y = 2a_0\boldsymbol{\varepsilon}_y$. This is indeed true, as $\mathrm{z} = 0$ and at $\mathrm{z} = \mathrm{L}$ we get "nodes" where the particle reach the same point on both ways, and at $\mathrm{z} = \mathrm{L}/2$, which is an "antinode" the position of the particles on both ways are separated by $2a_0$ along the y-direction. See figure4. It will be interesting to compare the expression obtained for the wave-function according to Schrodinger's equation in the region of the well, which is $\Psi = \sqrt{\frac{2}{L}}\,\mathrm{Sin(kz)} = \sqrt{\frac{2}{L}}\,\mathrm{Sin}\left(\frac{\pi}{L}\cdot z\right)$ for the case of $\mathrm{n} = 1$, which matches except for the "normalization constant"!

Figure 2. Simulation results showing, between two arbitrary points A & B a distance L apart on an imaginary cylinder, one can obtain a family of helices (geodesics), which go around the imaginary cylinder once, twice, thrice, etc.

Figure 3 If we simulate the paths between the two points A and B, a distance L apart, when the particle gets reflected at the boundaries $(\mathrm{z} = 0$ and $\mathrm{z} = \mathrm{L})$ (infinite potential well) we get "standing-wave helices" frozen in time. Each one of these closed paths $[(\mathrm{i}), (\mathrm{ii}), \ldots$ (vi)] being traversed in opposite directions after going round the imaginary cylinder once. twice, thrice, etc., respectively.

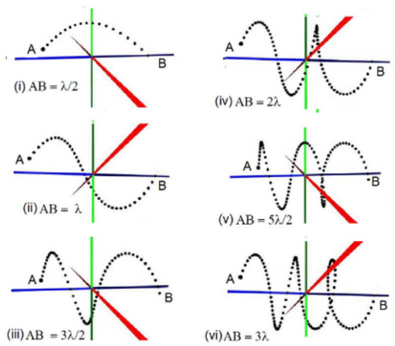


Figure 2

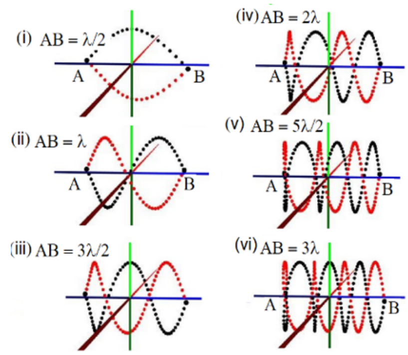


Figure 3

Thus the helical trajectories accommodate the "quantization" of energy. It is easy to visualize the closed orbits of electrons around, say, hydrogen atom would also conform to this "time independent standing wave" modes generating toroidal trajectories corresponding to paths of "least action'.

Figure 4. Represents the plot of the "standing wave" corresponding to the "fundamental" helix. It starts from the point $(\mathrm{a}_0, 0, 0)$ on the imaginary cylinder at $\mathrm{z} = 0$ and reaches the point $(-\mathrm{a}_\mathrm{o}, 0,\ \mathrm{L})$ at $\mathrm{z} = \mathrm{L}$, both the points being "nodes". At the midpoint, $\mathrm{z} = \mathrm{L}/2$, the particle would be crossing points $(0, -\mathrm{a}_\mathrm{o}, \mathrm{L}/2)$, and $(0, \mathrm{a}_\mathrm{o}, , \mathrm{L}/2)$ on either side (antinode) with a distance between them $2\mathrm{a}_\circ$ along the y-axis corresponding to "antinode".

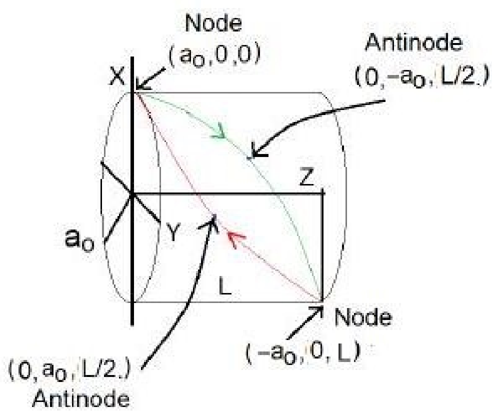


Figure 4

6. Expression for Energy

The total kinetic energy(KE) of the particle moving along a helical path consists of two parts. One is the kinetic energy of internal motion and the other that of external motion. For particles that travel with very small external velocities compared to ‘c’, the energy due to internal motion is a very large component. It is almost a constant for "small" variations in its external velocities. Hence it remains practically unchanged during interactions with similar particles having similar small velocities. In addition, this internal component of energy may not differ significantly for such low external velocities from particle to particle, when their masses are also comparable. Thus unless one is looking at the total energy, one can conveniently ignore this constant factor due to internal motion while dealing with such external interactions of these particles. We can describe the KE of such Newtonian particles with the classical expression

$$K = \frac{p^2}{2m} \tag{10}$$

where m is the mass and p external momentum along the z-axis. For such particles, the Hamiltonian can be written as

$$H = \frac{p^2}{2m} + V(z) \tag{11}$$

where $V(z)$ is the one-dimensional potential under which the particle is moving. For a “free particle”, the potential $V(z) = 0$ and hence $H = \frac{p^2}{2m}$ would represent the KE due to external motion which is “unconstrained” and hence not subjected to any quantization condition.

If the particles have external velocities large and comparable to ‘c’, then the energy contribution due to internal and external motions would also be comparable. The internal energy will tend to change significantly whenever the particle’s external motion changes due to collisions, etc., with other similar particles. We can compute the energies due to internal and external motions from postulate III. The action performed during travel through one wavelength is $p.\lambda = h$. Then the total action performed which is equivalent to the energy content per unit time(in seconds) due to external motion is

$$\mathrm{E} = h\left(\frac{\omega}{2\pi}\right) = \hbar\omega \tag{12}$$

where $\left(\frac{\omega}{2\pi}\right)$ is the frequency of this internal motion. Similarly, the total action performed per second due to internal motion alone is

$$\left(\frac{h}{2}\right)\left(\frac{\omega}{2\pi}\right) = \frac{1}{2}\hbar\omega \tag{13}$$

which is the “zero-point energy” when the particle has no external motion.

7. Schrodinger's Wave Equation

"Where did we get that (equation) from? Nowhere. It is not possible to derive it from anything you know. It came out of the mind of Schrödinger"-Richard Feynman.

Once we associate the "complex" wave functionΨ to the helical motion of fundamental particles we understand why Schrodinger's wave equations, work so amazingly well! It is essential to recognize that in the function $\Psi = Ae^{i(kz-\omega t)}$, the first term in the exponent relates to external kinetic energy $\frac{\hbar^2 k^2}{2m}$ (through $\mathbf{p} = \hbar\mathbf{k}$) while the second term corresponds to the total energy $E = \hbar\omega$

8. Energy due to only External Motion

Here we do not consider the very large component of the energy due to internal motion as it is a constant for particles with similar low velocities $(v \ll c)$ and comparable masses. The total energy of a Newtonian system due to its external motion is

$$E_{ext} = \frac{p^2}{2m} + V(z) \tag{14}$$

Where $V(z)$ is the one-dimensional potential. Thus, if we differentiate the wave function $\Psi = a_0 e^{i(kz-\omega t)}$ twice with respect to z, we get: $\frac{\partial^2\Psi}{\partial z^2} = -k^2 a_0 e^{i(kz-\omega t)} = -k^2\Psi$ Now if we multiply by $-\frac{\hbar^2}{2m}$ on both sides we get $-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial z^2} = -\frac{p^2}{2m}\Psi$ Hence we can write

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial z^2} + V(z)\Psi = E_{\text{ext}}\,\Psi \text{ or } \mathrm{H}\Psi = E_{\text{ext}}\,\Psi \tag{15}$$

This is Schrodinger's Time independent equation (TISE).

9. Energy including Internal Motion

If we wish to obtain the total energy including the large contributions from internal motion, then we may have to obtain the time derivative of the wave function as only that will extract the energy contributed by the high internal oscillations of frequency ω from which we can compute the total energy given by $E_T = \hbar\omega$.

If we perform the first-order time derivative of the helix equation (wave function):

$$\frac{\partial\Psi}{\partial t} = -i\omega Ae^{i(kz-\omega t)} = -i\omega\Psi$$

And multiplying by $i\hbar$ both sides we get, $i\hbar\frac{\partial\Psi}{\partial t} = \hbar\omega\Psi$ corresponding to Schrodinger's equation (TDSE)

$$i\hbar\frac{\partial\Psi(z,t)}{\partial t} = E_T\Psi(z,t) \tag{16}$$

Thus the new interpretation of the wave function as representing the helical trajectory of fundamental particles can restore a simple picture of quantum mechanics.

10. Time Evolution of the system

Since we saw the helix as a geodesic on an imaginary cylinder, we can interpret the "stationary states" as corresponding to a closed path with "least action" having constant total energy. This enables us to separate the space and time part of the "wave function" as $\Psi = \psi(z)T(t)$. Since the internal motion is the one that describes the "time evolution" of the path corresponding to its "proper time", and as the closed orbit is characterized by a constant energy state, we can represent the time evolution simply by the expression

$$T(t) = e^{-i\omega t} = e^{-i\frac{E}{\hbar}t}$$

where E corresponds to constant energy and time-independent (of course here we assume as in QM the potential is also time-independent). We must recognize that $T(t) = e^{-i\omega t}$ also represents a helical path along the time axis! Thus the total path of the particle can be represented by the wave function

$$\Psi(z, t) = \psi(z)e^{-i\frac{E}{\hbar}t} \quad (17)$$

11. Uncertainty

In this model, a simple picture of one-dimensional description of the motion of a fundamental particle is completely lost. Since a helix is evolving on an imaginary cylinder of radius equal to half the Compton wavelength the instantaneous position of the particle will be somewhere on the surface of this imaginary cylinder. For an observer seeing the particle moving along a "straight line", this leads to an uncertainty in its location equal to $\pm a_0$, which is equivalent to half the Compton Wavelength. For heavier particles the uncertainty in position Δx or Δy is negligible. But for a micro-particle, it becomes quite significant. Hence there is a finite uncertainty in determining its location in a plane transverse to its external motion (say, xy-plane for a particle moving along the z-axis). The minimum uncertainty in its position is $\pm a_0$. That is the magnitude of Δx or Δy would be $\geq \pm a_0$. The product '$m_0 c$' gives the maximum tangential momentum that the particle has in the direction normal to the external motion. Since external motion along the z-axis cannot alter this momentum in the xy plane, it is a constant. Therefore, if we equate the uncertainty in the particle's momentum to this value itself, then we can write Δp_x or $\Delta p_y = m_0 c$. Hence the uncertainty product

$$\Delta x \cdot \Delta p_x \simeq \pm a_0 m_0 c \quad \text{or} \quad \Delta y.\Delta p_y \simeq \pm a_0 m_0 c \quad \text{but} \quad a_0 m_0 c = \hbar/2 \quad (18)$$

Hence

$$\Delta x \cdot \Delta p_x \simeq \pm\hbar/2 \quad \text{or} \quad \Delta y.\Delta p_y \simeq \pm\hbar/2 \quad (19)$$

12. Time Reversal and Parity

The internal motion which is independent of external interactions helps in reckoning "proper time" in the "rest frame" of the particle. Since $\psi = Ae^{i\omega t}$ represents CCW rotation to correspond to motion of the particle "forward" in time, then, $\psi = Ae^{-i\omega t)} = Ae^{i\omega(-t)}$ which corresponds to CW rotation, indicates motion "backward" in time. The helix shown in Figure 5 represents a trajectory of a free electron (say) moving along the positive z-axis. Let z_0 be its instantaneous position at some time t_o. Then, z_1 and z_2 show their positions at some earlier and later times, say at t_1 and t_2 seconds, respectively. From z_0, to reach z_1 (i.e., forwards in time) one has to move along the CCW direction while to reach z_2 (i.e., backward in time) from z_0, one needs to move along CW direction.

Another interesting fact is that the two distinct helices correspond to mirror images of each other or correspond to "parity transformation". Thus the motion of particles such as electrons does not obey parity conservation and Time reversal symmetry because of its helical structure.

Figure 5. To move from an instantaneous position z_0 to a point z_1 later in time (forward in time) you go along the helix in the CCW direction. However, if you wish to move from z_0 to z_2 earlier in time (backward in time) you have to move along in the CW direction. Thus the right-handed and left-handed helices correspond to "time-reversed" trajectories!

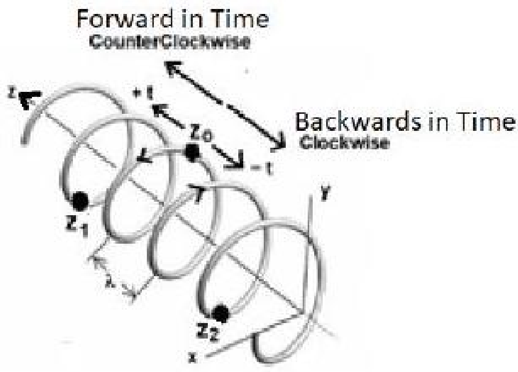


Figure 5

13. Results of Special Relativity

Consider two inertial frames S and S' with their origins coinciding at t=0, as shown in Figure 6. Frame S' is moving along z-direction with a relative velocity v with respect to S. Consider an electron "at rest" in frame S' [Figure 6(i)]. This means, for an observer in this (proper) frame, the electron does not have any external motion $v_{ex} = 0$. However, due to its internal motion, it will be executing a circular motion (in xy -pane) with a radius equal to half the Compton wavelength and instantaneous tangential velocity of magnitude c. Now, since frame S' is moving with a uniform velocity v parallel to the z-axis, to any observer in S which is at rest, the same electron would appear to be moving along a helix with an external velocity v [Figure 6(ii)] and its internal tangential velocity (in xy-plane) would now be $\sqrt{c^2 - v^2}$ so that the resultant tangential velocity along the helical trajectory would still be c for him/her in S. After a time dt' the electron would have traveled a distance $ds' = c.dt'$ for the observer in S' while for the one in S, it would be $ds = c.dt$, since their instantaneous velocities are the same and equal to c. Their time scales, however, are different since their internal velocities are different (i.e., c for S' and $\sqrt{c^2 - v^2}$ for S). Hence we can write for observers in S'

$ds'^2 = dx'^2 + dy'^2 + dz'^2 = c^2 dt'^2$. We must note that since here the particle is at "rest", $dz' = 0$, and it travels along a circle in the xy-plane, hence effectively the above equation reduces to $dx'^2 + dy'^2 = c^2 dt'^2$. For observers in S, we write the displacement ds during time dt for the same electron as $ds^2 = \mathrm{d}x^2 + dy^2 + dz^2 = c^2 dt^2$ and in this frame, it represents a segment along a cylindrical helix. Based on these, we can write a condition for the particles such as electrons,

$$dx^2 + dy^2 + dz^2 - c^2 dt^2 = 0 = dx'^2 + dy'^2 + dz'^2 - c^2 dt^2, \tag{20}$$

This is identical to the Einstein-Lorentz condition, now valid for even particles such as electrons!

Figure 6. (i) At $\mathrm{t} = \mathrm{t}' = 0$; observers in both S and S' would see an electron "at rest" executing only internal motion with instantaneous velocity c. (ii) At a later time, for the observer in S' the electron would still be "at rest" while for the one in S since S' is moving with a velocity v along the z-axis, the electron would be tracing a helix with an instantaneous internal velocity $\sqrt{c^2 - v^2}$, since the resultant velocity of both internal and external velocities is still c for observers in S'and in S.

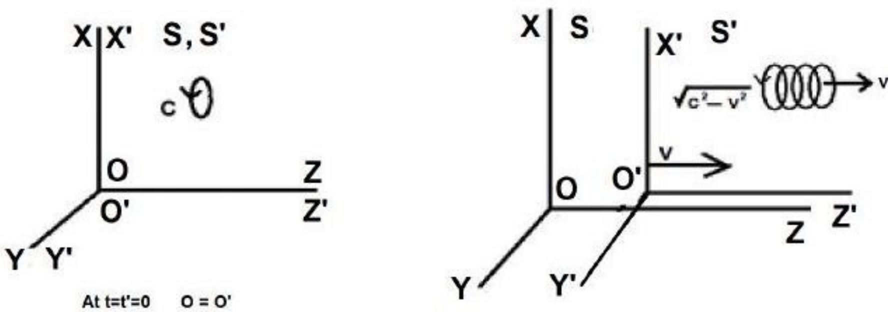


Figure 6(i) Figure 6(ii)

From postulate V, the internal periodic motion is a clock to reckon "proper" time in the rest frame of such particles. The smallest time interval in the "proper frame" of the fundamental particle is the time taken by the point-particle to complete one revolution around its internal motion through a distance $2\pi a_0$, with instantaneous velocity 'c' and radius a_0. The same interval measured from the other inertial frame would be larger than this since the speed along the internal motion for that frame would be $\sqrt{c^2 - v^2}$ as our particle would also have an external velocity v along the axis of the helix. Based on the above facts, we can write for the two frames

$$\frac{2\pi a_0}{c} = T';\ and \frac{2\pi a_0}{\sqrt{c^2 - v^2}} = T\ Hence, 2\pi a_0 = c.T' = \sqrt{c^2 - v^2}.T\ orT = \gamma T' \quad (21)$$

The smallest "time interval" or 'one tick' corresponding to the 'proper time' of the electron is $T = \frac{2\pi a_0}{c} =\sim 5 \times 10^{-20}$ second (50 Zepto seconds).

At present, in Relativity, the infinitesimal space-time interval is given as

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$$

If we apply this to an instantaneous rest frame (replacing normal time 't' with the proper time T') one may write $ds^2 = c^2 dT'^2 - dx_{T'}^2 - dy_{T'}^2 - dz_{T'}^2 = c^2 dT'^2$, since here the particle or the frame itself is at rest, $dx_{T'} = dy_{T'} = dz_{T'} = 0$. Taking the square root of the above yields

$$ds = cdT', or\ dT' = \frac{ds}{c} \quad (22)$$

This is indeed significant since it says that the "proper time interval" is computed by assuming the spatial "interval" ds along the "world line" traversed at the speed of light! This cannot be sustained in SR as no material particles can travel at this velocity. However, in the present model, it is consistent with the internal motion postulated. Another significant idea coming out of this model is the fact that space and time are intertwined within every individual particle! At present we do not know how the "proper time" is reckoned in particles such as muons!

Further, as already asserted, the mass increase with a velocity of a relativistic particle must come as a result of some sort of constraint due to a possible internal structure of the electron. If m is the mass of the particle in the "rest" frame S', then the angular momentum in this proper frame S' is $m_o.c.a_o$. The observer in S would find the angular momentum of the same particle to be $m \cdot \sqrt{c^2 - v^2} \cdot a_0$. Since a_0 does not change for other observers only mass has to change with changes in the internal velocity. Therefore, from the principle of conservation of angular momentum, we get m_0.c. $a_0 = m \cdot \sqrt{c^2 - v^2}.a_o$. This leads to the expression

$$m = \gamma m_o \quad (23)$$

This increase in mass can also be understood conceptually as follows: The restriction that the resultant velocity of the particle is c always implies that any increase in the external velocity of the particle must result in a corresponding decrease in its internal velocity. However, since such variations in internal velocity would alter the angular momentum of the particle, the system opposes any attempt to increase its external velocity which increases its inertial mass. Of course, this effect is significant only when external velocities are close to ‘c’. Hence mass increase with velocity is just a consequence of the conservation of the angular momentum of internal motion postulated.

We can obtain an expression for the total kinetic energy (KE) of a fundamental particle traveling with a resultant velocity ‘c’. The change in KE (dK) over a small displacement ds is $dK = \frac{d(mv)}{dt} \cdot ds$ since v = c, and $\frac{ds}{dt} = c, \quad dK = c^2 dm$ where dm is the change in the mass. If we integrate this expression between the limits $\mathrm{m_0}$ and m, respectively the masses of the particle when it is “at rest” and when traveling with external velocity v then,

$$K = \int_{m_0}^{m} c^2 .dm \quad \text{Hence KE,} \quad K = (m - m_0)\, c^2 \tag{24}$$

This expression corresponds to the KE due to external velocity alone and is identical to the expression obtained in SR. Thus the KE of a particle moving with external velocity is the difference between the two quantities mc^2 and m_0c^2. Here m_0c^2 and mc^2 correspond respectively to the energies of particles at "rest" and in motion. Hence the total energy E of the particle while in motion is $E = mc^2$ which is the well-known result of SR but again with a significant difference! It is not the energy equivalent during “annihilation” of the mass but the energy content at all times! When these particles get converted to energy during interaction we must account for this energy strictly according to the law of conservation of energy! We need to acknowledge that all energies unleashed are ultimately kinetic and the potential energy is only the “stored” energy. Thus by assigning a new fundamental role to the constant ‘c’ we can obtain a much deeper understanding of the results of SR without introducing any new mathematical framework!

14. Conclusions

By incorporating an internal motion to fundamental particles such as electrons consistent with ZB we can obtain amazing clarity in the following ways:

(i) It has replaced the counter-intuitive postulate of SR which only considers a limited role to ‘c’. Here fundamental particles such as electrons, positrons, etc., also move always with velocity ‘c’.

(ii) The internal motion postulated helps in keeping “proper time” in the rest frame of the particles. Thus it establishes how space and time are inseparably intertwined as Space-Time in the structure of these particles.

(iii) the concept of “time dilation” can be understood in the case of say, muons in cosmic ray showers! We also realize the mass increase with velocity, etc., of SR as a consequence of the conservation principles as applied to this internal motion!

(iii) There is **no** absolute state of rest for particles such as “electrons” which justifies the “zero point energy” of quantum mechanics.

(iv) The ‘complex wave-function of quantum mechanics is interpreted as representing a helical trajectory of particles like ‘electrons”. Helix is identified as a 3-dimensional wave having all the characteristics of our familiar 2 -dimensional sine/cosine waves.

(v) We must perhaps try simulating helical waves in the famous ‘double slit’ experiment. We need to consider not only the transverse width of the slit but also the longitudinal width of the slit (thickness of the opaque sheet having the slits) which can accommodate several “wavelengths” of the helix! This will introduce a periodicity in the points at which the particles will suffer reflections during their travel through the slits.

(vi) Due to the internal motion postulated, electrons do not move along a ‘straight line’ but along a 3-dimensional curve, namely, a helical path. Thus it helps us understand the ‘uncertainty principle’ in a more elegant way.

(vii) We can understand with this simple model, the actual meaning of equations such as $\mathrm{E} = \mathrm{mc}^2$; $\mathrm{E} = \hbar\omega$; and $\mathbf{p} = \hbar\mathbf{k}$, and in addition why Schrodinger’s equation works so well.

(x) The wave function is given a geometric interpretation as representing a helical motion which is a geodesic on a cylinder. Hence it holds promise to be unified with Einstein’s general theory of relativity which is also based on the geometrical structure of space-time.

What is more significant is that it has given a new realistic interpretation to the existing impeccable but abstract mathematics of Einstein, Schrodinger, and Dirac! With the modern amazing instruments which help us view objects at the nano and sub-nano scales, it must be possible to observe some of the features of the structure predicted here.

It appears all the difficulties that we faced so far in fundamental physics may be due to our ignoring possible internal structures for particles such as electrons, etc.! At least in this respect, this attempt here to explore such a structure must deserve wide circulation instead of rejection purely because it appears too simple to be true without any sophisticated abstract mathematics!

Every new idea is indeed a very small beginning and it might be too easy to choke it with all the data it cannot explain and reject it immediately. However, in the true spirit of science, it might be useful to bring it to light so that other fertile minds can pour over it and enrich it to better levels soon. We also must remember, the philosophy of Occam’s Razor namely, the simplicity of a model and mathematical parsimony are the hallmarks of successful theories.

References

1. Einstein, A., 1905. On the Electrodynamics of Moving Bodies. Annalen der Physik, Volume 17, pp. 891-921.
2. Dirac, P. A. M., 1981. The Principles of Quantum Mechanics (International Series of

Monographs on Physics). s.l.: Clarendon Press; 4th edition (7 January 1981).

3. Schroedinger, E., 1930. Preuss. Akad. Wiss. Phys.-Math. Kl, Volume 24, pp. 418-428.
4. Hestenes, D., 1990. Zitterbewegung Interpretation of Quantum Mechanics. Foundations of Physics, pp. 1213-1232.
5. Hestenes, D., 2010. Zitterbewegung in quantum mechanics. Foundations of Physics, Volume 40, pp. 1-54.
6. Riewe, F., 1972. Relativistic classical spinning-particle mechanics. Il Nuovo Cimento, Volume 8B, pp. 271-277.
7. Rivas, M., 1989. Classical relativistic spinning particles. J. Math. Phys., Volume 30, p. 318.
8. Consa, O., 2018. Helical Solenoid Model of the Electron. PROGRESS IN PHYSICS, 14 (4), pp. 80-89.
9. Beck, J. L., 2021 August. Relativistic Mechanics Theory for Electrons that Exhibits Spin, Zitterbewegung, Superposition and Produces Dirac's Wave Equation. [Online] Available at: http://arxiv.org/abs/2108.07267v1
10. Natarajan, T. S., 1996. A Unified Conceptual Foundation for Special Relativity and Quantum Mechanics. Physics Essays, 9(2), pp. 301-310.
11. DjSadhu, 2015. Solar System 2.0 - the helical model. [Online] Available at: https://www.youtube.com/watch?v=mvgaxQGPg7I&t=142s
12. Crenshaw, H. C., 1996. A New Look at Locomotion in Microorganisms: Rotating and Translating. AMER. ZOOL, Volume 36, pp. 608-618.
13. DeSimone, G. C. A., 2019. Modeling biological and bio-inspired swimming at microscopic scales: Recent results and perspectives. Computers & Fluids, Volume 179, pp. 799-805.
14. Zimmermann, U., 2020. complex sine wave. [Online] Available at: https://www.youtube.com/watch?v=YVLEsxq2kEA
15. Academy, N., 2016. Exponential Signals (Real and Complex). [Online] Available at: https://www.youtube.com/watch?v=IFaliBPD1UY
16. Khutoryansky, E., 2016. Quantum Wave Function Visualization -physics videos. [Online] Available at: https://www.youtube.com/watch?v=KKr91v7yLcM
17. Mcgowan, R., 2019. Geodesic on a Cylinder. [Online] Available at: https://www.youtube.com/watch?v=NO8QTtxV20o