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Original Paper

Parity Violation as a Possible Indication of a Pre-spatial Order Underlying the Standard Model

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Abstract: A codification of the internal degrees of freedom of the elementary fermions of the Standard Model is proposed in terms of overlapped triads of semiaxes in a three-dimensional Euclidean pre-space. The common orientation of these triads and the common length of their semiaxes in turn encode the fermion position in spacetime. The connection between the pre-spatial description of the fermion and its quantum description adopted by the Standard Model is established through a one-to-one correspondence between configurations of the system of triads and positional eigenstates of the fermion. The effect of the discrete symmetries C, P, T on triadic configurations in pre-space is discussed, and it is shown that the connection between the chirality of triads and ordinary space provides an intuitive explanation of the violation of parity in the weak interactions and of its conservation in the strong interactions.

Keywords: parity violation; chirality; elementary fermions; Standard Model

1. Introduction

The Standard Model (SM) is the commonly accepted theoretical framework in the discussion of problems related to the systematics and dynamics of elementary particles. Its predictive success is beyond question and is continually confirmed by new experiments. However, several experimental facts ascertained during the tumultuous development of high-energy physics in the past decades and gradually incorporated into the SM through a judicious choice and a convenient adjustment of the terms of its Lagrangian [1–3], still remain without a physical explanation convincing. For example, it is not known why the SM elementary fermions (quarks and leptons) appear multiplied in three distinct families or generations. It is not known why the phenomenon of flavor mixing exists in weak interactions mediated by charged current (and only in them). Despite numerous efforts on the subject, a satisfactory theoretical demonstration of hadronic confinement has not yet been produced. In this short note we will essentially consider two of these unsolved problems: 1) the reason for the invariance of strong interactions with respect to the combination of charge conjugation and parity inversion; 2) the reason for the violation of parity in weak interactions. The most accredited answer to the first problem today takes into consideration the breaking of the (hypothetical) Peccei-Quinn symmetry with the consequent appearance of axions [4–6], therefore an additional dynamical scenario with respect to the conventional SM. At present, however, the existence of these elusive particles has not yet been confirmed under controlled laboratory conditions. As regards the second problem, we must note that more than sixty years after the discovery of the parity violation [7,8], there are no theoretical indications concerning its possible physical cause. The most embarrassing aspect seems to be constituted by the fact that in interactions other than the weak one parity is instead conserved; an oddity that was immediately grasped by Pauli [9].

We propose here an interpretation of these phenomena that does not go beyond the framework of the usual SM dynamics, following a line of reasoning that, to our best knowledge, still remains unexplored. In our opinion, the violation of a spatiotemporal invariance in a specific class of interactions is an indication of the existence of some deep relationships between the internal and spatiotemporal degrees of freedom of the SM elementary fermions. These relationships manifest themselves experimentally as spatiotemporal invariances exhibited by the interactions of SM fermions, or as violations of such invariances.

If this idea is correct, then the problem becomes that of elaborating a *structural* model of elementary fermion, which satisfies the following conditions: 1) the model must reproduce the systematics of elementary fermions reported by the SM, giving a clear representation of at least some of the internal quantum numbers used by the SM for their description; 2) it must elucidate the relationship between the internal structure of the fermion and its spatiotemporal degrees of freedom (spacetime coordinates); 3) the model must clarify the relationship between this description and the usual representation of fermions through their spinor wave function (or field operator, which is conceptually the same thing); 4) the model must be minimal, *ie* it must not introduce changes in the commonly accepted formulation of the SM; 5) the model should hopefully provide an answer to the two problems listed above, or at least indicate a possible way to their solution.

It is clear, however, that the structure we are talking about cannot be linked to any chirality of the fermions in the ordinary space of physical experience. This would require the internal quantum numbers of the fermion be associated with a spatial "shape" of the fermion that would exhibit such chirality. The existence of this form, in turn, would require a spatial extension of the fermion. This direction of investigation seems to be a dead end because, even apart from the well-known causality problems involved with the concept of spatially extended particle, the high-energy scattering experiments clearly indicate a point-like nature of elementary fermions. The structure sought must therefore be compatible with this nature.

With these aims in mind, we pass now to present the general ideas of our specific model. It interprets the fundamental fermions of SM as particular triadic systems of semiaxes placed in a three-dimensional space, which we will call the "pre-space", distinct from spacetime and not directly accessible from observations. Fermion color and flavor are encoded in the combinatorics of the semiaxes, while the spatiotemporal position of the fermion is encoded both in the orientation of the semiaxes in pre-space and their length; the position of the triadic system in pre-space does not play any role in this scheme.

The connection between this pre-spatial configuration of the fermion and its usual quantum representation is established by postulating a one-to-one correspondence between the configuration of the triadic system and the quantum state of definite color, flavor and position, corresponding to the color, flavor and position encoded in the configuration. This correspondence is assumed to exist at the instant of proper time encoded in the configuration. This correspondence allows a connection with the wave function, which as we know is a quantum superposition of the position eigenstates. Specifically, since we are dealing with fermions of spin $1/2$, represented by a four-component spinor by virtue of the relativistic covariance requirements, we actually have four distinct wave functions corresponding to these components. The extension of the corrispondence to a system of several elementary fermions and to their global wave function (possibly entangled) is immediate.

The situation here is very similar to that which occurs in the quantization of the motion of an extended body in non-relativistic quantum mechanics, for example in the quantization of a rigid rotator (spinning top). Each orientation of the rotator in space, which is in this case the space usually understood, becomes, in the Hilbert space of states of the rotator, a state associated with that orientation. Unlike orientations, the quantum states associated with them can be linearly combined. Naturally, the structural relationships between the ontological elements of the rotator, for example the axes of inertia, which are manifested in the usual space, remain unchanged in this correspondence process. Although these relations do not appear explicitly in the rotator wave function, it is easy to realize that symmetry operations on ontological elements (for example, permutations or inversions of the axes of inertia) have reverberations on the wave function and on the Lagrangian which determines its evolution [10]. They therefore induce effects in the Hilbert space,

and influence the quantum description. For example, symmetries related to permutations or inversions of the axes will be expressed as symmetries of the Lagrangian.
Returning to the case of our triadic configurations in pre-space, the link between the chirality of the triadic system, the fermion-antifermion distinction and the definition of spacetime position leads, under appropriate assumptions of invariance under chiral inversion, to the CP invariance of the weak interactions and to the grouping of the elementary fermions in singlets or doublets of definite helicity connected by these interactions. This grouping, which appears in a somewhat mysterious way in the SM, is therefore originated from the deep connection between internal quantum numbers of elementary fermions and spacetime. Connection that exists at the level of an antecedent, constituted by the pre-space.
This short note is structured as follows. The geometrical objects corresponding to fermions are defined in Section 2, with reference to both the internal and spatiotemporal degrees of freedom. The passage to quantum description is commented in Section 3. In Section 4 the structural counterparts of the usual symmetries C, P, T are discussed in the language of the representation proposed here. Section 5 is dedicated to the conclusions.

2. The model

We assume as the physical state of "vacuum" a three-dimensional Euclidean space not accessible to direct observation: a pre-space. This pre-space must therefore not be confused with the common space of physical experience. The elements of matter are represented, in this pre-space, by monometric triples of orthogonal and directed semiaxes x, y, z, having the same point as their common origin. The position of this point in pre-space is considered irrelevant. We will assume two fundamental triples: a right-handed triad consisting of the positive semiaxes x^+, y^+, z^+ and a right-handed triad consisting of the negative semiaxes x^-, y^-, z^-. All the other triples of semiaxes ascribable to forms of matter will be obtained from these by reversing the direction (and therefore the sign) of one, two or three semiaxes. Each inversion will change the chirality of the triple. Triples of semiaxes x, y, z of undefinite direction, but of definite chirality, are also admitted.

The geometric object associated with an elementary fermion consists of three monometric triads of semiaxes $A = (x^a, y^b, z^c)$, $B = (y^d, z^e, x^f)$, $C = (z^g, x^h, y^i)$, of identical chirality and length of the semiaxes, overlapped so as to originate the semiaxes $X = (x^a, y^d, z^g)$, $Y = (y^b, z^e, x^h)$, $Z = (z^c, x^f, y^i)$. X, Y, Z are therefore distinct from cyclic permutations of order three. The signs $a, b, c, d, e, f, g, h, i$ follow the following conventions: 1) the signs of the same triad can all be null (for example $a = b = c = 0$, by this meaning that the direction of the related semiaxes is not definite, although the chirality of the triad is) or all non-null; 2) the non-null signs of the semiaxes x, y, z overlapped on the same semiaxis X, Y, Z are equal, with value $+$ or $-$. We identify the semiaxes

X, Y, Z with chromodynamic colors R, G, B respectively if the overlapping triads are right-handed. Instead, we identify these semiaxes with chromodynamic anti-colors $\bar{R}, \bar{G}, \bar{B}$ if the overlapped triads are left-handed. The number of overlapped triads having non-zero sign semiaxes will provide the generation of the fermion: first generation in the case of a single triad, second generation in the case of two triads, third generation in the case of three triads. It is not contemplated the case in which all three triads have semiaxes with null sign.

The sign of the semiaxis X, Y, Z can then be defined as the sign of the (non-null) semiaxes that are overlapped on it. The generic triad (X, Y, Z) can be denoted, unambiguously, by the ordered succession (a', b', c') where $a' = 1$ if $X = +$, $a' = 0$ if $X = -$ and analogously for b', c'. Each fermion therefore corresponds to an ordered sequence of three bits such as $(1, 0, 0)$, $(1, 1, 0)$ and so on. Of course, with the exception of successions $(1, 1, 1)$, $(0, 0, 0)$ there will always be a minority bit. For example, in succession $(1, 0, 0)$ the minority bit is "1" while in succession $(1, 1, 0)$ the minority bit is "0". The color of the semiaxis associated with the minority bit will define the color of the fermion (or the anti-color of the antifermion). The successions $(1, 1, 1)$, $(0, 0, 0)$ are not colored and are identified respectively with the charged lepton and with the neutral lepton (neutrino) in an eigenstate of mass; the colored successions constitute the quarks.

With reference to the first generation, the SM fermion classification complies with the following table:

Table I. First generation fermions

Particle	R	G	B	$\bar{R}$	$\bar{G}$	$\bar{B}$
electron e	1	1	1	-	-	-
quark u_B	-	-	-	1	1	0
quark u_G	-	-	-	1	0	1
quark u_R	-	-	-	0	1	1
quark d_R	1	0	0	-	-	-
quark d_G	0	1	0	-	-	-
quark d_B	0	0	1	-	-	-
neutrino ν_I	-	-	-	0	0	0

The corresponding antifermions are obtained from these by inverting the colors. There are in addition the following two generations of fermions:

Table II. Second generation fermions

Particle	R	G	B	$\bar{R}$	$\bar{G}$	$\bar{B}$
muon μ	1	1	1	-	-	-
quark c_B	-	-	-	1	1	0
quark c_G	-	-	-	1	0	1
quark c_R	-	-	-	0	1	1
quark s_R	1	0	0	-	-	-
quark s_G	0	1	0	-	-	-
quark s_B	0	0	1	-	-	-
neutrino ν_{II}	-	-	-	0	0	0

Table III. Third generation fermions

Particle	R	G	B	$\bar{R}$	$\bar{G}$	$\bar{B}$
tau τ	1	1	1	-	-	-
quark t_B	-	-	-	1	1	0
quark t_G	-	-	-	1	0	1
quark t_R	-	-	-	0	1	1
quark b_R	1	0	0	-	-	-
quark b_G	0	1	0	-	-	-
quark b_B	0	0	1	-	-	-
neutrino ν_{III}	-	-	-	0	0	0

and the corresponding antifermions. The neutrinos shown in this diagram are identified with the eigenstates of mass, instead of the states generated in weak interactions (electron, muon and tau neutrino). It should be noted that the neutrino of each generation is different from the corresponding antineutrino, because the latter has the opposite internal coloration and chirality. In other words, the proposed scheme is compatible with Dirac neutrinos, while it is not compatible with Majorana neutrinos.

As it is possible to verify, starting from the charged lepton, by inverting a semiaxis and changing its sign, the u-type quarks are obtained, according to the transformation: $(X^+, Y^+, Z^+)_R \rightarrow (X^-, Y^+, Z^+)_L, (X^+, Y^-, Z^+)_L, (X^+, Y^+, Z^-)_L$. Here the suffixes R and L mean right-handed or left-handed triad, respectively. By performing the operation on two semiaxes simultaneously, the d-type quarks are obtained: $(X^+, Y^+, Z^+)_R \rightarrow (X^+, Y^-, Z^-)_R, (X^-, Y^+, Z^-)_R, (X^-, Y^-, Z^+)_R$. By performing the operation on three semiaxes simultaneously, the neutrino is obtained: $(X^+, Y^+, Z^+)_R \rightarrow (X^-, Y^-, Z^-)_L$. The sequence of transformations valid for antifermions is obtained from these by inverting the signs of the semiaxes. Starting from the antineutrino, the antiquarks d-type, u-type and the charged antilepton are generated respectively. The three bits associated with each fermion are clearly related to its electric charge. Let us indicate with the index i = 1, 2, 3 the first, second or third bit of a fermion or antifermion. We set $a(i) = +1$ if the bit is "1" and it corresponds to a colored semiaxis, $a(i) = -1$ if the bit is "1" and it corresponds to an anti-colored semiaxis, $a(i) = 0$ if the bit is "0". The electric charge Q of the fermion (antifermion) is then defined, in units of the elementary charge, by the relation $Q = -(1/3)\sum_i a(i)$, as it can be checked immediately [11].

Summarizing, the object associated with the elementary fermion consists of a triad of orthogonal semiaxes (X, Y, Z). The orthogonality and monometricity conditions identify this set of three semiaxes unless of a rotation and a homothety. This rotation and this homothety can be described, as usual, associating a right-handed triad of quaternionic units $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ to the three semiaxes. The generic quaternion:

$$\mathbf{v} = a_0 + a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k} \tag{1}$$

where the coefficients a_i are assumed to be real numbers, has the modulus:

$$v = (a_0^2 + a_1^2 + a_2^2 + a_3^2)^{1/2} \tag{2}$$

This modulus can be considered as the homothetic factor which, by multiplying the semiaxes having a unit length (conventionally chosen) reproduces the semiaxes of length corresponding to that of the semiaxes making up the triad associated with the fermion. With this convention, homothety is therefore encoded in the quaternion $\mathbf{v}$. On the other hand the quaternionic units associated with the semiaxes of the triad can be multiplied by the quaternion of unit modulus $\mathbf{u} = \mathbf{v}/v$, which will thus induce a rotation of the triad. This quaternion can be associated with the rotation of the triple of semiaxes by an angle ϕ around a unit vector $\mathbf{n}$ through the usual rules involving the Hamilton-Rodriguez parameters:

$$\mathbf{u} = cos(\frac{\phi}{2}) + sin(\frac{\phi}{2})(l\mathbf{i} + m\mathbf{j} + n\mathbf{k}) \tag{3}$$

where $l = \mathbf{n} \cdot \mathbf{i}$, $m = \mathbf{n} \cdot \mathbf{j}$, $n = \mathbf{n} \cdot \mathbf{k}$.

Jointly, homothety and rotation identify a configuration of the triad of semiaxes. We now want to connect this configuration to a specific spatiotemporal position of the fermion. We do this through the following rules: 1) the instant of proper time of the fermion (considered in this configuration) is defined by the relation:

$$t = log(v) \tag{4}$$

where v is the modulus of $\mathbf{v}$ defined as in Equation (2), *ie* the homothetic factor; 2) the spatial position of the fermion (considered in this configuration) is:

$$\mathbf{x} = |\phi|\mathbf{n} \tag{5}$$

where the free end of the unit vector $\mathbf{n}$ runs over the entire surface of the unit sphere and the angle ϕ varies along the entire real axis. By convention, each negative value of ϕ will correspond to the configuration defined by the opposite value $-\phi$, with $\mathbf{n}$ reversed.

3. Quantization

The configuration of the triad (X^a, Y^b, Z^c) is therefore essentially defined by the following parameters: the chirality, the signature (a, b, c), the orientation in the pre-space, the common length of the semiaxes, the multiplicity of the semiaxes (that is, the number of semiaxes of non zero sign superimposed on the same resulting semiaxis; this parameter establishes the generation the fermion belongs to). We now postulate a one-to-one correspondence between the configurations thus defined and the quantum amplitudes constituting the basis of the state space of elementary fermions of the Standard Model. Unlike configurations, their corresponding amplitudes can be combined linearly, giving rise to wave functions that represent states of potential realization of a multiplicity of different configurations, to which each configuration contributes with its own statistical weight (and relative phase).

We start by fixing all the parameters of the fermionic configuration except the orientation of the triad in pre-space. This means that we limit ourselves to considering a fermion of definite flavor and color, at a well-defined instant of proper time, established by the value of the module v of the quaternion $\mathbf{v}$, according to Equation (4). What remains variable is therefore the unimodular quaternion $\mathbf{u}$, and then the spatial position $\mathbf{x}$, which can be connected to it through Equations (3),(5).

The spatial position $\mathbf{x}$ is associated with the eigenstate $|\mathbf{x}\rangle$ of the position operator in the usual three-dimensional space, *ie* the one in which our ordinary physical experience takes place. The positional dependence of the ket representative of the fermion is then expressed by a linear superposition of these eigenstates:

$$|\boldsymbol{\Psi}\rangle = \sum_{\mathbf{x}} \langle \mathbf{x}|\Psi\rangle |\mathbf{x}\rangle \tag{6}$$

This superposition is considered at the proper time instant t encoded in $\mathbf{v}$ with the modalities described in the previous section. The expression $\Psi(x) = \langle \mathbf{x}|\Psi\rangle$ is the wave function at that instant, evaluated in the fermion rest frame.

The relativistic covariance actually requires the fermions of spin $1/2$, as are all the elementary fermions of the SM, be described by four wave functions. The specification of the state of the fermion, in other words, requires not only one orientation of the triad, but four. These four functions must constitute the four components of a Dirac spinor [12]; we must therefore contemplate, as an effective wave function of the fermion in its rest frame, the complete spinor with four components. The wave function in an arbitrary frame of reference is then obtained by applying to this spinor the Lorentz transformation which changes the rest frame into that frame of reference. This spinorial wave function describes the additional degrees of freedom of the fermion, consisting of the spin and the sign of the energy [12].

Returning to the analogy of the spinning top, discussed in the Introduction, it can be seen how the gauge group of the SM, which acts on the space of quantum states, *ie* on the spinors, is clearly connected with the symmetry operations on the fermionic configurations in the pre-space . By virtue of the correspondence postulated at the beginning of this Section, the color of a quark corresponds to a color eigenstate. Therefore to cyclical permutations of order three that change the fundamental color R, G, B ($\bar{R}$,$\bar{G}$,$\bar{B}$) of a quark (anti-quark) in another fundamental color correspond, in the Hilbert space of the color, the transformations of the unitary symmetry group $SU(3)$ which convert the corresponding color eigenstates into each other. More generally, the transformations of this group will convert superpositions of color eigenstates into other superpositions [13].

Similarly, pre-spatial configurations of opposite weak isospin, such as the quarks d and u or the leptons e and ν, can be associated with the components of isospinors [13]. The transformations of these components will then be elements of the group $U(2) = U(1) \times SU(2)$ acting in the isospace.

Finally, we note that the model allows an internal description of the quantum discontinuity. In fact, it is possible to consider a situation in which the domain of existence of the triadic configurations of a fermion is limited by the following conditions (only one or both), formulated on ordinary spacetime [let us assume, for simplicity of exposition, a (1+1)-dimensional situation]:

$$t < \frac{t\prime_{fin} - \frac{x\prime V_{fin}}{c^2}}{\sqrt{1 - \frac{V_{fin}^2}{c^2}}} \tag{7}$$

$$t > \frac{t\prime_{in} - \frac{x\prime V_{in}}{c^2}}{\sqrt{1 - \frac{V_{in}^2}{c^2}}} \tag{8}$$

In the first case, $x\prime$ is the locus where the fermionic state is annihilated at time $t\prime_{fin}$. The $x\prime$, $t\prime_{fin}$ coordinates are evaluated in an arbitrary frame of reference O moving with velocity V_{fin} with respect to the fermion rest reference.

In the second case, $x\prime$ is the locus where the fermionic state is created at time $t\prime_{in}$. The $x\prime$, $t\prime_{in}$ coordinates are evaluated in an arbitrary frame of reference O moving with velocity V_{in} with respect to the fermion rest reference.

In the case of a fermionic state created at time $t\prime_{in}$ and annihilated at time $t\prime_{fin}$, these conditions imply that the four components of the associated spinor must contain the common factor:

$$\Theta(t - T_{in})[1 - \Theta(t - T_{fin})] \tag{9}$$

Where T_{in} and T_{fin} are the right-hand members of the previous inequalities. The discontinuities represented by the theta functions originate from the limitations of the domain of existence of pre-spatial triadic configurations associated with the fermionic state. They constitute "quantum jumps". The evolution of the spinor between two successive jumps is continuous and unitary.

Of course, the creation of a fermionic state is simultaneous with the destruction of a pre-existing state, and vice versa. For example, a free electron captured by an atom will not have configurations at instants following that of capture. The captured electron will be in effects a second fermion, devoid of configurations related to instants prior to capture. In order to preserve causality, a single fermion can be involved in only one creation event and only one subsequent annihilation event. In a new generated state, the coordinate $x\prime$ is normally defined with an uncertainty given by the volume of the spatial region that contributes significantly to the dot product of the pre- and post-collapse wave functions. The latter is essentially the interaction or capture region. It is certainly also possible to include the scenario of the so-called "null interaction", in which the discontinuity originates from the absence of interactions on a part of the wavefront, as in the case of crossing a simple or multiple slit in a screen.

4. Considerations on symmetries

4.1. Spacetime homogeneity and isotropy

We are now in a position to examine the action of the symmetry operations in the pre-space, relating it to the action induced on the Hilbert space. With reference to the continuous transformations of coordinates, it can be seen that the homogeneity of space and time is already present at the pre-spatial level. Indeed a redefinition of the unitary length in the pre-space implies, in Equation(1), the multiplication of the coefficients a_i by a common positive factor q. According to Equation (2), v is therefore subject to the scale transformation $v \rightarrow qv$, which replaced in Equation (4) generates the translation $t \rightarrow t + log(q)$ of the zero of the fermion proper time. This operation leaves the quaternionic units unchanged, and therefore has no effect on the spatial scale by virtue of Equation (5). On the other hand, a redefinition of the zero of the angular scale ($\phi \rightarrow \phi + \phi_0 = \phi \pm |\phi_0|$) induces (assuming, without loss of generality, $\phi \geq 0$ as requested by the convention) the spatial translation:

$$\mathbf{x} \rightarrow |\phi \pm |\phi_0||\mathbf{n} = |\phi|\mathbf{n} \pm |\phi_0|\mathbf{n} = \mathbf{x} \pm |\phi_0|\mathbf{n} \tag{10}$$

which has no effect on the scale of t. The transformations undergone by $\mathbf{x}$ and t are linear, and this demonstrates a substantial homogeneity of space and time already at the pre-spatial level. In Hilbert space this homogeneity takes the form of the covariance of the fermionic wave function for spatiotemporal translations, whose generators are the momentum and energy operators [10].

The problem of the spacetime isotropy is different, because to pose it we need the definition of an invariant metric of the spacetime, a concept that is simply not represented in pre-space. This aspect comes about through the definition of four distinct fermionic wavefunctions that form the components of a Dirac spinor. Such an object, in fact, is by definition relativistically covariant and it is at this level that the isotropy of spacetime appears as covariance of the description with respect to spatial rotations and boosts.

4.2. Parity and time inversion

The parity inversion operator P is represented, at the structural level, as the operator that inverts the signs of the angles of rotation of the triad X, Y, Z representing the fermionic configuration in pre-space. Recalling the convention on the negative signs of ϕ in Equation (5), the action of the P operator consists in the inversion $\mathbf{n} \to -\mathbf{n}$, which in turn implies $\mathbf{x} \to -\mathbf{x}$.

Similarly, the time inversion operator T is transcribed as the operation $v \to 1/v$ which implies $t \to -t$ in virtue of Equation (4).

4.3. Charge conjugation

The charge conjugation C represents the inversion of the chirality of the triad of the semiaxes (X^a, Y^b, Z^c), which nevertheless retain their signs a, b, c. The triad becomes left-handed if it was initially right-handed; instead it becomes right-handed if it was originally left-handed. Consequently, its coloring is reversed: the triad becomes anti-colored if it was initially colored; instead it becomes colored if it was originally anti-colored. This, in turn, induces the change of sign of the electric charge of the fermion.

We remark that leptons have no minority bits and therefore do not externally manifest color; however, the C operator reverses the coloring of their semiaxes and transforms them into enantiomers of the original form. This makes the neutrino different from the corresponding anti-neutrino.

As can be easily verified by an examination of Tables I-III, the inversion of the value of the bits, accompanied by the inversion of the coloring of the semiaxes, corresponds to the inversion of the weak isospin (and of the strong isospin in the case of quarks). This operation does not reverse the fermionic color. The counterpart of this operation in isospin space is the operator $exp(i\pi T_2)$, where T_2 denotes the second Pauli matrix in that space

[14].

4.4. Parity violation in weak interactions

An important observation can be made about the rotations XY, YZ, ZX. If the X and Y axes are swapped, these rotations become YX, XZ, ZY respectively. In other words, all rotations are reversed. Therefore the inversion of the chirality, which involves the change of sign of the charge and then an operation C, is unavoidably accompanied by an inversion operation P of the spatial coordinates, as it can be easily seen as a consequence of Equation (5). This means that the switch from a fermionic configuration to the corresponding anti-fermionic configuration is performed by a CP operation, rather than just the C operation. Therefore, if we admit that at the pre-spatial level the weak interactions are invariant with respect to the chirality inversion, this must translate in the SM description into an invariance of these interactions with respect to the combined CP transformation, and not to the C transformation alone. To understand the topic, let's focus on the charged weak current which is the one involved in the phenomenon of parity violation (the neutral weak current is self-conjugate). This phenomenon is associated, in the quantum description, with the $V - A$ structure of this current, and with the fact that the reversal of the coordinates changes the sign of V and not that of A [14,15]. At the level of triadic configurations, the conversion, performed by a boson W, of a right-handed triad R into a left-handed triad L can be schematised through an operator $L\bar{R}$. If we indicate with Γ the chirality inversion operator, then $\Gamma(L\bar{R})\Gamma^{-1} = (\Gamma L\Gamma^{-1})(\Gamma\bar{R}\Gamma^{-1}) = R\bar{L}$ transforms the action of the W into that of the oppositely charged W boson. Basically, Γ transforms the weak current into its conjugate Hermitian. The hypothesis of invariance of the weak interactions under chirality inversion, *ie* under CP, is substantially equivalent to that of the Hermiticity of the total current. We note that, for example, $\Gamma L\Gamma^{-1} = R \Longrightarrow PCLC^{-1}P^{-1} = R \Longrightarrow CLC^{-1} = P^{-1}RP$. That is, already at a structural level, there are relationships between P and C. We note that P reverses the helicity; therefore the weak current will couple a fermionic doublet (singlet) of given helicity with itself, and an antifermionic doublet (singlet) of opposite helicity with itself. This is consistent with the actually implemented Lagrangian of the Standard Model.

4.5. CP violation in weak interactions

It is possible to accommodate, in the proposed scheme, the breaking of the CP invariance, allowing negative values of v in the radical at the right side of Equation (2). Equation (4) should then be generalized as follows:

$$t = sign(v)log(|v|) \tag{11}$$

The transition to a negative value of the homothetic coefficient determines the inversion of all three semi-axes by inducing an operation $\Gamma = CP$. At the same time we have the transformation $t \to -t$. In this sense the chiral inversion involves a time reversal. A further operation T leads back to "pure" chiral inversion.

In general, the current that inverts chirality (weakly charged current) will have a component due to a pure inversion of chirality ($v > 0$) and a component due to $v < 0$. This second component must be related to a difference between the two components imaginaries of the current, the one linked to normal time and the one linked to inverted time. If these two components are equal, the invariance CP remains valid. The violation of CP invariance is relevant in some kaonic decays [15].

4.6. Strong interactions

Strong interactions induce the color change of a quark. At the pre-spatial level, the transformation of one color into another is a cyclical permutation of the signs of the semiaxes X, Y, Z of the quark and this operation does not change the chirality. The strong interaction must therefore be P invariant, because by inverting the rotations of the triad (X, Y, Z) the color transformations remain unaffected.

If the chirality of the triad (X, Y, Z) is reversed, the quark is transformed into the corresponding antiquark of the opposite color. At the same time, the transformation of one color into another is converted into the transformation that connects the corresponding anticolors, while the parity is reversed. The chirality inversion operation therefore corresponds to CP, as in the weak interactions. Again, assuming the invariance of the strong interactions by inversion of chirality means assuming their CP-invariance. However, unlike weak interactions, strong interactions are P invariant, so the operation $(CP)P = P(CP) = C$ converts a quark into the corresponding antiquark, and a color transformation into the transformation that connects the two corresponding anticolors, with no influence on the spatial coordinates. To better understand the subject, let us consider the action of a gluon which destroys the color B and creates the color G; it can be schematized by the operator $G\bar{B}$. The P operator has no effect on $G\bar{B}$: $PG\bar{B}P^{-1} = (PGP^{-1})(P\bar{B}P^{-1}) = G\bar{B}$. Therefore $\Gamma G\bar{B}\Gamma^{-1} = (\Gamma G\Gamma^{-1})(\Gamma\bar{B}\Gamma^{-1}) = (CPGP^{-1}C^{-1})(CP\bar{B}P^{-1}C^{-1}) = (CGC^{-1})(C\bar{B}C^{-1}) = C(G\bar{B})C^{-1} = \bar{G}B$. At the pre-spatial level the strong interactions are therefore separately invariant for C and P, if their invariance under chirality inversion is admitted. At the level of the SM description this implies in particular the non-relevance of the fermionic helicity in these interactions.

5. Conclusions

Since its discovery in 1956, parity violation in weak interactions has generated perplexity. It is not only a question of understanding through which mechanism an operation on spatial coordinates can influence a material process such as beta decay, but also of understanding why this operation has no effect on other phenomena, such as strong interactions. Many years after its discovery, the phenomenon remains unexplained, in the sense that links with other established physical facts have not been elucidated. In this sense, the phenomenon, although correctly and effectively implemented in the theoretical framework of the SM, remains fundamentally misunderstood.

We have proposed a model that links the parity violation in weak interactions to other established physical facts, such as the systematics of elementary fermions of the Standard Model and the existence of chromodynamic color. This model also illustrates possible reasons why the phenomenon does not manifest itself in other processes, such as strong interactions. In this sense we believe that our proposal can at least be of help in putting the phenomenon into context.

Our approach is not based on modifications of the dynamics of the SM, for example of its Lagrangian, but on the proposal of an unobservable pre-space inhabited by objects whose configurations correspond to eigenstates of compatible quantities associated with the quantum description of elementary fermions. Certainly, the quantum description represented by the SM can be considered complete in the sense that, once the conditions defining the initial (final) state of the system are specified, the subsequent (precedent) evolution of all the physical quantities accessible to the experimenter is exhaustively reproduced. This does not prevent, however, that there may be *structural* elements of physical reality which constrain the very forms of the quantum mechanical description. Following the analogy of the rotator taken from elementary quantum mechanics, here we have suggested the pre-spatial triadic configurations associated with fermions as an example of such structural elements.

The answers to the questions we asked ourselves in the Introduction are given, in this context, by the effects on the spatial coordinates of the chirality inversions in the pre-space, in the hypothesis of the invariance of physics with respect to these operations. The structural relationships between the elements that define the configurations in pre-space represent structural constraints of the quantum description. We find these constraints reflected in the structure of the SM Lagrangian; for example: in the selection of the basic states (the elementary fermions); in the role played by the fermionic helicity in the electroweak sector of the Lagrangian; in the gauge group itself.

The construction rests in an extremely critical way on the existence of two classes of triadic configurations of the same chirality but opposite signature of the semiaxes. One class constitutes the elementary fermions (Tables I-III), the other the corresponding antifermions. Within each class, the fermionic flavors can be sorted by decreasing (increasing) number of nonzero bits. In this sequence, each fermion is obtained from the previous one by inverting a semi-axis and consequently changing its sign. This operation reverses the chirality. An inevitable consequence is that each fermion is an enantiomer of the corresponding antifermion. This conclusion also applies to neutrinos, with the consequence that the proposed model is not compatible with Majorana's neutrino hypothesis. If it were experimentally confirmed that the neutrinos existing in nature are Majorana neutrinos, the entire construction would be denied at its root. As can easily be seen from the geometric scheme, even the explanation proposed here of the violation of parity, which is based on the same key principle, would then become untenable.

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