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Extraction of the Planck Length From Cosmological Redshift Without Knowledge of G or $\hbar$

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Abstract: We demonstrate how one can extract the Planck length from cosmological redshift without any knowledge of Newton's gravitational constant G or the Planck constant h. This result strongly suggests that there is a direct link between the cosmic scale and the Planck scale. We will also shortly discuss why this outcome will probably bring us closer to a unified quantum gravity theory that is closely linked to quantum cosmology. We have good reasons to think our findings are significant and should be of great interest to anyone trying to unify gravity and cosmology with the Planck scale.

Keywords: Cosmological redshift, Planck length, Planck scale, quantum gravity, quantum cosmology, gravitational redshift, Compton wavelength

1. Introduction

Max Planck [1,2] in 1899 and 1906 introduced the Planck units. He assumed there were three important universal constants: Newton's gravitational constant G, the speed of light c, and the Planck constant, and he used dimensional analysis in relation to this and found a unique length: $l_p = \sqrt{\frac{G\hbar}{c^3}}$, time $t_p = \sqrt{\frac{G\hbar}{c^5}}$, a mass $m_p = \sqrt{\frac{\hbar c}{G}}$, and a temperature $T_p = \sqrt{\frac{\hbar c^5}{Gk_B^2}}$. However, Max Planck did not say much about what they could represent, except that they were likely some important natural units.

Already, in 1916, Einstein [3], in one of the same papers he discussed general relativity theory, suggested that the next step in understanding gravity theory was likely to develop a quantum gravity theory. Eddington [4] in 1918 was perhaps the first to suggest that gravity theory ultimately had to be linked to the Planck length. However, this was far from easily accepted. For example: Bridgman [5] in 1931 ridiculed the idea that the Planck units would play an important role in physics as he looked at them more like mathematical artifacts coming out of the dimensional analysis. Today most researchers [6–9], and particularly those researchers working with quantum gravity theory, assume the Planck length is the minimum length and that it will play an important role in a quantum gravity theory that could unify gravity with quantum mechanics. Also, in superstring theory, the Planck length normally plays an important role. However, there are still physicists that hold a view similar to that of Bridgman's; that the Planck units are basically mathematical artifacts coming out of the dimensional analysis; see, for example [10]. The critics' point is that if the Planck scale can never be detected, nor even its indirect consequences, why then not simply abandon the concept that it plays an important role? It is very similar to the ether. If one cannot come up with any experiment to show the ether exists, why not simply abandon it as Einstein did in 1905? However, Haug has recently shown that the Planck length can be extracted from a Newton force spring [11] as well as from many other gravity phenomena [12,13] without any knowledge of the Newton gravitational constant G or the Planck constant h; in other words, by using a more direct approach. We will claim this result is an indirect detection of the Planck scale. Here we go one step further and even show that the Planck length can be extracted from cosmological redshift observations without any knowledge of G or h. If this result is possible, as we will demonstrate, this outcome supports a series of more theoretical papers that have claimed there is likely a link between the Planck scale and gravity, and even the cosmological scale.

A series of researchers have mentioned a possible link between the cosmological scale and the Planck scale [14,15]. Seshavatharam and Lakshminarayana [16] have suggested that it is likely needed to implement the Planck scale in the entire cosmic evolution as an important step in quantum cosmology. Calmet [17] suggested in an interesting paper that:

> *"Finally, using hand waving arguments, we show that a minimal length might be related to the cosmological constant which, if this scenario is realized, is time-dependent."*

Here we will demonstrate that we can extract the Planck length directly from the cosmological redshift and use the Planck length and the speed of light as the only two constants to correctly predict a series of gravity phenomena cosmological parameters, such as the Hubble constant. This result does not give any more precise predictions, but it does, in our view, provide a deeper insight as it strongly supports the link between the macroscopic and even the cosmological scale and the Planck scale.

2. The Compton wavelength

The Compton wavelength will play a central role in how we also get to the Planck length from cosmology, so we will have to spend some time on how it can be extracted from any mass, even the critical mass of the universe, without any knowledge of G and $\hbar$. The Compton [18] wavelength was introduced in 1923 by Arthur Compton. The Compton wavelength of any rest-mass, if we already know the mass in terms of kg, can be calculated by the following well-known formula:

$$\lambda = \frac{h}{mc} \tag{1}$$

This calculation involves the Planck constant, and we need to know the mass in terms of the kg. However, it is important to note that we also can solve the above equation with respect to m. This solution gives:

$$m = \frac{h}{\lambda}\frac{1}{c} = \frac{\hbar}{\bar{\lambda}}\frac{1}{c} \tag{2}$$

where $\hbar$ is the reduced Planck constant $\frac{h}{2\pi}$ and $\bar{\lambda}$ is the reduced Compton wavelength $\frac{\lambda}{2\pi}$. Any mass in terms of a kilogram can be expressed from the Planck constant, the speed of light, and the Compton wavelength of the mass. We will soon return to how all masses have an indirect, measurable Compton wavelength.

One can also find the Compton wavelength directly from Compton scattering. Here we shoot photons at an electron. We measure the photon's wavelength before and after it hits the electron and the incoming and outgoing photon angle. Based on these three measurements, the Compton wavelength of the electron is given by:

$$\begin{aligned}
\lambda_{\gamma,1} - \lambda_{\gamma,2} &= \frac{h}{mc}(1-\cos\theta) \\
\lambda_{\gamma,1} - \lambda_{\gamma,2} &= \frac{h}{\frac{h}{\lambda_e}\frac{1}{c}c}(1-\cos\theta) \\
\lambda_{\gamma,1} - \lambda_{\gamma,2} &= \lambda_e(1-\cos\theta) \\
\lambda_e &= \frac{\lambda_{\gamma,1} - \lambda_{\gamma,2}}{1-\cos\theta}
\end{aligned} \tag{3}$$

where θ is the angle between the incoming and outgoing beam, and $\lambda_{\gamma,1}$ and $\lambda_{\gamma,2}$ are the wavelength of the incoming and outgoing photon. So, in this way, we can measure the Compton wavelength of the electron without any knowledge of the Planck constant, and also without knowledge of G.

Next, we want to find the Compton wavelength of a proton. We here utilize the cyclotron frequency ratio of a proton and electron to equal the Compton wavelength ratio. This is because the cyclotron frequency is given by:

$$f = \frac{qB}{2\pi m} \tag{4}$$

where q is the charge and, since electrons and protons have the same charge, their cyclotron frequency ratio must be equal to:

$$\frac{f_e}{f_P} = \frac{\frac{qB}{2\pi m_e}}{\frac{qB}{2\pi m_P}} = \frac{m_P}{m_e} = \frac{\lambda_e}{\lambda_P} \approx 1836.15 \tag{5}$$

This result is more than just theory. Cyclotron experiments are one of the methods to find the proton-electron mass ratio. See for example Gräff et al. [19]. So the proton Compton wavelength is simply the electron Compton wavelength divided by approximately 1836.15. The first to discuss the Compton wavelength of the proton was likely Levitt [20] in 1958, where he pointed out that "*that many of the experimentally measured lengths pertaining to fundamental nuclear forces appear to be integral multiples of the Compton wavelength*". There has recently been a growing interest in the proton Compton wavelength. For example, Trinhammer and Bohr [21] have shown a likely relationship between the Compton wavelength of the proton and the proton radius. Some will perhaps question how a composite particle like the proton can have a Compton wavelength. Is it not only elementary particles like an electron that can have it? Personally, we think that only elementary particles have a physical Compton wavelength. However, since any composite mass ultimately consists of elementary particles, we can aggregate the Compton wavelengths of these elementary particles to get the Compton wavelength of the composite mass. We must have:

$$\begin{aligned} m &= m_1 + m_2 + m_3 + \cdots + m_n \\ \frac{\hbar}{m} &= \frac{\hbar}{m_1 + m_2 + m_3 + \cdots + m_n} \\ \frac{\hbar}{mc} &= \frac{\hbar}{(m_1 + m_2 + m_3 + \cdots + m_n)c} \end{aligned} \tag{6}$$

This type of mass aggregation is consistent for non-bound elements. However, for bound-elements it is well known that such a mass aggregation of components will typically slightly overestimate the mass due to ignoring binding energy; see, for example [22,23]. For example, hydrogen has no nuclear binding energy as it only consists of a proton and an electron, while nickel-62 has the highest known binding energy of about 0.94% (8.79 MeV per nucleus). Ignoring binding energy in bound elements can therefore give a predicted mass of that is maximum about 1% too high, and thereby a Compton wavelength that is 1% too short. This is not a lot when working with cosmology, as we will use the Compton wavelength in relation to the mass of the universe, where the uncertainty is, in general, considered to be in order of percentage points. Further, the error in the Compton wavelength for the universe using the method we will suggest will likely be much less than 1% even if ignoring binding energy, as most of the mass in the universe is non-bound and in the form of mass-equivalent energy. However, we should also be easily able to adjust for nuclear binding energy and other binding energies by simply treating them as mass equivalents $m = E/c^2$ and then incorporating the binding energy this way in the formula above.

Next, if we replace the masses on the right side in equation 6 with the equation 2 we get:

$$\begin{aligned} \frac{\hbar}{mc} &= \frac{\hbar}{(\frac{h}{\lambda_1}\frac{1}{c} + \frac{h}{\lambda_2}\frac{1}{c} + \frac{h}{\lambda_3}\frac{1}{c} + \cdots + \frac{h}{\lambda_n}\frac{1}{c})c} \\ \lambda &= \frac{1}{\frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3} + \cdots + \frac{1}{\lambda_n}} \end{aligned} \quad (7)$$

This result means that if we are only interested in the mass's kg size and describe the mass this way, the only variable that distinguishes the different masses is the Compton wavelength. And again, even if the mass does not have a single physical Compton wavelength, it consists of many elementary particles. To find the Compton wavelength of Earth or the sun, we "simply" need to count the number of protons and neutrons in them, as doing so will give a very good approximation. The Compton wavelength of a large mass, for example, Earth, is simply the Compton wavelength of the proton divided by the number of protons in the earth; as an approximation, we can even treat neutrons as the same mass as protons. Well, we should also count the number of electrons, but as a good approximation, we can ignore them as they only make up $\frac{1}{1836.15}$ of the mass, or we could also easily include these. Still, how do we count the number of protons in a large mass like Earth? A direct count of the number of protons in Earth is in principle naturally possible but practically impossible. We could take the mass of Earth in kg and divide it by the proton mass in kg. However, to know the kg mass of Earth, we would need to know the gravity constant. We aim to be independent of the gravitational constant and the Planck constant, so we want to avoid this method. We can count the number of protons and neutrons (atoms) in a small mass when we know the type of substance of which it consists. In recent times, accurate methods have been developed to count the number of atoms in a silicon sphere [24–26]. This was one of the competing methods to become the new kilogram standard, that one kilogram should be defined as an "exact" number of atoms of a certain kind. Also, other methods exist to count the number of atoms in macroscopic masses. See, for example, [27,28]. Again, when we know the number of atoms (protons and neutrons) in the mass, we know its Compton wavelength, as it is simply to take the Compton wavelength of a single proton, that we described how to find above, and divide it by the number of protons (and neutrons) in the mass. When we know the Compton wavelength of a small (but macroscopic size) mass, we can easily find the Compton wavelength of something as large as, for example, Earth. This is because the relative Compton wavelength in two masses is equal to, for example, the ratio of the gravitational acceleration fields multiplied by the ratio of the square of the radius of the objects; therefore, we must have:

$$\frac{g_1 r_1^2}{g_2 r_2^2} = \frac{\lambda_2}{\lambda_1} \tag{8}$$

The gravitational acceleration field of an object can be predicted by $g = \frac{GM}{R^2}$, so then we need to know the gravitational constant, but that is not the case if we want to measure the gravitational acceleration field. The gravitational acceleration field of Earth can be easily found by simply dropping a ball from a height H and measuring the time it takes to fall to the ground. This task is particularly simple today, when one even can buy balls with a built-in stopwatch precisely for this purpose (a so-called g-ball). After making these measurements, one can ascertain the gravitational acceleration field of Earth by the following well-known formula:

$$g = \frac{2H}{T^2} \tag{9}$$

where H is the ball drop height and T is the time it took for the ball to fall from that height H. So, one will easily find that the gravitational acceleration field of Earth is approximately $9.8\, m/s^2$. When we have counted the atoms for a small object, we also need to know g, which we can measure with a Cavendish apparatus. For example, we can use silicon spheres to count the numbers of atoms as the large balls in the apparatus. Next, we need to measure the oscillation time and the angle θ when the arm in the apparatus is deflected. We also need to know the distance between the two small balls in the apparatus L, and from this calculation we can estimate the gravitational acceleration field from the silicon sphere by plugging these measured values into the following formula:

$$g \approx \frac{2\pi^2 L\theta}{T^2} \tag{10}$$

It is important to note here that no gravitational constant is needed, nor the Planck constant. A common misunderstanding is that the Cavendish apparatus was designed to measure Newton's gravitational constant G, which was not the case; see also Clotfelter [29], Hodges [30]. It was Jon Mitchell [31] that in 1783/84 designed what today is known as a Cavendish apparatus (torsion balance), but he died before being able to use it. Cavendish [32] in 1798, with full credit to Mitchell, used it to measure the density of Earth. Neither Mitchell nor Cavendish mentioned or used a gravitational constant. The main idea behind the Cavendish apparatus is that it is sensitive enough to measure the gravitational effect from a mass that is so small that we can control from what it is made. This is because one can make sure that the gravitational object in the apparatus are of a uniform known substance; for example, the large balls are made of lead, gold, or iron. Cavendish used lead. That a Cavendish apparatus, in addition, can be used to find G, is true. However, the so-called Newton gravitational constant was actually never in the original Newton formula, which stated, in Newton's own words, [33] in Principia as simply $F = \frac{Mm}{r^2}$. However, Newton used a very different mass definition than is used today. The gravitational constant was first introduced in 1873 by Cornuand and Baille [34]. It was introduced largely because the kilogram definition of mass had become the standard, at least in large areas of Europe. Cornuand and Baille used the notation f for the gravity constant. Today's notation, G, was likely first introduced in 1894 by Boys [35]. Naturally, if one uses the symbol f or G for the gravitational constant or as Einstein k is just cosmetic, the important point here is that the gravity constant did not play an important role in the Cavendish apparatus, not even in the original Newton gravity force formula. This fact is part of the reason why we can measure the gravitational acceleration field in a Cavendish apparatus with no prior knowledge of G; see also [36].

3. Planck length from Cosmological redshift

The cosmological redshift is given by (see for example [37])

$$z_H = \frac{\lambda_{obs} - \lambda_{emit}}{\lambda_{emit}} \approx \frac{dH_0}{c} \tag{11}$$

where H_0 is the Hubble constant, and d is the distance to the object we are studying. The critical mass of the universe that we get from the Freidmann [38] equation is given by $M_c = \frac{c^3}{2GH_0}$; see, for example, [39,40]. The critical mass of the universe does not distinguish between energy and mass, for energy is treated as mass equivalent. How much is energy and how much is baryonic matter, is naturally of great interest, but is outside the scope of this paper. Next, we solve the critical mass equation with respect to H_0, and this gives:

$$H_0 = \frac{c^3}{2GM_c} \tag{12}$$

and next we are replacing H_0 with this in the cosmological redshift equation 11 and we get

$$\begin{aligned} z_H &\approx \frac{d\frac{c^3}{2GM_c}}{c} \\ z_H &\approx \frac{1}{\frac{2GM_c}{c^2 d}} \end{aligned} \tag{13}$$

Please pay attention to the fact that this looks very much like one divided by standard gravitational redshift $z = \frac{GM}{c^2 r}$, except it is multiplied by 2, and that d acts as a r. We can measure the gravitational redshift of Earth by, for example, sending a laser beam from a tower and measuring the wavelength at r_2 and r_1 ($r_1 > r_2$). A similar experiment was already done in 1959 by Pound and Rebka [41]. The predicted gravitational redshift in such an experiment is given by:

$$z = \frac{\lambda_{obs} - \lambda_{emit}}{\lambda_{emit}} = \frac{\sqrt{1 - \frac{2GM}{c^2 r_1}}}{\sqrt{1 - \frac{2GM}{c^2 r_2}}} - 1 \approx \frac{1 - \frac{GM}{c^2 r_1}}{1 - \frac{GM}{c^2 r_2}} - 1 \approx \frac{GM}{c^2 r_1} - \frac{GM}{c^2 r_2} \tag{14}$$

Next, we multiply the cosmological redshift with the gravitational redshift we can measure from a laser beam (and predict) on Earth. This calculation gives:

$$\begin{aligned} zz_H &= \left(\frac{GM}{c^2 r_1} - \frac{GM}{c^2 r_2}\right)\frac{1}{\frac{2GM_c}{c^2 d}} \\ zz_H &= \left(\frac{M}{r_1} - \frac{M}{r_2}\right)\frac{1}{\frac{2M_c}{d}} \end{aligned} \tag{15}$$

Next, we take advantage of the fact that the Mass of Earth in kg can be expressed as $M = \frac{\hbar}{\bar{\lambda}_E}\frac{1}{c}$ and that the mass of the critical universe can also be expressed as $M = \frac{\hbar}{\bar{\lambda}_c}\frac{1}{c}$, and input these into the equation above to give:

$$\begin{aligned} zz_H &= \left(\frac{\frac{\hbar}{\bar{\lambda}_E}\frac{1}{c}}{r_1} - \frac{\frac{\hbar}{\bar{\lambda}_E}\frac{1}{c}}{r_2}\right)\frac{1}{\frac{2\frac{\hbar}{\bar{\lambda}_c}\frac{1}{c}}{d}} \\ zz_H &= \left(\frac{\frac{1}{\bar{\lambda}_E}}{r_1} - \frac{\frac{1}{\bar{\lambda}_E}}{r_2}\right)\frac{1}{\frac{2\frac{1}{\bar{\lambda}_c}}{d}} \end{aligned} \tag{16}$$

If we now solve this with respect to the Compton wavelength of the critical mass of the universe, we get:

$$\bar{\lambda}_c = \frac{2\bar{\lambda}_E r_1 r_2 z z_H}{d(r_1 - r_2)} \tag{17}$$

We do not need to know G to find the Compton wavelength of the mass of the critical universe, as G cancels out. We need to know the observed cosmological redshift, the distance to the object from which we measure the cosmological redshift, the two radiuses from which we measure the gravitational redshift on Earth, and the Compton wavelength of Earth.

The Planck length formula $l_p = \sqrt{\frac{G\hbar}{c^3}}$ can be solved with respect to G and this gives:

$$G = \frac{l_p^2 c^3}{\hbar} \tag{18}$$

One could, in other words, claim the gravitational constant is a composite constant that consists of even more fundamental constants as suggested by [42,43]. However, this would be of little use if the Planck length is always dependent on G, as it would just lead to a circular problem. Actually, in 1984 Cahill [44,45] already suggested that the Planck units could be more fundamental than the gravitational constant and suggested that one could represent G by the Planck mass, by the form $G = \frac{\hbar c}{m_p^2}$. This formula one simply gets by solving the Planck mass formula with respect to G. However in 1987 Cohen [46], suggesting the same formula as Cahill and pointed out that this simply seems to lead to a circular problem, for one had to know G to know the Planck mass m_p, so there was no point in thinking the Newton gravitational constant could be represented by Planck units. This is a view held by the physics community until recently, and that is still dominant; see, for example, McCulloch [47] 2016.

However, as we [11,12,48] have already demonstrated and will demonstrate in a different way here, the Planck length can be measured independently of any knowledge of G. This composite form of G also contains the Planck constant. But almost any gravity phenomena that can be both predicted and observed contains GM and never GMm (real, two-body problems contain $\mu = GM1 + GM_2$); that is, the small m in the Newton gravity force formula cancels out in calculations when dealing with predictable observable phenomena, something we will look at more closely in the last section of this paper, before the conclusion. Let us input the composite form of G in the cosmological redshift formula and also replace the critical mass with: $M_c = \frac{\hbar}{\bar{\lambda}_c}\frac{1}{c}$.

This gives:

$$\begin{aligned} z_H &\approx \frac{1}{\frac{2GM_c}{c^2 d}} \\ z_H &\approx \frac{1}{\frac{\frac{2l_p^2 c^3}{\hbar}\frac{\hbar}{\bar{\lambda}_c}\frac{1}{c}}{c^2 d}} \\ z_H &\approx \frac{d\bar{\lambda}_c}{2l_p^2} \\ l_p^2 &\approx \frac{d\bar{\lambda}_c}{2z_H} \\ l_p &\approx \sqrt{\frac{d\bar{\lambda}_c}{2z_H}} \end{aligned} \tag{19}$$

Alternatively, we can use the following relation:

$$l_p = \sqrt{\frac{c\bar{\lambda}_c}{2H_0}} \tag{20}$$

We also notice that we must have:

$$H_0 = \frac{\bar{\lambda}_c}{2t_p l_p} \tag{21}$$

Further, the Hubble time (time since the Big Bang) is given by:

$$T_H = \frac{2l_p}{\bar{\lambda}_c} t_p \approx 13.7 \text{ billion years} \tag{22}$$

and the Hubble radius is given by:

$$r_H = \frac{2l_p^2}{\bar{\lambda}_c} \approx 13.7 \text{ billion years} \times c \approx 1.32 \times 10^{26}\ m \tag{23}$$

That is, the Hubble constant, the Hubble time, and the Hubble length can all be expressed as a function of the reduced Compton wavelength of the critical universe and the Planck time and the Planck length. It is worth noting that we [49] have recently also shown how one can find the Planck time independent of G, and $\hbar$ from a Huygens [50] pendulum clock, and also with a Newton force spring [11].

4. Numerical Example

First, we find the Compton wavelength of the electron by using Compton scattering and inputting our findings in formulas 3. It is about 2.43×10^{-12} m. Next, we measure the cyclotron frequency of a proton and electron and take the ratio of these. It is about 1836.15. We now divide the Compton wavelength of the electron by this number and get a Compton wavelength of the proton equal to approximately 1.32×10^{-15} m. Next, we make a silicon crystal sphere, and due to the uniformness of the sphere and the crystal structure, we can count the numbers of atoms in the sphere. We need to count the number of atoms in a very small part of this sphere and know the volume of that area, and next measure the radius of the sphere accurately, and then we will know the number of protons in the whole sphere. Assume it is 3×10^{26} protons (for simplicity we treat neutrons as protons too) in this silicon sphere that is of a size we can easily hold in our hand (approximately half a kg, but we do not need to know anything about how many kg is in this mass, but just as a reference point as most researchers are used to think in kg.) Its Compton wavelength is therefore the Compton wavelength of a single proton divided by this number, and this gives $\lambda_{\text{silicon sphere}} = \frac{\lambda_P}{N} = \frac{1.32\times10^{-15}}{3\times10^{26}} \approx 4.4 \times 10^{-42}$ m.

We next make two of these silicon spheres and use a Cavendish apparatus to measure the gravitational acceleration from these balls on even smaller balls. Assume the smaller balls in the Cavendish apparatus are 5 cm distance from the large ball. We measure a gravitational acceleration of $1.34 \times 10^{-08}\ m/s^2$, and we are measuring the oscillation time and the angle θ in the apparatus and inputting this in formula 10. And again, this formula does not require any knowledge of G or $\hbar$. Next, we measure the gravitational acceleration field on the surface of Earth, for example, by simply dropping a ball from height H and measuring how long it takes for it to fall to the ground T. Then we get g from $g = \frac{2H}{T^2}$, which is approximately $9.81\ m/s^2$. The radius of Earth is approximately 6371000 meters. We can now find the Compton wavelength of Earth. It is given by:

$$\lambda_E = \frac{g_1 r_1^2}{g_E r_E^2}\lambda_1 = \frac{1.34 \times 10^{-08} \times 0.05^2}{9.81 \times 6371000^2} \times 4.4 \times 10^{-42} \approx 3.7 \times 10^{-67}\ m \tag{24}$$

Next, we measure the gravitational redshift from a laser beam sent down an 800-meter tower[1] and get $z = 8.76 \times 10^{-14}$. Next, we measure the cosmological redshift of an object at a distance of 10 Gpc (3.08×10^{23} m). This value will be approximately $z_H \approx 0.00233$. To find the Compton wavelength of the critical mass of the cosmos, we now use formula 17 and get:

$$\begin{aligned}\bar{\lambda}_c &= \frac{2\bar{\lambda}_E r_1 r_2 z z_H}{d(r_1 - r_2)} \\ &= \frac{2\frac{3.7\times 10^{-67}}{2\pi} \times 6371800 \times 6371000 \times 8.76 \times 10^{-14} \times 0.00233}{3.08 \times 10^{23} \times (6371800 - 6371000)} \approx 3.96 \times 10^{-96}\ m\end{aligned}$$

We now have all the input to plug into formula 19, and we get:

$$l_p \approx \sqrt{\frac{d\bar{\lambda}_c}{2z_H}} \approx \sqrt{\frac{3.08 \times 10^{23} \times 3.96 \times 10^{-96}}{2 \times 0.00233}} \approx 1.617 \times 10^{-35}\ m$$

which is very close to the CODATA (2019) value of the Planck length $1.616255 \times 10^{-35}\ m$. Naturally, one can discuss how accurately one can measure the cosmological redshift, especially for large distances. There is considerable uncertainty in such measurements, as also reflected in the considerable differences in recent estimates of the Hubble constant [51–56]. So, our point is not that this is easily done in practice or improves our precision in Planck length estimates, but that this method is fully possible. All the different elements in the method described above have been done, so we have just put it all together. This result strongly supports the idea that the Planck length is linked to gravity and cosmology, and more information about relations here can be found in Haug [57]. Hence, a quantum gravity theory linked to the Planck scale is also a quantum cosmology, and we think great progress has been made in recent years.

5. Discussion

We have earlier demonstrated that we can extract the Planck length as well as the Planck-time (as it is simply the Planck length divided by c) from a series of gravity phenomena without knowledge of the gravitational constant by using a Cavendish apparatus [13,58], a Newton force spring [11], and a Huygens [50] clock. We have also

[1] For example, from the tower in Dubai.

shown how the Planck length can be extracted from cosmological redshift without knowing the gravitational constant and the Planck constant, in strong contrast to the view that one needs to know the Planck constant and the gravity constant in addition to the speed of light and that one can find the Planck units from these using dimensional analysis. An important question is if this has any important implications. We think so. Table 1 (see the end of the paper) shows a series of observable gravitational phenomena.

All of the observable gravitational and cosmological phenomena in the table contain GM and none of them contain GMm. In our view, G is a composite constant of the form $G = \frac{l_p^2 c^3}{\hbar}$ and, when multiplied by the kg definition of mass, the Planck constant cancels out; that is, in all the formulas below, we have:

$$GM = \frac{l_p^2 c^3}{\hbar} \frac{\hbar}{\bar{\lambda}} \frac{1}{c} = c^2 \frac{l_p^2}{\bar{\lambda}} \tag{25}$$

In our view, the Planck constant embedded in G is needed to get the Planck constant out of the kg mass definition. We likely have an incomplete mass definition when defined as kg that is fixed with multiplying it with G. One needs to get $\hbar$ out of the kilogram mass, and the Planck length one need to get in because gravity phenomena are linked to the Planck scale, a view we recently have discussed in more detail [59–61]. G contains the Planck length, not based on assumption, as the gravity constant was invented before the Planck units, but it is contained indirectly from calibration to gravity phenomena. In the table, all the formulas in the most-right column give the same output units and the same output predictions as the traditional way to write the gravity formulas using G and M. As we can see from the table, we only need two constants, l_p and c, in addition to variables, to predict any of these gravity phenomena.

In comparison, the standard view is that we need knowledge off G, c, and $\hbar$. However, this can lead to a long discussion. The main purpose of this paper was to show that we can extract the Planck length from cosmological redshift with no knowledge of G or h. This outcome should hopefully give rise to curiosity as to why this can be the case.

There is likely no need for G in physics [59,60] as we can replace G, $\hbar$, and c with simply l_p and c. Still, it is interesting to note that G can also be found from the Hubble constant and the critical mass of the universe, as already suggested by Bleksley [62] in 1951 when he suggested the following relation:

$$G = \frac{R_u c^2}{M_u} \tag{26}$$

where R_u is the universe radius and M_u is the universe mass. This formula will actually give $2G$ and not G if one uses the Friedman critical mass (energy) as the universe mass; see Hoyle [63] and Velev [64]. That is, $M_u = M_c = \frac{c^3}{2GH_0}$. What is interesting here is that this can be re-written as:

$$G = \frac{R_u c^2}{2M_c} = \frac{c^3}{H_0 2M_c} = \frac{l_p^2 c^3}{\hbar} \tag{27}$$

where M_c is the critical mass of the universe. It is worth noting that in recent modified gravity theory [61,65] that does not ignore relativistic mass, as is done in standard theory, the critical mass is twice that of this, and in that theory one has $G = \frac{c^3}{H_0 M_h} = \frac{l_p^2 c^3}{\hbar}$ (where M_h is the universe mass given by this new model); that is, the 2 disappears. Be aware that we can find both H_0 and M_c (or M_h) without any knowledge of G, as the Friedmann critical mass is also given by $M_c = \frac{c^3}{2GH_0} = \frac{\hbar}{2l_p^2 H_0}$. The reason we can find G from the Hubble constant and the critical mass is that the Hubble constant embedded contains the Planck length. It is the Planck length and the speed of light (gravity) [12,66] that is important for observable gravity phenomena, not G. This naturally means one must be able to find l_p independently of G as is clearly demonstrated is possible, including in this paper.

6. Conclusion We have shown how one can extract the Planck length from the cosmological redshift. Our findings strongly strengthen support for a relation between the Planck scale and cosmology, and could be important information for the development of full quantum cosmology. Our findings, in our view, also support the idea that most, if not all, gravity phenomena are an indirect detection of the Planck scale.

Competing interest and conflict of interest statement

- Competing interests: no competing interest and no conflict of interest.
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Data Availability Statement

No data has been used for this paper, except the CODATA 2019 Planck length number that is publicly available at https://physics.nist.gov/cgi-bin/cuu/Value?plkl

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Table 1. The table shows that all observable gravity phenomena are linked to the Planck length and the speed of gravity, which is equal to the speed of light. For all observable gravity phenomena, we have GM and not GMm. This means that the embedded Planck constant cancels out, and all observable gravity phenomena are linked to the Planck length and the speed of gravity, that again are identical to the speed of light. When this is understood, one can, as we have demonstrated, extract the Planck length directly from gravity phenomena, such as cosmological redshift.

	Standard:	**Deeper level:**
Mass	M (kg)	$M = \frac{\hbar}{\bar{\lambda}_M}\frac{1}{c}$ (kg)
Gravitational constant	G	$G, \left(G = \frac{l_p^2 c^3}{\hbar}\right)$
Non observable (contains GMm)		
Gravity force	$F = G\frac{Mm}{R^2}\ (kg \cdot m \cdot s^{-2})$	$F = \frac{l_p^2 c^3}{\hbar}\frac{Mm}{R^2}\ (kg \cdot m \cdot s^{-2})$
Observable predictions, identical for the two methods: (contains only GM)		
Gravity acceleration	$g = \frac{GM}{R^2}$	$g = \frac{c^2}{R^2}\frac{l_p^2}{\bar{\lambda}_M}$
Orbital velocity	$v_o = \sqrt{\frac{GM}{R}}$	$v_o = cl_p\sqrt{\frac{1}{R\bar{\lambda}_M}}$
Orbital time	$T = \frac{2\pi R}{\sqrt{\frac{GM}{R}}}$	$T = \frac{2\pi\sqrt{\bar{\lambda}R^3}}{cl_p}$
Velocity ball Newton cradle	$v_{out} = \sqrt{2\frac{GM}{R^2}H}$	$v_{out} = \frac{cl_p}{R}\sqrt{\frac{H}{\bar{\lambda}}}$
Periodicity Pendulum (clock)	$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi R\sqrt{\frac{L}{GM}}$	$T = \frac{2\pi R}{cl_p}\sqrt{L\bar{\lambda}}$
Frequency Newton spring	$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi R}\sqrt{\frac{GM}{x}}$	$f = \frac{cl_p}{2\pi R}\sqrt{\frac{1}{\bar{\lambda}x}}$
Observable predictions (from GR): (contains only GM)		
Gravitational redshift	$z = \frac{\sqrt{1-\frac{2GM}{R_1c^2}}}{\sqrt{1-\frac{2GM}{R_2c^2}}} - 1$	$z = \frac{\sqrt{1-\frac{2l_p^2}{R_1\bar{\lambda}_M}}}{\sqrt{1-\frac{2l_p^2}{R_2\bar{\lambda}_M}}} - 1$
Time dilation	$T_R = T_f\sqrt{1-\sqrt{\frac{2GM}{R}}^2/c^2}$	$T_R = T_f\sqrt{1-\frac{2l_p^2}{R\bar{\lambda}_M}}$
Gravitational deflection (GR)	$\delta = \frac{4GM}{c^2R}$	$\delta = \frac{4}{R}\frac{l_p^2}{\bar{\lambda}_M}$
Advance of perihelion	$\sigma = \frac{6\pi GM}{a(1-e^2)c^2}$	$\sigma = \frac{6\pi}{a(1-e^2)}\frac{l_p^2}{\bar{\lambda}_M}$
Micro lensing	$\theta = \sqrt{\frac{4GM}{c^2}\frac{d_s-d_L}{d_sd_L}}$	$\theta = 2l_p\sqrt{\frac{1}{\bar{\lambda}_M}\frac{d_s-d_L}{d_sd_L}}$
Indirectly/"hypothetical" observable predictions: (contains only GM)		
Escape velocity	$v_e = \sqrt{\frac{2GM}{R}}$	$v_e = cl_p\sqrt{\frac{2}{R\bar{\lambda}_M}}$
Schwarzschild radius	$r_s = \frac{2GM}{c^2}$	$r_s = 2\frac{l_p^2}{\bar{\lambda}_M}$
Gravitational parameter	$\mu = GM$	$\mu = c^2\frac{l_p^2}{\bar{\lambda}_M}$
Two body problem	$\mu = G(M_1 + M_2)$	$\mu = c^2\frac{l_p^2}{\bar{\lambda}_1} + c^2\frac{l_p^2}{\bar{\lambda}_2}$
Cosmology: (contains only GM)		
Cosmological redshift	$z_H \approx \frac{dH_0}{c} = \frac{1}{\frac{2GM_c}{c^2d}}$	$z_H \approx \frac{d\bar{\lambda}_c}{2l_p^2}$
Hubble constant	$H_0 = \frac{c^3}{2GM_c}$	$H_0 = \frac{\bar{\lambda}_c c}{2l_p^2} = \frac{\bar{\lambda}_c}{2t_pl_p}$
Hubble radius	$R_H = \frac{c}{H_0} = \frac{2GM_c}{c^2}$	$R_H = \frac{2l_p^2}{\bar{\lambda}_c} = \frac{2ct_pl_p}{\bar{\lambda}_c}$
Quantum analysis:		
Constants needed	G, $\hbar$, and c or l_p, $\hbar$, and c	l_p and c, for some phenomena only l_p
Variable needed	one for mass size	one for mass size