Quantum Mysteries for No One?

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Abstract

In "Quantum Mysteries for No One," *Journal of Modern Physics* **12**, 1366-1399 (2021), Frank Lad claims to have found and corrected errors in David Mermin's famous paper, "Quantum Mysteries for Anyone," *The Journal of Philosophy* **78**(7), 397-408 (1981). In his paper, Mermin merely shows how a specific prediction of quantum mechanics cannot be accounted for by a specific instantiation of "local realism," so in that sense it is irrefutable. Herein we will show that Lad's paper is a non-starter, but his analysis can be made relevant to Mermin's paper. In that case, far from refuting Mermin's paper, Lad's mathematical analysis can be viewed as an unnecessarily convoluted special case of Mermin's very simple analysis, so Lad's results support Mermin's paper.

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I. INTRODUCTION

In "Quantum Mysteries for Anyone" [1] and "Bringing home the atomic world: Quantum mysteries for anybody" [2], David Mermin introduces the mystery of quantum entanglement for the "general reader." Richard Feynman wrote to Mermin complimenting this work saying [3, p. 366-7], "One of the most beautiful papers in physics that I know of is yours in the American Journal of Physics." This is the work that Frank Lad would have us believe is erroneous [4], so let us briefly review Mermin's presentation in these two papers (it's the same in both papers, so we will refer to these papers as "Mermin's paper" hereafter).

Before we explain Mermin's result, it is important to explain what he did not claim. Mermin's result has nothing to do with experimental tests of Bell's inequality, e.g., ruling out violations of statistical independence regarding hidden variables or ruling out all forms of "local realism." Mermin defines precisely a hypothetical underlying (hidden) physical situation regarding spin-entangled particles that, given well-defined assumptions, produces experimental outcomes that do not agree with a prediction of quantum mechanics (QM). Whether or not his theoretical situation can be instantiated physically or tested experimentally is absolutely irrelevant to his result. In other words, as long as one does not dispute how to compute and understand measurement outcome probabilities per textbook QM, Mermin's result is a mathematical fact that cannot be refuted. Herein, we assume the standard textbook understanding of QM and note that Lad does not dispute this in his paper. With that caveat, let us start with an overview of Mermin's result.

In his paper, Mermin introduces a device (Figure 1) and explains how it is operated and what it produces. The reader does not need to understand anything about QM to appreciate that this device is indeed mysterious. We will relate the operation of this device immediately to measurements of a Bell spin-$\frac{1}{2}$ singlet state (or "singlet state" for short), since this paper is written for those with a knowledge of the physics rather than for a "general reader."

The Mermin device contains a source (middle box in Figure 1) that emits a pair of spin-$\frac{1}{2}$-entangled particles in a singlet state in each trial of the experiment that is measured by Alice and Bob (Figure 2). The two detectors (boxes on the left and right in Figure 1) controlled by Alice and Bob make measurements at settings (1, 2, or 3) corresponding to one of three coplanar Stern-Gerlach (SG) magnet angles ($0^\circ, 120^\circ, \text{or} -120^\circ$) (Figure 3). The settings (1, 2, 3) are selected randomly and independently by Alice and Bob, so we can assume each of

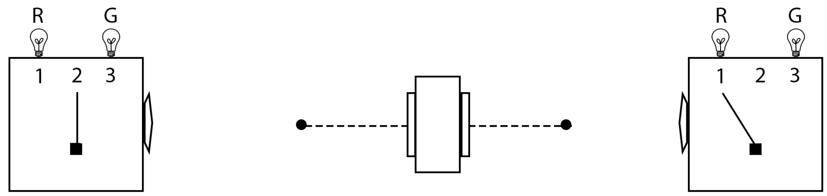

FIG. 1. **The Mermin Device**. Alice has her measuring device on the left set to 2 and Bob has his measuring device on the right set to 1. The particles have been emitted by the source in the middle and are in route to the measuring devices. Figure reproduced from Stuckey et al. [5]

the nine setting pairs (11, 12, 13, 21, 22, 23, 31, 32, 33) accounts for about $\frac{1}{9}$ of the data. Each measurement at each detector in each trial produces either a result of R (red) or G (green), corresponding to spin up(down) or spin down(up) for Alice(Bob). These are the two quantum-mechanical facts that produce the mystery:

Fact 1. When Alice and Bob's settings happen to be the same in a given trial ("case (a)"), their outcomes are always the same, $\frac{1}{2}$ of the time RR (Alice's outcome is R and Bob's outcome is R) and $\frac{1}{2}$ of the time GG (Alice's outcome is G and Bob's outcome is G).

Fact 2. When Alice and Bob's settings happen to be different in a given trial ("case (b)"), the outcomes are the same $\frac{1}{4}$ of the time, $\frac{1}{8}$ RR and $\frac{1}{8}$ GG.

Thus, the Mermin device has totally correlated R-G device outcomes corresponding to the totally anti-correlated up-down spin outcomes of the spin singlet state. Mermin writes [2]:

> Why do the detectors always flash the same colors when the switches are in the same positions? Since the two detectors are unconnected there is no way for one to "know" that the switch on the other is set in the same position as its own.

Thus, Mermin introduces "instruction sets" to account for Fact 1 per "local realism." That is, in each trial of the experiment each particle in the pair carries the same rule or property for producing an outcome in each setting (1, 2, or 3). Essentially, we're supposing that the perfect singlet-state correlations in Fact 1 are accounted for by their possessing the same (hidden) instruction set (GGR, RRG, GRR, RGG, GRG, RGR, GGG, or RRR) to account for outcomes in any of the three joint settings (11, 22, 33) in local fashion. Concerning the use of instruction sets to account for Fact 1, Mermin writes [1], "I cannot prove that it is the only way, but I challenge the reader, given the lack of connections between the detectors, to suggest any other." Again, it is important to understand that Mermin is assuming this particular instantiation of local realism, so his result is only applicable to his particular use of instruction sets, as we now detail.

Mermin notes first, that the particles cannot "know" what settings they will encounter until they arrive at the detectors. Second, they cannot communicate their settings and outcomes with each other superluminally. Mermin imposes these constraints by stating there are no connections between the detectors or between the detectors and the source, so there is no information flow – superluminal, retrocausal, or otherwise – between elements of the Mermin device, except as instantiated by the exchange of the particles themselves. He also tacitly rules out hidden violations of statistical independence for his instruction sets, i.e., "superdeterminism" per Hossenfelder [6], by assuming that each instruction set produced is ultimately measured with equal frequency in any single one of the nine detector setting pairs (example below). Finally, Mermin rules out statistical accidents when he writes [1]:

> The statistical character of the data should not be a source of concern or suspicion. Blaming the behavior of the device on repeated, systematic, and reproducible accidents, is to offer an explanation even more astonishing than the conundrum it is invoked to dispel.

The instruction sets do account for Fact 1, but Mermin shows that they are not compatible with Fact 2, given his assumptions. As Mermin explains, instruction sets with two R(G) and one G(R) will produce agreement in $\frac{1}{3}$ of all case (b) trials. This is where Mermin tacitly rules out superdeterminism. In superdetermism, it's possible that a dynamical mechanism in accord with the initial conditions of the universe causes, for example, Alice and Bob to select setting pairs 23 and 32 with twice the frequency of 21, 12, 31, and 13 in those trials where the source is caused to emit particles with the instruction set RRG or GGR (produced with equal frequency). You can see that this "conspiracy" would indeed satisfy Fact 2 – although the detector setting pairs would not occur with equal frequency, so we need to fix that. Notice that a similar disparity in the frequency of the detector setting pair

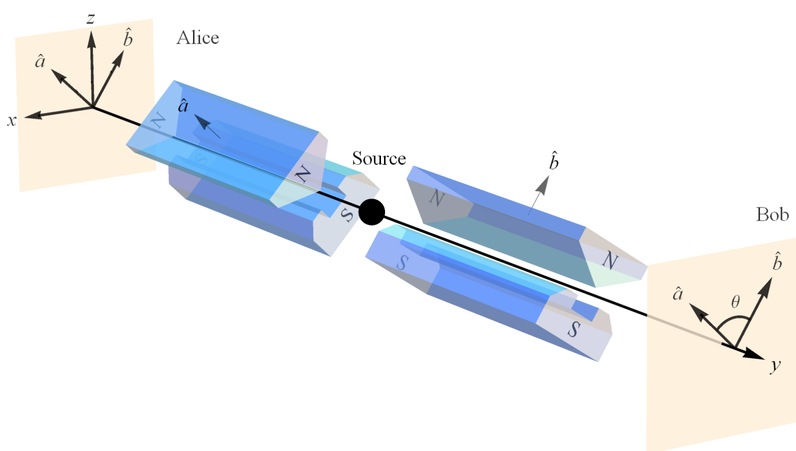

FIG. 2. Alice and Bob making spin measurements on a pair of particles in a Bell spin-$\frac{1}{2}$ singlet state in the x-z plane with their SG magnets and detectors. Figure reproduced from Silberstein et al. [7].

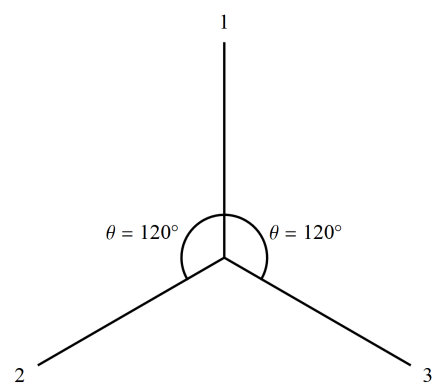

FIG. 3. Three possible planar orientations of Alice and Bob's SG magnets for a singlet state measurement corresponding to the Mermin device. Figure reproduced from Stuckey et al. [5]

measurements for RGR/GRG (12 and 21 frequencies doubled) and RGG/GRR (13 and 31 frequencies doubled) also satisfy Fact 2. So, if these six instruction sets are produced with equal frequency, then the six detector setting pairs will occur with equal frequency overall, accounting for Fact 2 of the Mermin device in accord with this particular version of local realism. We still need to account for Fact 1 and the equal frequency of occurrence for all nine detector setting pairs.

Add the following assumptions to those above: detector setting pair 11 occurs with twice the frequency of 22 and 33 for RRG/GGR, detector setting pair 22 occurs with twice the frequency of 11 and 33 for RGR/GRG, and detector setting pair 33 occurs with twice the frequency of 22 and 11 for RGG/GRR. Then, we will have accounted for Facts 1 and 2 of the Mermin device in accord with local realism with all nine detector setting pairs occurring with equal frequency overall. So, Mermin's assumption that any instruction set that is produced by the source will be measured with equal frequency in all nine setting pairs is necessary to rule out this superdeterministic conspiracy. As we will see, Lad's complicated analysis imposes this constraint de facto and his results conform exactly to Mermin's simple analysis with instruction sets. Let us continue with that analysis.

Adding instruction sets RRR and GGG to the mix then increases this $\frac{1}{3}$ fraction of agreement. Therefore, the Bell inequality [8] for the Mermin device is that we expect to get agreement in more than $\frac{1}{3}$ of all case (b) trials. But, Fact 2 for the Mermin device as required by QM says you only get the same outcomes in $\frac{1}{4}$ of all case (b) trials, which violates this Bell inequality. Thus, the mystery of quantum entanglement per the Mermin device is that the instruction sets (with Mermin's assumptions) apparently needed to explain Fact 1 cannot yield the outcomes QM requires for Fact 2. Mermin leaves it as a "challenging exercise to the physicist reader to translate the elementary quantum-mechanical reconciliation of cases (a) and (b) into terms meaningful to a general reader struggling with the dilemma raised by the device" [2]. In other words, we know the (elementary) formalism of QM predicts the observed Facts 1 and 2, but we don't have any (consensus) physical mechanism or principle [9] to explain or account for that elementary QM formalism. This summarizes the mystery of quantum entanglement and Mermin's challenge as introduced in Mermin's paper.

To remind the reader, "the elementary quantum-mechanical reconciliation of cases (a) and (b)" is obtained from the joint probabilities for Alice and Bob's measurements of a singlet state with R and G representing spin up(down) and spin down(up) for Alice(Bob):

$$P(R,R \mid \theta) = P(G,G \mid \theta) = \frac{1}{2}\cos^2\left(\frac{\theta}{2}\right) \tag{1}$$

and

$$P(R,G \mid \theta) = P(G,R \mid \theta) = \frac{1}{2}\sin^2\left(\frac{\theta}{2}\right) \tag{2}$$

where θ is the angle between Alice and Bob's SG measurement settings. That is, $\theta = 0^\circ$ for Fact 1 of case (a), so $P(R,R \mid \theta) = P(G,G \mid \theta) = \frac{1}{2}$, and $\theta = \pm 120^\circ$ for Fact 2 of case (b), so $P(R,R \mid \theta) = P(G,G \mid \theta) = \frac{1}{8}$. Obviously, Mermin's device is an accurate representation of the elementary QM, and instruction sets per Mermin's assumptions do not produce the QM predictions, so his result is a mathematical fact that cannot be refuted. Thus, we know that Lad must be mistaken when he claims [4]:

> Mermin's exposition concludes with a description of the contextual quantum experiment that the parable is meant to portray, emphasizing that such detail can be conveniently ignored while the significance of the mystery is absorbed in awe. ... Rather than ignoring the experimental physics as suggested, the remainder of my exposition now is oriented to a detailed assessment of the exact specification of this experiment and the proclamations of quantum theory that concern it. We shall find that the parable fails to represent the situation adequately, ...

We now reveal Lad's mistakes.

II. LAD'S MISTAKES

Despite what "Literally thousands upon thousands of people" (Lad's wording) find quite clear about Mermin's explanation of the experimental procedure, Lad misunderstands what Mermin is saying regarding the conduct of the experiment with instruction sets. He believes Mermin is describing two different experiments, e.g., concerning case (b) Lad writes [4]:

> Rather than counting the spin products as each pair of balls enters the machine at a dial setting, [Mermin] is counting the spin products for each pair of balls as it would pass all six of the mixed dial settings. [Mermin's] reported lighting statistics pertains to one [experimental procedure], and his counting of the matching colours pertains to another, two completely different [experimental procedures].

So, concerning Mermin's statement [1]:

> Suppose, for example, that both particles carry the instruction set RRG. Then out of the six possible case (b) settings, 12 and 21 will result in both detectors flashing the same colour (red), and the remaining four settings 13, 31, 23, and 32, will result in one red flash and one green. Thus, both the detectors will flash the same color for two of the six possible case (b) settings.

Lad writes [10]:

> [Mermin] is proposing very clearly the gedanken procedure of sending a single selected pair of balls to all six of the mixed dials settings, and counting the numbers of various possible light signals that will result.

But, if Lad's inference is correct, then these statements by Mermin make no sense [1]:

> Let us now consider the totality of all case (b) runs. In none of them do we ever learn what the full instruction sets were, since the data reveal only the colors assigned to two of the three settings. (The case (a) runs are even less informative.)

and

> In the case of my device, three such properties are involved for each particle. We will call them the 1-color, 2-color, and 3-color of the particle. The n-color of the particle is red if a detector with its switch set to n flashes red when the particle arrives. The three n-colors of a particle are complementary properties. The switch on a detector can be set to one of only three positions, and the experimental arrangements for measuring the 1-, 2-, or 3-color for a particle are mutually exclusive. (We may assume, to make this point quite firm, that the particle is destroyed by the act of triggering the detector, which is, in fact, the case in many recent experiments probing the principles that underly the device.)

Clearly, as explained above and confirmed directly by Mermin himself [11], there is only one experimental procedure in which each pair of particles is measured only once in one of the nine detector setting pairs. Again, Mermin is simply showing that QM Facts 1 and 2 for this experiment cannot be accounted for by assuming the particles have instruction sets per his assumptions (and this is a mathematical fact). As Mermin explains, while we don't know exactly which instruction set was being measured in each case (b) trial, we do know the Bell inequality for the case (b) trials will be satisfied if the particles in the singlet state possess any distribution of the hidden instruction sets (being measured as described above). Since Fact 2 violates this Bell inequality, we know that the QM prediction is not consistent with Mermin's instantiation of instruction sets.

Again, Facts 1 and 2 for the Mermin device are in accord with coplanar SG spin measurements at (0, 120, or -120) degrees for a singlet state, i.e., in accord with QM. In this

QM experiment, everyone understands (including Lad) that each particle pair is in the same singlet state and each particle pair is measured randomly in one (and only one) of the nine setting pairs (the three of case (a) and the six of case (b)). Contrary to Lad's claim, Mermin is not changing anything about the conduct of that experiment when he introduces instruction sets in an attempt to account for Facts 1 and 2 of said experiment. Lad (correctly) understands that while there are multiple measurements, there is only a single measurement of each particle pair. But for some reason when instruction sets are assumed to exist in that experiment, he erroneously believes that each pair of particles is subsequently measured in all nine setting pairs, always responding as specified by its original instruction set. Thus, in Lad's instruction-set model the instruction set for each particle pair survives unaltered from one of the nine measurements to another.

This mistake leads Lad to believe that each pair of particles emitted in accord with local realism is subsequently measured in all nine setting pairs. Thus, he organizes the data for his (not Mermin's) locally real account of the experiment into 9-dimensional vectors of data he calls "G9 vectors." Each G9 vector contains the measurement results for each particle pair in each setting pair. Each component of each G9 vector is +1 or -1 depending on whether the corresponding measurement produced the same outcome (+1) or a different outcome (-1) in each setting pair (11, 12, 13, 21, 22, 23, 31, 32, 33). Lad writes:

> Suppose we order the detector dial settings as 11, 12, 13, 21, 22, 23, 31, 32, 33, and send each pair of identical uncoded balls to all of them. Designating matching-light-colour observations by a +1 and mixed-light-colour observations by -1, the experimental results would be recorded not merely by something like 13RG, but rather something like (+1,-1,-1,-1,+1,+1,-1,+1,+1). [± notation switched to match the singlet state per the Mermin device.]

Lad then argues that there are only four unique G9 vectors that satisfy Fact 1 and local realism. He doesn't relate these to Mermin's instruction sets directly, but they are related as shown in Table I (assuming we're talking about each instruction set, not each pair of particles, being sent to all nine detector setting pairs). Note again, we are using the flipped R-G meaning for Alice and Bob (same outcomes for same settings) in accord with the Mermin device, rather than the singlet state (different outcomes for same settings). Additionally, note that Lab also uses the following notation for the setting pairs of Table I:

Instruction Sets	Setting Pair and G9 Data									Data Vector
	11	12	13	21	22	23	31	32	33	
GGR RRG	+1	+1	−1	+1	+1	−1	−1	−1	+1	G9-1
GRR RGG	+1	−1	−1	−1	+1	+1	−1	+1	+1	G9-2
GRG RGR	+1	−1	+1	−1	+1	−1	+1	−1	+1	G9-3
GGG RRR	+1	+1	+1	+1	+1	+1	+1	+1	+1	G9-4

TABLE I. **Instruction Sets and Their Corresponding G9 Vectors.**

$(A_nB_n, A_nB_z, A_nB_p, A_zB_n, A_zB_z, A_zB_p, A_pB_n, A_pB_z, A_pB_p)$. We will stick to (11, 12, 13, 21, 22, 23, 31, 32, 33) per the Mermin device, since there is no substantive reason to alter the conventions of the device.

So, while Lad's motivation for G9 vectors is misguided and irrelevant for the Mermin device, we are free to consider the implications of instruction sets via G9 vectors, since Mermin is assuming each instruction set is measured with equal frequency in all nine detector setting pairs, i.e., no hidden violation of statistical independence. Therefore, we will review Lad's analysis with G9 vectors in a manner consistent with Mermin's instruction sets, so as to render Lad's analysis relevant (although, the G9 vectors lack the precision of Mermin's R-G outcomes). Otherwise, Lad's paper is a complete non-starter.

Lad generates 1,000,000 G9 vectors in each of his twelve Monte-Carlo simulations and 1,000,000 G9 vectors equates to 9,000,000 trials with the Mermin device. Notice that it is possible to find distributions of the four G9 vectors (and therefore, of the instruction sets) so as to satisfy Fact 2 in four of the six case (b) setting pairs (Table II). Of course, the total for such distributions is still $\frac{1}{3}$ (Bell inequality), so it fails to account for Fact 2 (must be $\frac{1}{4}$ for every case (b) setting pair per QM) exactly as Mermin explained.

Now let's look at what Lad did with these G9 vectors. He notes that columns 2 and 3 of the four G9 vectors in Table I (corresponding to setting pairs 12 and 13) are exactly the four possible ± 1 Cartesian combo pairs, so these two columns can be viewed as the domain of a function $23 \to 1456789$ generating the other seven columns (and therefore the four G9 vectors). He notes there are twelve such functions over the G9 vector and describes them as "symmetric functional relations mapping $\{-1, +1\}^2$ into $\{-1, +1\}^7$." We'll refer to each such mapping via its domain, e.g., $23 \to 1456789$ will be the "23 functional mapping."

Fraction of +1 Results								
11	12	13	21	22	23	31	32	33
1.00	0.25	0.25	0.25	1.00	0.50	0.25	0.50	1.00

TABLE II. **1:2:1 Distribution Ratio of G9-1:G9-2:G9-3 Data Vectors.**
The total fraction of +1 results for case (b) is $\frac{1}{3}$, but there are several case (b) setting pairs with exactly a fraction of $\frac{1}{4}$ in agreement with QM. There are no G9-4 vectors used here.

Total Number of +1 Results for 23 $\rightarrow$ 1456789								
11	12	13	21	22	23	31	32	33
1000000	250191	250332	250191	1000000	625225	250332	625225	1000000

TABLE III. **Lad's Results.**
This shows the total number of +1 results for each pair setting for Lad's Monte-Carlo simulation generating 1,000,000 G9 vectors for the 23 functional mapping.

Lad seeks a distribution of 1,000,000 G9 vectors that satisfies Fact 2, so he writes a random number generator to create these ±1 pairs such that same outcomes in settings 12 and 13 occur approximately 25% of the time for the 23 functional mapping (similarly for the other eleven functional mappings). Of course, each ±1 pair of the domain corresponds to a particular G9 vector, so he can then add up the occurrence of +1 outcomes in any column of his trials to obtain the number of same outcome measurements for any particular detector setting pair. Table III is what his Monte-Carlo simulation produced for the 23 functional mapping (the other eleven are very similar, see Lad's Table 2).

Of course, we know from Mermin's very simple and general analysis that any distribution of instruction sets will not reproduce Fact 2 per QM. Thus, we know that Lad's unnecessarily convoluted instantiation of instruction sets via Monte-Carlo distributions of G9 vectors related by "symmetric functional relations mapping $\{-1,+1\}^2$ into $\{-1,+1\}^7$" will satisfy the Bell inequality precisely as explained by Mermin. And indeed, Lad notes that each of his twelve distributions produces the same outcomes in about $\frac{3}{8}$ (0.375) of the case (b) trials.

You can see that Table III is similar to Table II, so we wanted to know the exact distribution for each of his twelve Monte-Carlo simulations to see how each distribution varies from the 1:2:1 distribution of (G9-1, G9-2, G9-3) shown in Table II. If N1 is the total number of G9-1 occurrences, N2 is the total number of G9-2 occurrences, N3 is the total number of G9-3 occurrences, and N4 is the total number of G9-4 occurrences, then we can use Lad's Table 2 to generate four equations in N1, N2, N3, and N4 for each of the twelve functional mappings. For example, those equations for the 23 functional mapping (using Table III above) are:

$$\mathrm{N1} + \mathrm{N4} = 250191$$
$$\mathrm{N3} + \mathrm{N4} = 250332$$
$$\mathrm{N2} + \mathrm{N4} = 625225$$
$$\mathrm{N1} + \mathrm{N2} + \mathrm{N3} + \mathrm{N4} = 1000000$$

The solutions are given in the "23" column of Table IV with the distributions of the G9 vectors for the other eleven functional mappings. Instead of 25%, 50%, and 25% of some ordering of G9-1, G9-2, and G9-3 (as in Table II), Lad is getting (roughly) 19%, 56%, and 19% of some ordering of G9-1, G9-2, and G9-3 while adding 6% of G9-4, which is responsible for the case (b) agreement increasing from $\frac{3}{9}$ (0.333) to approximately $\frac{3}{8}$ (0.375). All of this is exactly in accord with Mermin's analysis with his easily understood instruction sets per local realism.

	G9 Occurrences for Each Functional Mapping											
Data Vector	23	26	27	28	34	36	38	46	47	48	67	78
G9-1	187317	187114	187815	187434	188021	562898	561997	187683	187334	187207	562911	563037
G9-2	562351	187385	561911	187306	561582	187253	187384	187445	562950	187135	187379	187227
G9-3	187458	562974	187993	562506	187641	187288	187726	562462	187410	563382	187115	187282
G9-4	62874	62527	62281	62754	62756	62561	62893	62410	62306	62276	62595	62454

TABLE IV. **Distribution of G9 Vectors for All Twelve Functional Mappings.**

III. CONCLUSION

So, we see that Lad's second mistake is that he believes his Monte-Carlo distributions of G9 vectors related by "symmetric functional relations mapping $\{-1,+1\}^2$ into $\{-1,+1\}^7$"

in some way refute Mermin's paper when in fact, Lad's mathematical results are in total agreement with Mermin's analysis using instruction sets. For example, Lad concludes [4]:

> We have created a Monte-Carlo simulation of results of a scenario which is both wholly consistent with quantum theory and also respects the restrictive symmetric functional relations that govern the structure of the experiment. It generates proportions of matching lights on the order of 0.375, precisely on the order of magnitude that the professor would have us suspect on account of his error of neglect.

But, as we showed, Lad's Monte-Carlo simulations are just a rococo version of Mermin's instruction sets, which explain Lad's 0.375 result precisely in accord with Mermin's analysis. So, Lad's results do not reproduce the 0.25 rate of case (b) agreement per QM (Fact 2 of the Mermin device) and therefore are not "wholly consistent with quantum theory." Essentially, Lad's second mistake arises because he (erroneously) believes his analysis with G9 vectors is in accord with QM when in fact, his analysis with G9 vectors is in accord with instruction sets. For some reason, Lad fails to recognize that when Mermin talks about Fact 2, Mermin is talking about the QM prediction, which instruction sets per Mermin's assumptions fail to reproduce. Again, that is a mathematical fact, so it cannot be refuted. We are simply trying to render Lad's analysis relevant to Mermin's paper, otherwise Lad's paper is a colossal non-starter.

Accordingly, Lad writes, "the repeated count of 625225 corresponding to spin products [for detector settings 23 and 32] differs markedly from the claims of Professor Mermin that the experiment should yield 250000 in every spin-product column representing" case (b). That is, Lad's 0.625 rate of agreement for some case (b) detector settings per his G9 vector analysis agrees perfectly with Mermin's explanation of how instruction sets fail to account for Fact 2 of QM. Lad also concludes [4]:

> However, if you do a long sequence of simulated experiments that gedankenly subjects the electrons to all nine paired magnet angle directions in the way local realism restricts them, you would find the proportion of spin-products equal to +1 at about 0.375 whenever the relative angle between the magnets equals $-120°$ or $+120°$. This happenstance governs the counts displayed [for detector setting pairs 12, 13, 21, 23, 31, and 32]. *The result has nothing to do with*

> *Mermin's proposed explanation of "the mystery" involving colour-encoded balls.* It derives from a recognition of the functional relations embedded into spin-product possibility vectors in the gedankenexperiment. [Italics added.]

On the contrary, again, his 0.375 case (b) agreement can be understood precisely via "Mermin's explanation of 'the mystery' involving colour-encoded balls." Lad continues this confusion in his polytope analysis when he concludes there [4]:

> Professor Mermin's proclaimed point of probabilities in these three dimensions, (0.25, 0.25, 0.25), is exterior to this polytope, while the simulation vector of probabilities (0.375, 0.375, 0.375) is a point well within the hull as a convex combination of its vertices.

and

> When the operation of Professor Mermin's machine is applied to the gedankenexperiment, the crude vector of quantum probabilities representing his provocative claims lies outside of the convex hull of probability vectors that are supported by the results of quantum theory.

Again, we see that Lad's conclusion is wrong way round. Instruction sets ("the simulation vector of probabilities") are inside the polytope while the QM prediction ("Mermin's proclaimed point of probabilities") lies outside, so Lad's polytope result obtained using his convoluted G9 vector analysis conforms exactly to Mermin's Bell inequality result from his simple instruction set analysis. Again, we see that Lad erroneously believes his G9 vector analysis supports "the results of quantum theory" when in fact, his G9 analysis supports the results of instruction sets.

In conclusion, Lad starts with an erroneous inference concerning Mermin's experimental procedure that "Literally thousands upon thousands of [readers]" did not make. He then performs an analysis of the data collected in erroneous fashion via his "G9 vectors." As it turns out, one can simply assume that Lad's G9 vectors represent the measurement of each instruction set, rather than each particle pair, in all nine detector setting pairs with equal frequency and that renders his G9 vectors relevant to Mermin's analysis. Ironically, Lad's analysis is then seen as a baroque version of Mermin's simple analysis with instruction sets, so Lad's analysis is in perfect agreement with Mermin's. Thus, Lad's argument does not refute Mermin's result (again, that is impossible), but it actually agrees with Mermin's result. Consequently, Lad's G9 vectors map to Mermin's instruction sets representing local realism (without superdeterminism) and fail to account for the "elementary quantum-mechanical reconciliation of cases (a) and (b)" thereby producing "quantum mysteries for anyone." In other words, Lad has been hoisted by his own petard.

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