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Original Paper

Natural Planck Units and the Structure of Matter and Radiation

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Abstract: Planck's constant and the gravitational constant embody natural units of length, mass, and time. When we replace the universal constants with natural Planck units, a hidden structure appears in the equations of physics comprising ratios of length, mass, and time to the Planck scale. It is the proportions of Planck units that define observable physical phenomena and not the composite values of the constants. Natural unit formulas offer more granular information about the structure of matter and radiation than equations written with $\hbar$ and G. These natural formulas reveal physical relationships explaining the correspondence between classical and quantum phenomena. Relationships between rest mass, velocity, and wavelength show how classical and quantum mechanical momentum and energy are related, suggesting that momentum is universally a function of wavelength and not velocity.

Keywords: Planck constant; gravitational constant; Planck units; natural units; symmetry

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1. Introduction

At the close of the 19th century, Max Planck unveiled the constant of proportionality that bears his name today [1–3]. Planck's discovery ushered in a new era of quantum physics and $\hbar$ became ubiquitous with equations describing the physical universe on small scales. At the same time, Planck showed that combinations of $\hbar$, G, and c produce natural quantities of length, mass, and time.

Planck units have long been regarded as the natural scale of the universe [4–17]. Natural unit systems take advantage of this scale by assigning a value of 1 to the Planck units and normalizing unit dimensions of length, mass, and time at the Planck scale.

In this paper we show that formulas restated in natural Planck units reveal certain relationships concealed by the composite values of Planck's constant and the gravitational constant. These relationships yield insights about the structure of matter and radiation.

Planck's constant and the gravitational constant can be expressed as ratios of Planck units according to the formulas [18], [19]

$$\hbar = l_P m_P c \tag{1}$$

$$G = \frac{l_P}{m_P} c^2. \tag{2}$$

The compositions of the two constants are shown in table 1 using 2021 CODATA values [20].

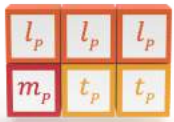
Table 1. Planck's constant and the gravitational constant are composite values of fundamental Planck units.

Constant		Unit	Value
$\hbar =$		l_P	$1.616255 \times 10^{-35}\ m$
	$\times$	m_P	$2.176434 \times 10^{-8}\ kg$
	$\times$	c	$299,792,458\ m/s$
	$=$		$\mathbf{1.054572 \times 10^{-34}}\ \boldsymbol{kgm^2/s}$
$G =$		l_P	$1.616255 \times 10^{-35}\ m$
	$\div$	m_P	$2.176434 \times 10^{-8}\ kg$
	$\times$	c	$299,792,458\ m/s$
	$\times$	c	$299,792,458\ m/s$
	$=$		$\mathbf{6.67430 \times 10^{-11}}\ \boldsymbol{m^3/kgs^2}$

2. Natural formulas

The benefits of replacing $\hbar$ and G with natural units of length, mass, and time become evident in certain quantum mechanical and gravitational formulas.

Figure 1. The gravitational constant is a mixture of fundamental Planck units



For example, the formula for gravitational acceleration

$$g = -\frac{GM}{r^2}$$

can be re-written in natural form by expanding the gravitational constant

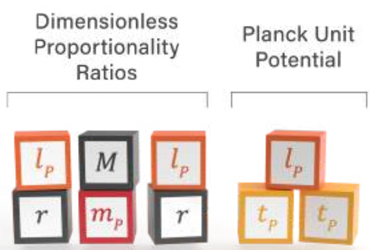
$$g = -\left(\frac{l_P^3}{m_P t_P^2}\right)\frac{M}{r^2}.$$

Re-grouping the terms of the equation highlights meaningful ratios

$$g = -\left(\frac{l_P}{r}\right)\left(\frac{M}{m_P}\right)\left(\frac{l_P}{r}\right)\left(\frac{l_P}{t_P^2}\right).$$

This natural formula can be characterized in two parts. One part consists of Planck units in the dimensions we are solving for–in this case, acceleration. The second part includes one or more dimensionless ratios between observable physical properties and the Planck scale.

Figure 2. The formula for gravitational acceleration produces three dimensionless proportionality ratios and the Planck acceleration



Applying these dimensionless ratios to the Planck unit dimensions provides the answer. We can simplify the equation to emphasize the two distinct parts of the natural formula

$$g = -\left(\frac{l_P}{r}\right)\left(\frac{M}{m_P}\right)\left(\frac{l_P}{r}\right)a_P. \tag{3}$$

In light of this granular structure, the gravitational constant can be interpreted as the emergence of fundamental quantities, encoded in the Planck units, into classical formulas. The following table summarizes several gravitational formulas restated in fundamental Planck units

Table 2. A summary of certain classical gravitational formulas in which G and c are restated in Planck units.

Physical quantity	Planck unit potential	Dimensionless ratios	Natural formula	Standard formula
Energy	E_P	$\frac{l_P}{r}, \frac{M}{m_P}, \frac{m}{m_P}$	$U_g = -\left(\frac{l_P}{r}\right)\left(\frac{M}{m_P}\right)\left(\frac{m}{m_P}\right)E_P$	$U_g = -\frac{GMm}{r}$
Acceleration	a_P	$\frac{l_P}{r}, \frac{M}{m_P}, \frac{l_P}{r}$	$g = -\left(\frac{l_P}{r}\right)\left(\frac{M}{m_P}\right)\left(\frac{l_P}{r}\right)a_P$	$g = -\frac{GM}{r^2}$
Force	F_P	$\frac{l_P}{r}, \frac{M}{m_P}, \frac{l_P}{r}, \frac{m}{m_P}$	$F = \left(\frac{l_P}{r}\right)\left(\frac{M}{m_P}\right)\left(\frac{l_P}{r}\right)\left(\frac{m}{m_P}\right)F_P$	$F = \frac{GMm}{r^2}$
Escape velocity	c	$\frac{l_P}{r}, \frac{M}{m_P}$	$v_e = \sqrt{2\left(\frac{l_P}{r}\right)\left(\frac{M}{m_P}\right)}\,c$	$v_e = \sqrt{\frac{2GM}{r}}$
Schwarzschild radius	l_P	$\frac{M}{m_P}$	$r_s = 2\left(\frac{M}{m_P}\right)l_P$	$r_s = \frac{2GM}{c^2}$

The same can be done for quantum formulas. Planck's constant is a convenient mix of fundamental Planck units for calculating quantum mechanical phenomena.

Figure 3. Planck's constant is a mixture of fundamental Planck units.

The following table restates quantum formulas in natural form by substituting Planck units for the composite Planck constant.

Table 3. A summary of certain quantum mechanical formulas in which $\hbar$ and c are restated in Planck units.

Physical quantity	Planck unit potential	Dimensionless ratios	Natural formula	Standard formula
Photon momentum	p_P	$\frac{l_P}{\lambda}$	$p = \left(\frac{l_P}{\lambda}\right)p_P$	$p = \frac{\hbar}{\lambda}$
Photon energy	E_P	$\frac{l_P}{\lambda}$	$E = \left(\frac{l_P}{\lambda}\right)E_P$	$E = \frac{\hbar c}{\lambda}$
Compton wavelength	l_P	$\frac{m_0}{m_P}, \frac{c}{c}$	$\lambda_C = \left(\frac{m_P}{m_0}\right)\left(\frac{c}{c}\right)l_P$	$\lambda_C = \frac{\hbar}{mc}$
de Broglie wavelength	l_P	$\frac{m_0}{m_P}, \frac{v}{c}$	$\lambda = \left(\frac{m_P}{m_0}\right)\left(\frac{c}{v}\right)l_P$	$\lambda = \frac{\hbar}{mv}$

The formulas suggest that $\hbar$ and G are not fundamental constants, but convenient ratios of Planck units for performing calculations. It is in the relationships between fundamental quantities of length, mass, and time that we obtain precious insights into the nature of equations featuring Planck's constant and the gravitational constant.

We also see that there is nothing inherently *quantum* about $\hbar$ or *gravitational* about G. Since the Planck units are responsible for producing formula outputs, a gravitational formula can be restated using $\hbar$, and a quantum formula can be written with G provided we assemble the correct mixture of Planck units. The relationship between the two constants is shown in Planck units as follows

$$G = \frac{l_P}{m_P}c^2 = \left(\frac{c}{m_P^2}\right) l_P m_P c = \left(\frac{c}{m_P^2}\right)\hbar. \tag{4}$$

3. Momentum

Natural Planck unit formulas lend new insights into the structure of matter and radiation. These formulas help explain physical concepts expressed in different momentum formulas. For example, the equation

$$p = mv \tag{5}$$

implies a kind of *mass in motion* in which momentum is proportional to velocity. But photons do not have rest mass and their velocity is fixed. The formula for calculating photon momentum

$$p = \frac{\hbar}{\lambda} \tag{6}$$

suggests that momentum is inversely proportional to wavelength irrespective of the particle's velocity.

Natural Planck unit formulas explain the relationship between mass, velocity, and wavelength in these two formulas, giving consonant meaning to momentum for matter and radiation.

Table 4 demonstrates two relationships that are discussed in the following section. Each column quantifies a physical property or dynamic of the three charged leptons.

3.1. Relationship between Compton wavelength and rest mass

The first relationship that clarifies the physical meaning of momentum is between a particle's Compton wavelength and its rest mass. This known relationship [21], [22], [23] is shown more explicitly by restating the Compton wavelength formula in Planck units

$$\lambda_C = \frac{\hbar}{m_0 c} = \left(\frac{m_P}{m_0}\right)\left(\frac{c}{c}\right) l_P. \tag{7}$$

Table 4. Planck unit formulas highlight physical relationships between wavelength and mass, and between wavelength and velocity.

lepton	λ_C	m_0	$\lambda_C m_0$	$l_P m_P$	v	λ	$\frac{\lambda_C}{\lambda}$	$\frac{v}{c}$
	m	kg	kgm	kgm	m/s	m	-	-
e	3.86×10^{-13}	9.11×10^{-31}	$\mathbf{3.52 \times 10^{-43}}$	$\mathbf{3.52 \times 10^{-43}}$	$2,997,924$	3.86×10^{-11}	**0.0100**	**0.0100**
μ	1.87×10^{-15}	1.88×10^{-28}	$\mathbf{3.52 \times 10^{-43}}$	$\mathbf{3.52 \times 10^{-43}}$	$1,000,000$	5.60×10^{-13}	**0.0033**	**0.0033**
τ	1.11×10^{-16}	3.17×10^{-27}	$\mathbf{3.52 \times 10^{-43}}$	$\mathbf{3.52 \times 10^{-43}}$	$100,000,000$	3.33×10^{-16}	**0.3336**	**0.3336**

2021 CODATA values are used for λ_C, m_0, l_P, m_P, and c. λ determined using equation 11. Velocities were selected arbitrarily and any velocity will yield similar results.

Arranging the simplified formula as an equality between wavelength and mass gives a relationship between each quantity and the Planck scale

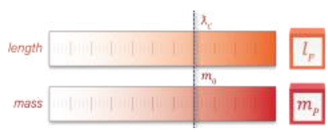
$$\frac{l_P}{\lambda_C} = \frac{m_0}{m_P}. \tag{8}$$

In addition, the Planck unit formula shows that the product of Compton wavelength and rest mass is equal to the product of Planck length and Planck mass, an important physical constant

$$\lambda_C m_0 = l_P m_P = 3.52 \times 10^{-43} kgm. \tag{9}$$

The formula describes a reciprocal relationship between wavelength and mass that is demonstrated in table 4 by the Compton wavelengths and rest masses of the charged leptons.

Figure 4. Ratios of Compton wavelength and rest mass to the Planck scale are equivalent, producing a natural unit scale in which Planck units represent physical limits and observable phenomena are proportional to these limits.



We can express a particle's rest mass as a ratio of the Planck mass where the Compton wavelength gives the correct proportion

$$m_0 = \frac{l_P}{\lambda_C} m_P. \tag{10}$$

3.2. Relationship between particle wavelength and velocity

The second relationship that clarifies the physical meaning of momentum is between a matter particle's wavelength and velocity. The wavelength of a matter particle is given by the de Broglie wavelength formula

$$\lambda\!\!\!^{-} = \frac{\hbar}{m_0 v} = \left(\frac{m_P}{m_0}\right)\left(\frac{c}{v}\right) l_P. \tag{11}$$

At the maximum limit of a matter particle's velocity, the de Broglie and Compton wavelength formulas coincide, giving a proportional ratio between particle wavelength and velocity. Evaluating 7 and 11 shows that velocity is the only variable between the two wavelengths

$$\frac{\lambda\!\!\!^{-}_C}{\lambda\!\!\!^{-}} = \frac{v}{c}. \tag{12}$$

This relationship is demonstrated in table 4 for the charged leptons.

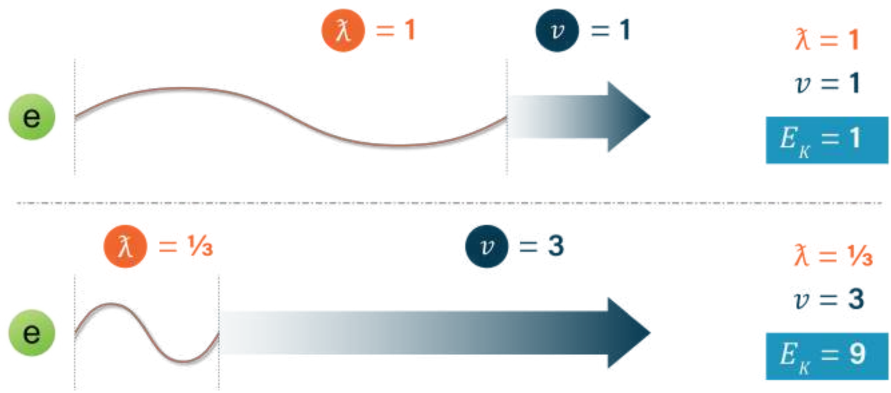
We can state a matter particle's velocity as a ratio of the speed of light, in which the ratio of Compton wavelength to de Broglie wavelength gives the correct proportion

$$v = \frac{\lambda\!\!\!^{-}_C}{\lambda\!\!\!^{-}} c \tag{13}$$

This relationship between wavelength and velocity explains why a change in kinetic energy is proportional to velocity squared, and not simply proportional to velocity. A change in velocity is accompanied by a proportional change in a matter particle's wavelength. Correlated changes in wavelength and velocity may be described as an equipartition of energy across two degrees of freedom, portrayed in figure 5 for an electron.

Photon energy is allocated over a single degree of freedom–the particle's wavelength–as velocity is fixed.

Figure 5. For particles of matter, wavelength and velocity are correlated such that a change in one accompanies a proportional change in the other. This correlation makes kinetic energy proportional to velocity squared.



3.3. Reconciling momentum formulas

The particle properties discussed in sections 3.1 and 3.2 show that both momentum formulas are equivalent to a single natural formula. The photon momentum formula can be stated in natural form by replacing Planck's constant with natural units

$$p = \frac{\hbar}{\lambda} = \frac{l_P m_P c}{\lambda} = \frac{l_P}{\lambda} p_P \tag{14}$$

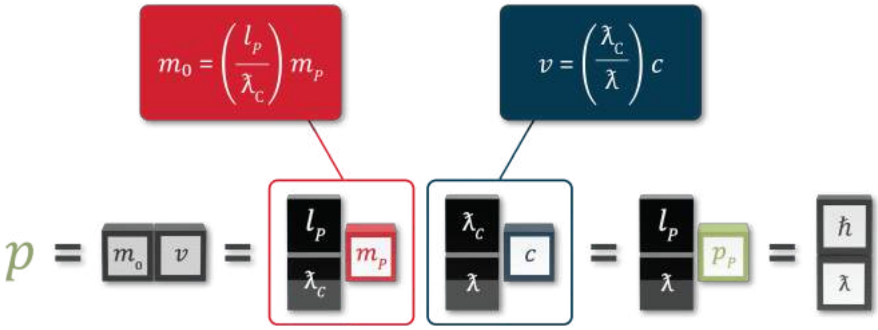
indicating that momentum is proportional to the Planck momentum, and identifying the ratio of Planck length to particle wavelength as the correct proportion.

We can restate the second momentum formula in natural units by quantifying rest mass and velocity using wavelength according to 10 and 13

$$p = m_0 v = \left(\frac{l_P}{\lambda_C}\right) m_P \left(\frac{\lambda_C}{\lambda}\right) c = \frac{l_P}{\lambda} p_P. \tag{15}$$

yielding the same natural formula. Figure 6 illustrates how the two standard momentum formulas are related to each other and to the natural formula.

Figure 6. Relationships between Compton wavelength and rest mass, and between wavelength and velocity show how the different momentum formulas are related.



Momentum is therefore the same phenomenon for both matter and radiation, and a single natural formula correctly describes both. It is pragmatic to calculate momentum for large particle systems using the bulk properties of rest mass and velocity, but only particle wavelength gives consistent meaning for matter and radiation.

Table 5 demonstrates this singular meaning of momentum with the charged leptons and a photon. The momentum of each particle is the same when the particles' wavelengths are the same–regardless of the presence or magnitude of rest mass, and without regard to whether the particle's velocity changes.

Table 5. The wavelength description of momentum works for matter and radiation.

particle	m_0	v	$\lambda\!\!\!^{-}$	p
	kg	m/s	m	kgm/s
e	9.109×10^{-31}	$2,997,924$	3.862×10^{-11}	2.731×10^{-24}
μ	1.884×10^{-28}	$14,499$	3.862×10^{-11}	2.731×10^{-24}
τ	3.168×10^{-27}	862	3.862×10^{-11}	2.731×10^{-24}
γ	-	$299,792,458$	3.862×10^{-11}	2.731×10^{-24}

2021 CODATA values used for m_0; velocities selected arbitrarily and any velocity will yield similar results; $\lambda\!\!\!^{-}$ of charged leptons determined using equation 11 and $\lambda\!\!\!^{-}$ of photon chosen to match the leptons; p determined using 5 and 6.

This singular concept of momentum shows that the textbook description of momentum as *mass in motion* is misleading. The momentum of a cement truck barreling down the highway is greater than when the truck is at rest because its particle wavelengths are shorter, *not* because the truck is moving faster. Momentum is a static property–a kind of density or energy potential represented by a particle's wavelength. Velocity, on the other hand, is the kinetic property that distinguishes energy from momentum.

3.4. Demonstrating momentum

The physical meaning underlying the two momentum formulas is reflected in the properties of a ground state electron. These properties are shown in table 6 and discussed below.

Table 6. Properties of a ground state electron demonstrate the relationship between classical and quantum mechanical momentum.

Parameter	Quantity	Units
Electron rest mass	9.109×10^{-31}	kg
Electron Compton wavelength	3.862×10^{-13}	m
Electron de Broglie wavelength	5.292×10^{-11}	m
Electron velocity	$2,187,691$	m/s
Reduced Planck constant	1.055×10^{-34}	kgm^2/s

The electron rest mass, Compton wavelength, and Planck constant are 2021 CODATA values [20] and the electron's velocity is calculated using $v = \frac{n\hbar}{m_e r}$ [24]. The electron wavelength is calculated using 11.

The ground state electron's momentum can be calculated using both formulas which yield the same result, suggesting that a single meaning is encoded in the different formulas.

Standard formula	**Quantum formula**
$p = mv$	$p = \frac{\hbar}{\lambda}$
$= (9.109 \times 10^{-31}\ kg)(2,187,691\ m/s)$	$= \frac{1.055 \times 10^{-34}\ kgm^2/s}{5.292 \times 10^{-11}\ m}$
$= \mathbf{1.993 \times 10^{-24}}\ kgm/s$	$= \mathbf{1.993 \times 10^{-24}}\ kgm/s$

Evaluating natural ratios between the electron's physical properties and the Planck scale show how the two formulas produce the same result.

3.4.1. Rest mass ratio

The ratio of Planck length to the electron's Compton wavelength is equal to the ratio of rest mass to the Planck mass. Each of these relationships gives the same dimensionless ratio

$$\frac{m_0}{m_P} = \frac{l_P}{\lambda\!\!\!^{-}_C} = \frac{9.109 \times 10^{-31}\ kg}{2.176 \times 10^{-8}\ kg} = \frac{1.616 \times 10^{-35}\ m}{3.862 \times 10^{-13}\ m} = 4.185 \times 10^{-23}.$$

3.4.2. Velocity ratio

The ratio of the electron's velocity to the speed of light is equal to the ratio of its Compton wavelength to de Broglie wavelength. Each relationship gives the same dimensionless ratio; in this case, the fine-structure constant

$$\frac{v}{c} = \frac{\lambda\!\!\!^{-}_C}{\lambda\!\!\!^{-}} = \frac{2,187,691\ m/s}{299,792,458\ m/s} = \frac{3.862 \times 10^{-13}\ m}{5.292 \times 10^{-11}\ m} = .0073.$$

The combined rest mass and velocity ratio is equal to the ratio of Planck length to wavelength

$$\left(\frac{l_P}{\lambda\!\!\!^{-}_C}\right)\left(\frac{\lambda\!\!\!^{-}_C}{\lambda\!\!\!^{-}}\right) = \frac{l_P}{\lambda\!\!\!^{-}} = (4.185 \times 10^{-23})(.0073) = \mathbf{3.054 \times 10^{-25}}.$$

We can confirm this result using values of Planck length and the ground state de Broglie wavelength

$$\frac{l_P}{\lambda\!\!\!^{-}} = \frac{1.616 \times 10^{-35}\ m}{5.292 \times 10^{-11}\ m} = \mathbf{3.054 \times 10^{-25}}.$$

4. Energy

Natural Planck units show how classical and quantum mechanical energy formulas are related. The formula for photon energy can be stated in Planck units as

$$E_\gamma = \frac{\hbar c}{\lambda\!\!\!^{-}} = \frac{l_P m_P c^2}{\lambda\!\!\!^{-}} = \left(\frac{l_P}{\lambda\!\!\!^{-}}\right) E_P. \tag{16}$$

indicating that photon energy is proportional to the Planck energy.

We can create a natural energy formula that is equivalent to the classical kinetic energy formula by building on the natural momentum formula (equations 14 and 15). Including a one-half coefficient and stating velocity as a ratio of the speed of light gives the formula

$$E_K = {}^1\!/_2 m v^2 = \frac{1}{2}\left(\frac{l_P}{\lambda\!\!\!^{-}}\right)\left(\frac{v}{c}\right) E_P \tag{17}$$

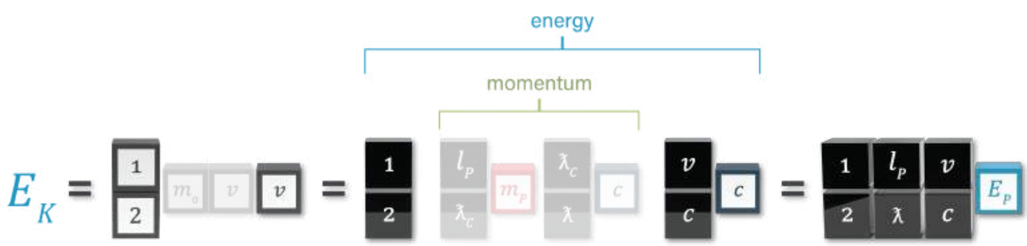
It is easy to show that this natural kinetic energy formula gives the same result as the classical kinetic energy formula. The following table compares the two formulas applied to the charged leptons.

Table 7. The charged leptons demonstrate that the natural kinetic energy formula is equivalent to the classical formula.

lepton	m_0	v	$\lambda\!\!\!^{-}$	E	E
	kg	m/s	m	kgm^2/s^2	kgm^2/s^2
	CODATA	arbitrary	$\frac{\hbar}{p} = \left(\frac{m_P}{m_0}\right)\left(\frac{c}{v}\right) l_P$	$1/2 m_0 v^2$	$\frac{1}{2}\left(\frac{l_P}{\lambda\!\!\!^{-}}\right)\left(\frac{v}{c}\right) E_P$
e	9.109×10^{-31}	$2,997,924$	3.862×10^{-11}	$\mathbf{4.094 \times 10^{-18}}$	$\mathbf{4.094 \times 10^{-18}}$
μ	1.884×10^{-28}	$1,000,000$	5.599×10^{-13}	$\mathbf{9.418 \times 10^{-17}}$	$\mathbf{9.418 \times 10^{-17}}$
τ	3.168×10^{-27}	$100,000,000$	3.329×10^{-16}	$\mathbf{1.584 \times 10^{-11}}$	$\mathbf{1.584 \times 10^{-11}}$

Figure 7 shows how the natural kinetic energy formula relates to classical momentum and energy formulas

Figure 7. Building on the natural momentum formula with a one-half coefficient and a particle's velocity produces a natural kinetic energy formula for matter.



4.1. Comparing the structure of matter and radiation

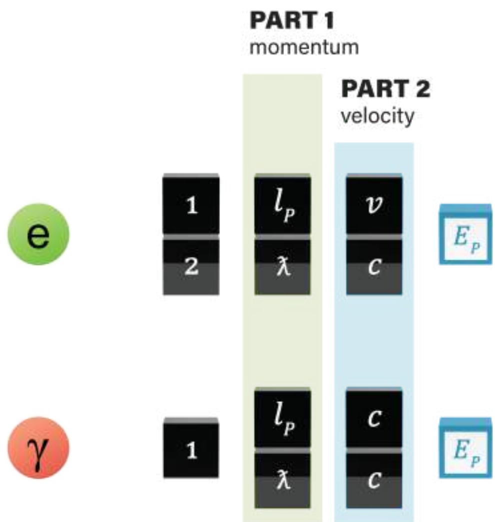
We can compare the structure of matter and radiation using the natural formulas. The natural formula for matter is given by 17 and the formula for radiation by 16. Equation 16 can be stated equivalently as

$$E_\gamma = 1\left(\frac{l_P}{\lambda\!\!\!^{-}}\right)\left(\frac{c}{c}\right) E_P \tag{18}$$

to match the structure of 17.

The following illustration compares the two formulas, highlighting the similarity in structure.

Figure 8. Natural formulas describe a 2-part energy mechanism consisting of 1) a particle's kinetic energy potential inversely proportional to its wavelength; and 2) a particle's velocity.



The two natural formulas give a physical description of kinetic energy that is hidden by the standard formulas. For matter and radiation, kinetic energy can be characterized in two parts:

1. The kinetic energy potential generated by a particle's wavelength, quantified by the ratio $l_P/ƛ$, where the Planck scale is the maximum potential.
2. The particle's velocity, where the speed of light is the maximum potential.

These two factors constitute a 2-part energy mechanism that gives simple, intuitive meaning to the kinetic energy of matter and radiation. Momentum is analogous to a payload quantified by the concentration of a particleâĂŹs wavelength, while velocity delivers the payload as energy.

The 2-part energy mechanism may be characterized as *reducing* or *diluting* the Planck energy potential over the 2 degrees of freedom represented by a particle's wavelength and velocity. These two factors can be consolidated into a single temporal ratio quantified by the particle's oscillation period. For both matter and radiation, the ratio of Planck time to oscillation period is equal to the ratio of the particle's energy to the Planck energy. The temporal formula for radiation is

$$E_\gamma = \frac{t_P}{T} E_P \tag{19}$$

and the matter formula is

$$E_K = \frac{t_P}{2T} E_P. \tag{20}$$

The natural kinetic energy formulas for radiation and matter have coefficients of one and one-half respectively. We propose that a one-half reduction in the kinetic energy of matter is due to quantum mechanical spin, distributing a half-spin particle's energy over an extended cycle.

5. Mass and symmetry

If we accept the quantum mechanical description of momentum as a proper description for large systems, and mv as a pragmatic tool for calculating particle wavelengths based on comparable proportions, then we must re-evaluate whether unit dimensions of momentum should include LT^{-1}. If velocity does not characterize the physical state of a particle's momentum, then unit dimension M may be a more natural expression of the *strength* or *density* given by a particle's wavelength.

Momentum has been a bedrock idea for centuries and the notion of changing unit dimensions will surely be met with strong historical bias. But the evidence warrants an objective evaluation. Unit dimensions of momentum were defined long before quantum mechanical properties of matter became known. Velocity squared seemed to fit the observational data and wavelength was an unknown and unobservable property at the time. It is easy to see why momentum was defined in terms of velocity. But a single, consistent explanation of momentum based on the quantum mechanical definition offers compelling evidence to the contrary.

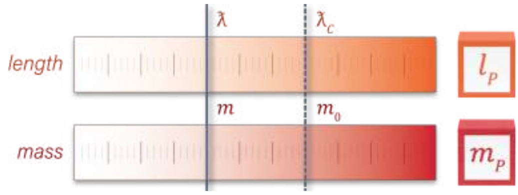
The case for restating momentum in units of mass is therefore presented here. The energy potential of a matter particle's wavelength can be represented in unit dimension M by applying a dimensionless transformation v/c or λ_C/λ to a particle's rest mass. By defining *inertial mass* m such that

$$m = m_0 \left(\frac{v}{c}\right) = \frac{p}{c}, \tag{21}$$

we get a number of results worth considering. First, we get a consistent physical description of momentum for matter and radiation as a function of wavelength and not velocity. Second, we get a general principle of length-mass symmetry that applies to all combinations of wavelength and inertial mass, and for which Compton wavelength and rest mass are a special case.

$$\lambda m = l_p m_P. \tag{22}$$

Figure 9. Inertial mass m makes length-mass symmetry a general principle in which Compton wavelength and rest mass are a special case.



Inertial mass is distinct from rest mass but the two quantities are related by the energy-momentum formula. Replacing traditional momentum $m_0 v$ with inertial mass m gives a vector sum of the two masses:

$$E = \sqrt{m_0^2 + m^2}\, c^2 \tag{23}$$

Inertial mass paints a clearer picture of how the individual properties of elementary particles conserve mass, momentum, and energy. The following table displays several properties and dynamics of the charged leptons given an arbitrary velocity. As before, any velocity will give similar results. The photon wavelength in the table was also chosen arbitrarily, and any wavelength will yield similar results. Conserved relationships are shown in bold and discussed below.

Using inertial mass in place of rest mass produces the same result as standard formulas for wavelength, momentum, oscillation period, and kinetic energy. In addition, the values in table 8 show three pairs of conserved quantities that help explain conservations of mass, momentum, and energy. These pairs are summarized in table 9.

On the surface these three relationships may appear as different symmetries but they are all expressions of the same length-mass symmetry represented in 9 and 22, and demonstrated in table 8. The additional two symmetries are easily explained in terms of the superfluous quantity c found in the classical momentum unit dimensions. Multiplying the conserved length-mass constant $l_P m_P$ by the constant c gives Planck's constant, which is conserved by combinations of wavelength-momentum

$$\lambda\!\!\!^{-} mc = l_P m_P c = \hbar. \tag{24}$$

Multiplying wavelength-momentum by the oscillation period T in both numerator and denominator gives time-energy which is therefore also conserved.

The following illustration explains how a conserved quantity of wavelength-mass produces conserved pairs of length-momentum and time-energy.

Table 8. Certain pairs of particle properties are conserved including wavelength-inertial mass, wavelength-momentum, and time-energy. All three relationships can be explained in terms of a single length-mass symmetry.

	λ_C	m_0	$\lambda_C m_0$	v	λ	m	λm
	m	kg	kgm	m/s	m	kg	kgm
	CODATA	CODATA	-	arbitrary*	$\frac{l_P m_P}{m}$	$\frac{l_P m_P}{\lambda}$	-
e	3.86×10^{-13}	9.11×10^{-31}	$\mathbf{3.52 \times 10^{-43}}$	$2,997,924$*	3.86×10^{-11}	9.11×10^{-33}	$\mathbf{3.52 \times 10^{-43}}$
μ	1.87×10^{-15}	1.88×10^{-28}	$\mathbf{3.52 \times 10^{-43}}$	$1,000,000$*	5.60×10^{-13}	6.28×10^{-31}	$\mathbf{3.52 \times 10^{-43}}$
τ	1.11×10^{-16}	3.17×10^{-27}	$\mathbf{3.52 \times 10^{-43}}$	$100,000,000$*	3.33×10^{-16}	1.06×10^{-27}	$\mathbf{3.52 \times 10^{-43}}$
γ	-	-	-	$299,792,458$	1.93×10^{-8}	1.82×10^{-35}	$\mathbf{3.52 \times 10^{-43}}$

	p	T	E	λp	$TE^{\S}$
	kgm/s	s	kgm^2/s^2	kgm^2/s	kgm^2/s
	mc	$\frac{\lambda}{v}$	$1^{\dagger} mvc$	-	-
e	2.73×10^{-24}	1.29×10^{-17}	4.09×10^{-18}	$\mathbf{1.05 \times 10^{-34}}$	$\mathbf{1.05 \times 10^{-34}}$
μ	1.88×10^{-22}	5.60×10^{-19}	9.42×10^{-17}	$\mathbf{1.05 \times 10^{-34}}$	$\mathbf{1.05 \times 10^{-34}}$
τ	3.17×10^{-19}	3.33×10^{-24}	1.58×10^{-11}	$\mathbf{1.05 \times 10^{-34}}$	$\mathbf{1.05 \times 10^{-34}}$
γ	5.45×10^{-27}	6.45×10^{-17}	1.63×10^{-18}	$\mathbf{1.05 \times 10^{-34}}$	$\mathbf{1.05 \times 10^{-34}}$

2021 CODATA values are used for λ_C, m_0, l_P, m_P, and c. λ determined using equation 11 and m determined using equation21.

§ 2TE for matter

† 0.5 for matter

Figure 10. Length-mass symmetry produces conserved pairs of wavelength-momentum and time-energy in standard unit dimensions of momentum. All three symmetries are the same expression of length-mass symmetry multiplied by constants.

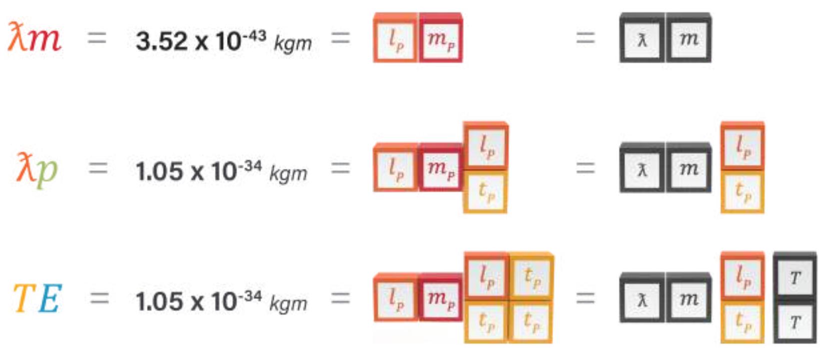


Table 9. A summary of symmetries accounting for conservations of mass, momentum, and energy in the properties of elementary particles.

Symmetry	Formula	Invariant	Conserved quantity
Length-Mass	$\lambda m = l_P m_P$	λm	$3.52 \times 10^{-43}\ kgm$
Length-Momentum	$\lambda p = l_P p_P$	λp	$1.05 \times 10^{-34}\ kgm^2/s$
Time-Energy	$TE = t_P E_P$	TE	$1.05 \times 10^{-34}\ kgm^2/s$

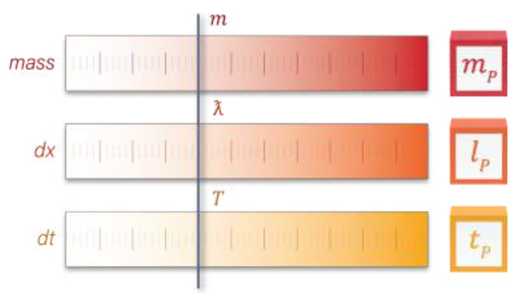
5.1. The structure of radiation

The physical structure of radiation can be explained in terms of inertial mass. A given photon wavelength has an inversely proportional quantity of inertial mass and a proportional oscillation period

$$\frac{l_P}{\lambda} = \frac{m}{m_P} = \frac{t_P}{T}. \tag{25}$$

These three ratios are evident from table 8 and represented in figure 11 as equivalent proportions between observable photon properties and the Planck scale.

Figure 11. In particles of radiation, ratios of wavelength, inertial mass, and oscillation period to the Planck scale are equivalent.



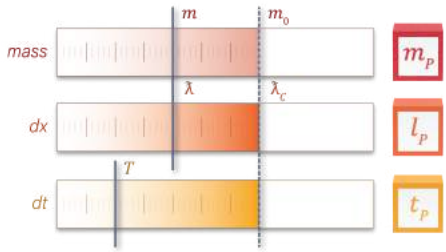
The photon's energy is the product of its inertial mass m and velocity c. A second instance of c simply states this result in standard unit dimensions of momentum and energy.

5.2. The structure of matter

The principal effect of rest mass is to limit the range of a particle's energy potential from the Planck scale down to the Compton wavelength scale. For particles of matter, the Compton wavelength quantifies the minimum wavelength and maximum inertial mass of the particle in non-relativistic terms.

Quantities of wavelength, inertial mass, and oscillation period are proportional to the Planck scale at the maximum limit of the particle's velocity, represented by the dashed line in figure 12. But since energy is allocated over two degrees of freedom, the particle's wavelength increases as its velocity decreases, and its oscillation period increases relative to the wavelength.

Figure 12. Wavelength and inertial mass are equal proportions of the Planck scale for matter, but rest mass limits the effective range of a particle's wavelength. Changes in kinetic energy are apportioned equally between changes in wavelength and velocity.



6. Conclusion

More than a century after Max Planck introduced his famous constant of proportionality, the universal constants $\hbar$ and G are considered fundamental units while Planck units of length, mass, and time are defined as their derivatives. But replacing the constants with natural Planck units reveals meaningful structure that explains relationships between classical and quantum phenomena. Planck's constant and the gravitational constant are better characterized as composite values of fundamental Planck units arranged in convenient ratios for performing calculations. The physical structure revealed by Planck unit formulas offers a deeper understanding of the natural world.

Natural unit formulas give new meaning to quantum and classical physics. The structure and relationships revealed by these formulas yield intuitive explanations of certain physical phenomena suggesting that quantum and classical systems are not so different. Furthermore, matter and radiation have more common structure than the standard equations imply. These insights offer new perspectives on existing theories and may stimulate new theories formulated in less abstract ways.

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