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Original Paper

Lorentz force, virtual photon, and electromagnetic radiation

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Abstract: By treating an electron as its own Electromagnetic (EM) field and generalizing the Lorentz force to be the field force between the electron's EM field and its external EM field, it is proved that the radiating field and the Coulomb-like field of an accelerated electron do interact, with the radiating field provides the exact momentum change needed by the Coulomb-like field. Thus, the radiating field of an accelerated electron fulfills the role of virtual photon in Quantum Electrodynamics (QED). By treating the radiating field as virtual photon, it is closely examined how the virtual photon is emitted and absorbed by the electron, and how the condition which leads to infinity in QED can be removed. Consequently, the necessity of Renormalization is removed. The conventional formula of the radiation power by an accelerated electron is questioned, and a new formula is given. Two experiments to test the new formula are proposed. When the electron is treated as its own EM field and its location is the center of mass of its EM field, it is explained why an electron does not radiate when it free-falls under the gravity.

Keywords: Lorentz force; Virtual photon; EM radiation; Renormalization; QED

1. Introduction

In the book "The Evolution of Physics", Albert Einstein stated "We cannot build physics on the basis of the matter concept alone. But the division into matter and field is, after the recognition of the equivalence of mass and energy, something artificial and not clearly defined. Could we not reject the concept of matter and build a pure field physics? What impresses our senses as matter is really a great concentration of energy into a comparatively small space. We could regard matter as the regions in space where the field is extremely strong. In this way a new philosophical background could be created. Its final aim would be the explanation of all events in nature by structure laws valid always and everywhere. There would be no place, in our new physics, for both field and matter, field being the only reality. Our ultimate problem would be to modify our field laws in such a way that they would not break down for regions in which the energy is enormously concentrated. But we have not so far succeeded in fulfilling this program convincingly and consistently. At present we must still assume in all our actual theoretical constructions two realities: field and matter" [1].

Quantum Field Theory (QFT) is one big step towards this goal [2] [3] [4] [5] [6]. In QFT, everything is described as field. The dynamics of a specific type of field is described by a partial differential field equation. A particle is simply treated as a field quantum which is a unit solution to the field equation [7]. For example, an electron is treated as a field quantum of the relativistic Dirac equation [8] [10].

In QFT, a particle has a self-field. The particle interacts with its self-field. For example, an electron has a self-field which is the electron's EM field. The electron's self-energy is the result of the interaction between the electron and its self-field. The calculation of the electron's self-energy results in an infinity [7]. But this does not cause any concern because QFT (or QED) uses the Renormalization procedure to cancel the infinity to yield a finite physical value which can be measured by experiments [9] [10].

Despite its enormous success, Richard P. Feynman lamented in his later book "QED" on Renormalization, "The shell game that we play is technically called renormalization. But no matter how clever the word, it is what I would call a dippy process! Having to resort to such hocus-pocus has prevented us from proving that the theory of quantum electrodynamics is mathematically self-consistent. It's surprising that the theory still hasn't been proved self-consistent one way or the other by now; I suspect that renormalization is not mathematically legitimate" [9].

Although Feynman is probably too harsh on Renormalization, Nature indeed does not allow any infinite physical value. An infinite value is most likely due to a poor postulation in physics, such as the ultraviolet catastrophe in the blackbody radiation before Max Planck solved it in 1901 [11].

Nevertheless, Renormalization is only applicable to renormalizable field theory. For nonrenormalizable field theory such as quantum gravity, physicists are still searching for a solution to overcome the infinite value problem [10].

In QFT, the particle's complete non-local wavefunction is treated as one physical field quantum [7]. To explain the single photon double slit experiment, the non-local physical field quantum passes through both slits, thus causes interference. However, when you put a photon detector at each slit, you can only hear one click when the photon passes through both slits [7][9]. If the complete non-local wavefunction is physical, then we could not explain why when a photon travels from the Sun to Earth, its wavefunction can instantly reach us, but we must wait for 8.3 minutes to feel its energy due to the speed of light.

In QFT (or QED more specifically), electron and its self-field which is the electron's EM field are treated as two separate identities. It is well known that the EM field has energy. Because of the equivalence of energy and mass, the electron's self-field also has mass. Two immediate questions arise: 1) what is the electron's mass made of? 2) how to differentiate between the inertia from the electron's mass and the inertia from the mass of its self-field when the electron is accelerated by a force? On the other hand, if we treat electron as its own EM field and its location is at the center of mass of its field, then these problems won't exist, and we do not need to worry about counting the electron's inertia twice.

We would like to address Einstein's quest with a different approach and publish our study in a series of two papers. The first paper of the series "Quantum Wavefunction Explained by the Sampling Theory" deals with the simplest case: an electron (or a photon) travels freely in an inertial coordinate system (CS). We treat the particle (electron or photon) as a local EM field and the particle's wavefunction as a non-local pseudo-EM wave which is formed by the particle's EM field and its non-physical image replicas. We successfully explain the single photon double slit experiment [12].

In this paper, we will consider yet another simple case: an electron moves in a constant EM field. We would like to see how the particle behave in acceleration.

People may find that the approaches in both papers are classical. The justification of using the classical approach is as follow. A century ago, when people studied the hydrogen atom, they found that classical physics completely failed. According to classical physics, when the electron moves around the atomic nucleus, it will constantly radiate energy, thus quickly falls into the nucleus. The classical physics also fails to explain why the electron can only occupy a discrete set of orbits in the atom. So, physicists abandoned the classical physics and went directly to the modern quantum physics. The consequence of that approach is that the link between classical physics and quantum physics is lost. So, people just bluntly say that classical physics only applies to the macro-world and quantum physics only applies to the micro-world. However, this does not explain why the same elementary particle electron only obeys the quantum rule inside an atom, but only obeys the classical rule in a cloud chamber. Is it because the cloud chamber is too big?

If we can solve both the radiation problem and the discrete energy problem in hydrogen atom, and re-create the particle's wavefunction by classical approach, then we may find the missing link between classical physics and quantum physics.

Since Lienard and Wiechert derived the EM field of a moving charge particle, it has become a textbook knowledge that an accelerated charge particle emits EM radiation [13] [14]. One can easily find the following description in a textbook on classical EM theory. The EM field of an accelerated charge particle is separated into two fields: a radiating field which decreases as $1/r$ and a Coulomb-like field which decreases as $1/r^2$. The radiating field exists in a spherical shell. The Coulomb-like field has a dislocation at the same spherical shell. The spherical shell which is like a ripple propagates at the speed of light. The center of the spherical shell back-traces to the particle's location when it was accelerated. Because the total power of the radiating field does not change with the radius of the spherical shell, the radiating energy escapes from the charge particle [14]. This process is depicted in Figure 1 in which an electron moves in the z direction. It experiences a sudden acceleration at location O' at time t'. At time $t > t'$, the electron travels to location O; the ripple travels to the spherical shell enclosed by two surfaces $\Sigma^+$and Σ^-.

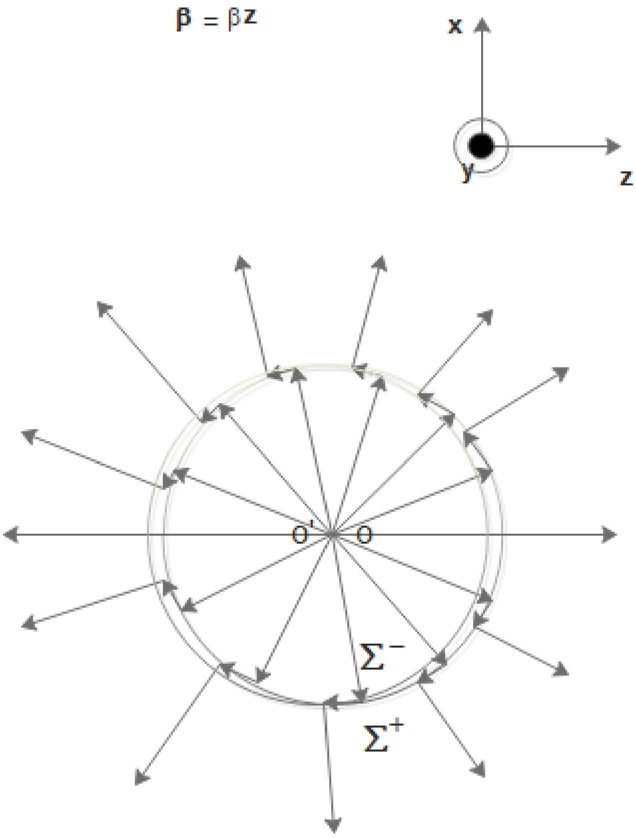

Fig. 1 Accelerated electron emits radiation

There is a hidden assumption in the above description: when the radiating field propagates, it does not interact with the Coulomb-like field and moves freely as though in an empty space. This assumption will be examined in later sections.

According to this theory, an accelerated electron radiates energy [14]. However, this conclusion is far from consistent with experiments. For example, consider electrons moving in a circular ring circuit. If the resistor of the circuit is high, there will be a lot of light and heat being radiated (a light bulb). However, if the resistor drops to zero so that the circuit becomes a super-conductor ring, then there will be no radiation, the electrons can move in the ring forever. It seems that the radiation depends more on the resistor of the circuit which tries to slow down the moving electrons than the circular motion of the electrons.

According to the Larmar formula on the radiation power by an accelerated charge particle, the radiation power is proportional to the power of 2 of the particle's acceleration [14] [15]. So, the radiation power is symmetric on acceleration and deceleration, electron radiates the same amount of energy in both scenarios. But it is very hard to find any direct experimental evidence to support this. It appears that a linearly decelerated electron radiates far more energy than a linearly accelerated electron. The radiation in deceleration is so strong that we even give it a special name, Bremsstrahlung radiation [14]. Theoretically, it is still an open debate on whether a linearly accelerated electron radiates or not [16][17][18][19]. Can we calculate and test the asymmetry of the electron's radiation between acceleration and deceleration?

In classical EM theory, electron is treated as a point of charge. The electron experiences the Lorentz force in an external EM field as

$$\mathbf{F} = \mathrm{e}\boldsymbol{E} + \mathrm{e}\boldsymbol{V}\mathrm{x}\boldsymbol{B} \quad (1)$$

in which $\boldsymbol{E}$ and $\boldsymbol{B}$ are the external electric field and magnetic field respectively, e is the electron's charge and $\boldsymbol{V}$ is its velocity [14]. In Classical Mechanics, the Lorentz force changes the electron's momentum and trajectory.

QED describes the EM force quite differently. In QED's view, the EM force is realized by emission and absorption of virtual photons by a charge particle. An electron changes its state by randomly emitting and absorbing virtual photons [9][10]. However, QED does not describe how a virtual photon is emitted and absorbed by the electron. It does not describe how this random process add up to the classical Lorentz force either.

Is there a link between the virtual photon and the radiating EM field of an accelerated electron? If the radiating EM field and the Coulomb-like EM field of an accelerated electron do interact, with the former provides the exact momentum change needed by the later, then can we say that the radiating EM field is the virtual photon? If so, how does it change the classical radiation theory?

Because we treat an electron as its own EM field [12], we must generalize the Lorentz force to be the field force between the electron's EM field and its external EM field. In our first paper, we demonstrate that a particle does not move randomly, and its wavefunction is not a probability wave but a pseudo-EM wave [12]. For the simple case in which an electron moves in a constant EM field, the interference between the electron and its image replicas is negligible in a large space, so we will discard the image replicas of the electron and treat it in the semi-classical way.

We will draw some analogy between the classical radiating field of an accelerated electron and the virtual photon in QED, and hopefully understand how the virtual photon is emitted and absorbed by the electron. We would also like to see if the condition which leads to infinity in QED can be completely removed.

It is not always true that an accelerated electron radiates. The famous paradox of the radiation by a charge particle in a gravitational field is described as follow. When a charge particle and a neutron particle both free-fall in a gravity field, General Relativity predicts that both fall at the same acceleration and speed. But if the charge particle radiates energy, then there will be a drag to slow it down compared to the neutron particle. Then they will not fall at the same speed.

A typical solution to this paradox is to admit that the Maxwell equations only hold true in an inertial CS. In a non-inertial CS where gravity prevails, the Maxwell equations as well as other classical EM theories need to be modified [20]. But is there a simpler explanation?

In the following sections these questions will be answered. The interaction between the radiating field and the Coulomb-like field of an accelerated electron will be calculated. It will be demonstrated when the radiation energy (emitting photons) be absorbed back by electron and when escape away.

2. Particle and field

In this section, we treat the electron as its own EM field with its position being at the center of mass of its EM field. Most formulas in this section are repeats from the first paper [12]. We include them here for a smooth description. In classical EM theory [14], the electron's electric field is described by the Gauss's law

$$\oiint \boldsymbol{E} \cdot d\boldsymbol{S} = \frac{e}{\varepsilon_0} \tag{2}$$

where $\boldsymbol{E}$ is the electric field, e is the electron's charge. In the first inertial CS, an electron rests at the origin $(x, y, z) = (0,0,0)$. From the spherical symmetry, it is easy to calculate that

$$\boldsymbol{E} = \frac{e}{4\pi\varepsilon_0}\frac{\hat{\boldsymbol{r}}}{r^2} \tag{3}$$

The energy density of the electric field is

$$\rho_E = \frac{\varepsilon_0}{2}E^2 \tag{4}$$

If we assume that Equation (3) only holds true at $r \geq r_0$, where r_0 denotes the radius of a very small spherical surface which serves as a boundary of the field, inside which $\boldsymbol{E} = 0$, then the total energy of the electric field

$$\mathcal{E}_E = \iiint \rho_E \, dV = \int_{r_0}^{\infty} r^2 \, dr \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\varphi \frac{\varepsilon_0}{2}\left(\frac{e}{4\pi\varepsilon_0 r^2}\right)^2 = \frac{e^2}{8\pi\varepsilon_0 r_0}$$

Let us equate it to the energy of electron's mass m_e

$$m_e c^2 = \frac{e^2}{8\pi\varepsilon_0 r_0} \tag{5}$$

Now consider a second CS moving at a constant speed $\boldsymbol{v} = -v\hat{\boldsymbol{z}}$ with respect to the first CS, where $\hat{\boldsymbol{z}}$ is the unit vector of z-axis. In the second CS, the electron moves at a constant speed $v\hat{\boldsymbol{z}}$. With Lorentz transform, the electron's electric field and magnetic field are [14]

$$\boldsymbol{E} = \frac{e}{4\pi\varepsilon_0}\frac{1-\beta^2}{[1-(\beta\sin\theta)^2]^{3/2}}\frac{\hat{\boldsymbol{r}}}{r^2} \tag{6}$$

$$\boldsymbol{B} = \frac{\mu_0 ec}{4\pi}\frac{\beta(1-\beta^2)\sin\theta}{[1-(\beta\sin\theta)^2]^{3/2}}\frac{\hat{\boldsymbol{\varphi}}}{r^2} \tag{7}$$

Equation (6) and (7) are written in the spherical coordinates (r, θ, φ). The energy density of the electric field and magnetic field are thus

$$\rho_E = \frac{\varepsilon_0}{2}E^2 = \frac{\varepsilon_0}{2}\left(\frac{e}{4\pi\varepsilon_0}\right)^2\frac{(1-\beta^2)^2}{[1-(\beta\sin\theta)^2]^3}\frac{1}{r^4} \tag{8}$$

$$\rho_B = \frac{1}{2\mu_0}B^2 = \frac{1}{2\mu_0}\left(\frac{\mu_0 ec}{4\pi}\right)^2\frac{[\beta(1-\beta^2)\sin\theta]^2}{[1-(\beta\sin\theta)^2]^3}\frac{1}{r^4} \tag{9}$$

The field boundary, a spherical surface

$$r_s = r_0 \tag{10}$$

in the first CS, then becomes an elliptical surface in the second CS due to the relativistic shrinking in z-axis. The surface is thus

$$r_s = \frac{r_0}{\gamma[1-(\beta\sin\theta)^2]^{1/2}} \tag{11}$$

Here $\beta = \frac{v}{c}$, $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ with c being the speed of light. The total energy of the electric field is

$$\begin{aligned}\mathcal{E}_E = \iiint \rho_E \, dV &= \int_{r_s}^{\infty} r^2 \, dr \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\varphi \frac{\varepsilon_0}{2}\left(\frac{e}{4\pi\varepsilon_0}\right)^2\frac{(1-\beta^2)^2}{[1-(\beta\sin\theta)^2]^3}\frac{1}{r^4} \\ &= \frac{e^2(1-\beta^2)^2}{16\pi\varepsilon_0}\int_0^{\pi}\frac{\sin\theta}{r_s[1-(\beta\sin\theta)^2]^3}d\theta \\ &= \frac{e^2(1-\beta^2)^2\gamma}{16\pi\varepsilon_0 r_0}\int_0^{\pi}\frac{\sin\theta}{[1-(\beta\sin\theta)^2]^{5/2}}d\theta\end{aligned}$$

Use

$$\int_0^\pi \frac{\sin\theta}{[1-(\beta\sin\theta)^2]^{5/2}}d\theta = \int_{-1}^{1}\frac{dx}{(1-\beta^2)^{\frac{5}{2}}(1+\frac{\beta^2x^2}{1-\beta^2})^{5/2}} = 2\gamma^4(1-\frac{\beta^2}{3})$$

Then

$$\mathcal{E}_E = \frac{e^2\gamma}{8\pi\varepsilon_0 r_0}\left(1-\frac{\beta^2}{3}\right) \quad (12)$$

Similarly, the total energy of the magnetic field is

$$\mathcal{E}_B = \iiint \rho_B\, dV = \int_{r_s}^{\infty} r^2\, dr \int_0^\pi \sin\theta\, d\theta \int_0^{2\pi} d\varphi \frac{1}{2\mu_0}(\frac{\mu_0 ec}{4\pi})^2 \frac{[\beta(1-\beta^2)\sin\theta]^2}{[1-(\beta\sin\theta)^2]^3}\frac{1}{r^4}$$
$$= \frac{\mu_0 e^2c^2\gamma}{8\pi r_0}\left(\frac{2\beta^2}{3}\right) \quad (13)$$

Use $c^2 = \frac{1}{\varepsilon_0\mu_0}$, Equation (5), (12) and (13), the total energy of the EM field of the electron is

$$\mathcal{E} = \mathcal{E}_E + \mathcal{E}_B = m_ec^2\gamma\left(1-\frac{\beta^2}{3}\right) + m_ec^2\gamma\left(\frac{2\beta^2}{3}\right) = m_ec^2\gamma\left(1+\frac{\beta^2}{3}\right) \quad (14)$$

Now let us calculate the electron's momentum in the second CS. The momentum density of the EM field is [14]

$$\boldsymbol{Q} = \varepsilon_0\, \boldsymbol{E}\text{x}\boldsymbol{B} \quad (15)$$

The total momentum of the EM field is

$$\mathbf{P} = \iiint \boldsymbol{Q}\, dV = \iiint \varepsilon_0\, \boldsymbol{E}\text{x}\boldsymbol{B}\, dV$$
$$= \int_{r_s}^{\infty} r^2\, dr \int_0^\pi \sin\theta\, d\theta \int_0^{2\pi} d\varphi\, \varepsilon_0\, \frac{e}{4\pi\varepsilon_0}\frac{1-\beta^2}{[1-(\beta\sin\theta)^2]^{3/2}}\frac{\hat{\boldsymbol{r}}}{r^2}\text{x}\frac{\mu_0 ec}{4\pi}\frac{\beta(1-\beta^2)\sin\theta}{[1-(\beta\sin\theta)^2]^{3/2}}\frac{\hat{\boldsymbol{\varphi}}}{r^2}$$
$$= \frac{e^2\gamma(1-\beta^2)^2\boldsymbol{v}}{8\pi\varepsilon_0 c^2 r_0}\int_0^\pi \frac{(\sin\theta)^3}{[1-(\beta\sin\theta)^2]^{5/2}}d\theta$$

Use

$$\int_0^\pi \frac{(\sin\theta)^3}{[1-(\beta\sin\theta)^2]^{5/2}}d\theta = \int_{-1}^{1}\frac{(1-x^2)}{(1-\beta^2)^{\frac{5}{2}}(1+\frac{\beta^2x^2}{1-\beta^2})^{5/2}}dx = \frac{4}{3}\gamma^4$$

Then

$$\mathbf{P} = \frac{e^2\gamma\boldsymbol{v}}{6\pi\varepsilon_0 c^2 r_0} = \frac{4}{3}m_e\gamma\boldsymbol{v} \quad (16)$$

In Equation (14) and (16), notice that from the field view, an electron not only has the traditional particle term

$$\begin{pmatrix}\mathcal{E}\\ \mathbf{P}\end{pmatrix} = \begin{pmatrix}m_e\gamma c^2\\ m_e\gamma\boldsymbol{v}\end{pmatrix} \quad (17)$$

but also, a new term

$$\begin{pmatrix}\mathcal{E}\\ \mathbf{P}\end{pmatrix} = \begin{pmatrix}\frac{1}{3}m_e\gamma v^2\\ \frac{1}{3}m_e\gamma\boldsymbol{v}\end{pmatrix} \quad (18)$$

The new term accompanies the traditional term and travels with it. This new term does not contradict Special Relativity which only describes the term in Equation (17) for a classical particle. Unlike a classical particle, electron radiates energy in deceleration and this new term plays an important role in electron's radiation. This will be discussed in more detail in the following sections.

3. Lorentz force in field view

In classical EM theory, an electron in an external EM field (EM field excluding its own field) experiences the Lorentz force in Equation (1). The force is exerting on the point of charge which is at the center of mass of the electron's EM field. In section 2, the electron is simply treated as its own field. Then we can ask the following question "Can the Lorentz force be expressed as a field force between the electron's EM field and its external EM field?"

If we use $(\mathbf{E}_1, \mathbf{B}_1)$ to denote the electron's EM field and $(\mathbf{E}_2, \mathbf{B}_2)$ to denote its external EM field, then we can combine Equation (1) and (2) as follow

$$\mathbf{F} = \mathrm{e}\mathbf{E}_2 + \mathrm{e}\boldsymbol{V}\mathrm{x}\mathbf{B}_2 = \varepsilon_0 \oiint (\mathbf{E}_2 + \boldsymbol{V}\mathrm{x}\mathbf{B}_2)\mathbf{E}_1 \cdot d\boldsymbol{S} \tag{19}$$

the integral is on any surface which encloses the electron's inner boundary surface described by Equation (11).

4. EM field of an accelerated electron

According to Lienard and Wiechert, if an electron has velocity $\boldsymbol{\beta}$ and acceleration $\dot{\boldsymbol{\beta}}$ at time t', then at time $t > t'$ its EM field will have two terms [14],

$$\begin{cases} \mathbf{E}_1 = \dfrac{e}{4\pi\varepsilon_0} \dfrac{(1-\beta^2)(\mathbf{n}-\boldsymbol{\beta})}{(1-\mathbf{n}\cdot\boldsymbol{\beta})^3 R^2} & (20a) \\ \mathbf{B}_1 = \dfrac{1}{c}\mathbf{n}\mathrm{x}\mathbf{E}_1 & (20b) \end{cases}$$

$$\begin{cases} \mathbf{E}_2 = \dfrac{e}{4\pi\varepsilon_0 c} \dfrac{\mathbf{n}\mathrm{x}[(\mathbf{n}-\boldsymbol{\beta})\mathrm{x}\dot{\boldsymbol{\beta}}]}{(1-\mathbf{n}\cdot\boldsymbol{\beta})^3 R} & (21a) \\ \mathbf{B}_2 = \dfrac{1}{c}\mathbf{n}\mathrm{x}\mathbf{E}_2 & (21b) \end{cases}$$

here $\mathbf{n}$ is the unit vector of $\boldsymbol{R}$ where $|\boldsymbol{R}| = c(t-t')$. As depicted in Figure 2(a), at time $t > t'$, the electron travels to location O. At time t', the electron was at location O' where it was accelerated and emitted a ripple which travels at the speed of light. At time $t > t'$, the ripple travels to the location in the hashed area.

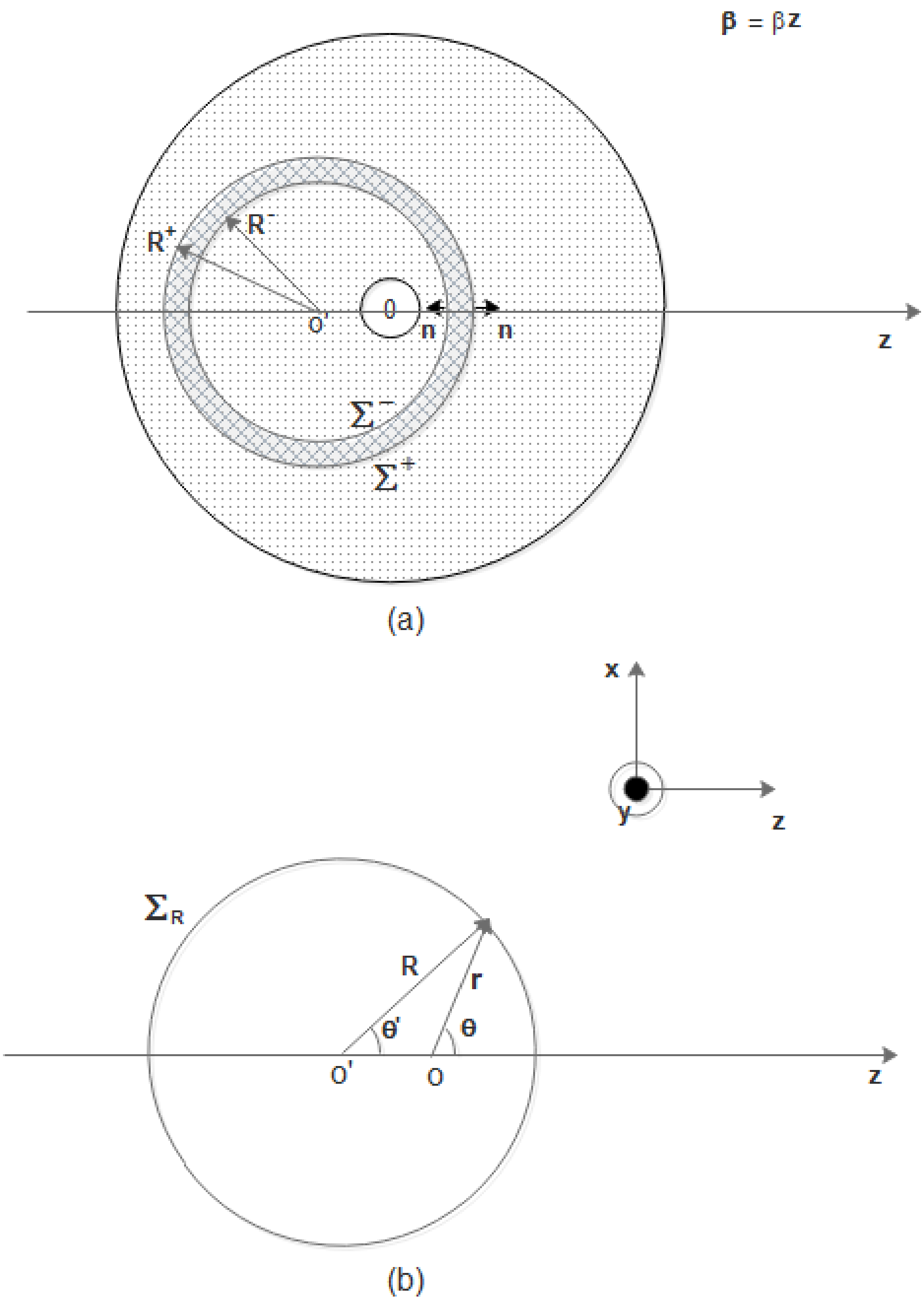

Fig. 2. Radiating field and Coulomb-like field of accelerated electron

From Figure 2(b), it is easy to derive

$$\begin{cases} r sin\theta = R sin\theta' \\ r cos\theta + \beta c(t - t') = R cos\theta' \\ R = c(t - t') \end{cases} \tag{22}$$

From Equation (22), it is easy to derive

$$\begin{cases} \dfrac{R}{r} = \gamma^2\{\beta cos\theta + [1-(\beta\sin\theta)^2]^{1/2}\} \\ 1-\mathbf{n}\cdot\boldsymbol{\beta} = \dfrac{[1-(\beta\sin\theta)^2]^{1/2}}{\gamma^2\{\beta cos\theta + [1-(\beta\sin\theta)^2]^{1/2}\}} \\ \mathbf{n}-\boldsymbol{\beta} = \dfrac{\boldsymbol{r}}{R} \end{cases} \tag{23}$$

Using Equation (23), Equation (20a) is reduced to Equation (6)

$$\mathbf{E}_1 = \frac{e}{4\pi\varepsilon_0}\frac{(1-\beta^2)(\mathbf{n}-\boldsymbol{\beta})}{(1-\mathbf{n}\cdot\boldsymbol{\beta})^3R^2} = \frac{e}{4\pi\varepsilon_0}\frac{1-\beta^2}{[1-(\beta\sin\theta)^2]^{3/2}}\frac{\hat{\boldsymbol{r}}}{r^2} \tag{24}$$

This is quite extraordinary, the Coulomb-like field of the electron has radial symmetry regardless of its acceleration.

Assume that the electron is only accelerated in a very short time interval $[t', t'+dt']$ during which its velocity is accelerated from $\boldsymbol{\beta}_1 = \boldsymbol{\beta}$ to $\boldsymbol{\beta}_2 = \boldsymbol{\beta} + \dot{\boldsymbol{\beta}}dt'$, then the electron emits a ripple which travels at the speed of light. At time $t > t'$, the ripple travels to the region between spherical surfaces $\Sigma^+$and Σ^- as depicted in Figure 2(a).

In the rest of the paper, let us define particle 1 to be the EM field described by Equation (20) and particle 2 to be the EM field described by Equation (21).

In classical EM theory, it is assumed that there is no interaction between particle 1 and particle 2. In Figure 2, the ripple travels freely as though it is in an empty space. But this assumption is not quite apparent.

In section 2, it is demonstrated that an electron with speed $\boldsymbol{\beta}$ has momentum of $\frac{4}{3}m_e\gamma c\boldsymbol{\beta}$ if the electron is treated as its own EM field. From Figure 1, it's apparent that when the electron is accelerated from $\boldsymbol{\beta}_1$ to $\boldsymbol{\beta}_2$ during $[t', t'+dt']$, its momentum does not immediately change from $\frac{4}{3}m_e\gamma c\boldsymbol{\beta}_1$ to $\frac{4}{3}m_e\gamma c\boldsymbol{\beta}_2$ during the same time interval. For example at time $t > t'$, inside the wave-front Σ^- the EM field of particle 1 is described by $\frac{e}{4\pi\varepsilon_0}\frac{1-{\beta_2}^2}{[1-(\beta_2\sin\theta)^2]^{3/2}}\frac{\hat{\boldsymbol{r}}}{r^2}$, but outside the wave-front Σ^+ it is described by $\frac{e}{4\pi\varepsilon_0}\frac{1-{\beta_1}^2}{[1-(\beta_1\sin\theta)^2]^{3/2}}\frac{\hat{\boldsymbol{r}}}{r^2}$. So, its momentum gradually changes from $\frac{4}{3}m_e\gamma c\boldsymbol{\beta}_1$ to $\frac{4}{3}m_e\gamma c\boldsymbol{\beta}_2$ as the ripple propagates through the entire field.

We are going to calculate the interaction between particle 1 and particle 2 by using the generalized Lorentz force formulated in section 3. It will be demonstrated that the force exerted on particle 1 by particle 2 provides the exact momentum change needed by particle 1 according to the Newton's second law.

5. EM radiation by electron in linear motion

In Figure 2, assume that the electron is in linear motion. At time t' its velocity is $\boldsymbol{\beta} = \beta\hat{\boldsymbol{z}}$ and its acceleration is $\dot{\boldsymbol{\beta}} = \dot{\beta}\hat{\boldsymbol{z}}$**.** At time $t > t'$, the energy of particle 2 is in a spherical shell between two spherical surfaces Σ^+ and Σ^-,

$$d\mathcal{E} = \iiint \rho\, dV = \int_{R^-}^{R^+} R^2\, dR \int_0^{\pi} \sin\theta'\, d\theta' \int_0^{2\pi} d\varphi'\, \varepsilon_0 |\boldsymbol{E}_2|^2$$

$$= \int_{R^-}^{R^+} R^2\, dR \int_0^{\pi} \sin\theta'\, d\theta' \int_0^{2\pi} d\varphi'\, \varepsilon_0 \left| \frac{e}{4\pi\varepsilon_0 c} \frac{\mathbf{n}\mathrm{x}[(\mathbf{n}-\boldsymbol{\beta})\mathrm{x}\dot{\boldsymbol{\beta}}]}{(1-\mathbf{n}\cdot\boldsymbol{\beta})^3 R} \right|^2$$

$$= \int_{R^-}^{R^+} R^2\, dR \int_0^{\pi} \sin\theta'\, d\theta' \int_0^{2\pi} d\varphi'\, \varepsilon_0 \left(\frac{e}{4\pi\varepsilon_0 c}\right)^2 \left| \frac{\mathbf{n}\mathrm{x}(\mathbf{n}\mathrm{x}\dot{\beta}\hat{\mathbf{z}})}{(1-\beta\cos\theta')^3 R} \right|^2$$

$$= \int_{R^-}^{R^+} R^2\, dR \int_0^{\pi} \sin\theta'\, d\theta' \int_0^{2\pi} d\varphi'\, \varepsilon_0 \left(\frac{e}{4\pi\varepsilon_0 c}\right)^2 \frac{\dot{\beta}^2(\sin\theta')^2}{(1-\beta\cos\theta')^6 R^2}$$

$$= \frac{e^2\dot{\beta}^2}{8\pi\varepsilon_0 c^2} \int_0^{\pi} \frac{dR(\sin\theta')^3}{(1-\beta\cos\theta')^6} d\theta'$$

Use $dR = c(1-\beta\, cos\,\theta')dt'$

$$d\mathcal{E} = \frac{e^2\dot{\beta}^2 dt'}{8\pi\varepsilon_0 c} \int_0^{\pi} \frac{(\sin\theta')^3}{(1-\beta\, cos\,\theta')^5} d\theta' = \frac{e^2\dot{\beta}^2 dt'}{8\pi\varepsilon_0 c} \int_{-1}^{1} \frac{(1-x^2)}{(1-\beta x)^5} dx = \frac{e^2\dot{\beta}^2 dt'}{8\pi\varepsilon_0 c} \frac{4}{3}\gamma^6$$

So

$$d\mathcal{E} = \frac{e^2\dot{\beta}^2\gamma^6}{6\pi\varepsilon_0 c} dt' \tag{25}$$

The momentum of particle 2 is

$$d\boldsymbol{P} = \iiint \boldsymbol{Q}\, dV = \int_{R^-}^{R^+} R^2\, dR \int_0^{\pi} \sin\theta'\, d\theta' \int_0^{2\pi} d\varphi'\, \varepsilon_0 \boldsymbol{E}_2 \mathrm{x} \boldsymbol{B}_2$$

$$= \int_{R^-}^{R^+} R^2\, dR \int_0^{\pi} \sin\theta'\, d\theta' \int_0^{2\pi} d\varphi'\, \varepsilon_0 \boldsymbol{E}_2 \mathrm{x} \left(\frac{1}{c}\mathbf{n}\mathrm{x}\boldsymbol{E}_2\right)$$

$$= \int_{R^-}^{R^+} R^2\, dR \int_0^{\pi} \sin\theta'\, d\theta' \int_0^{2\pi} d\varphi' \frac{\varepsilon_0}{c} |\boldsymbol{E}_2|^2 \mathbf{n}$$

$$= \int_{R^-}^{R^+} R^2\, dR \int_0^{\pi} \sin\theta'\, d\theta' \int_0^{2\pi} d\varphi' \frac{\varepsilon_0}{c} \left(\frac{e}{4\pi\varepsilon_0 c}\right)^2 \frac{\dot{\beta}^2(\sin\theta')^2}{(1-\beta\cos\theta')^6 R^2} (\cos\theta'\, \hat{\mathbf{z}}$$
$$+ \sin\theta' \cos\varphi'\, \hat{\boldsymbol{x}} + \sin\theta' \sin\varphi'\, \hat{\boldsymbol{y}}) = \frac{\mu_0 e^2\dot{\beta}^2 \hat{\mathbf{z}}}{8\pi c} \int_0^{\pi} \frac{dR(\sin\theta')^3\cos\theta'}{(1-\beta\cos\theta')^6} d\theta'$$

Use $dR = c(1-\beta\, cos\,\theta')dt'$

$$d\boldsymbol{P} = \frac{\mu_0 e^2\dot{\beta}^2 dt'\hat{\mathbf{z}}}{8\pi} \int_0^{\pi} \frac{(\sin\theta')^3\, cos\,\theta'}{(1-\beta\, cos\,\theta')^5} d\theta' = \frac{\mu_0 e^2\dot{\beta}^2 dt'\hat{\mathbf{z}}}{8\pi} \int_{-1}^{1} \frac{(1-x^2)x}{(1-\beta x)^5} dx = \frac{\mu_0 e^2\dot{\beta}^2 dt'\hat{\mathbf{z}}}{8\pi} \frac{4}{3}\gamma^6\beta$$

So

$$d\boldsymbol{P} = \frac{\mu_0 e^2 \boldsymbol{\beta}\dot{\beta}^2\gamma^6}{6\pi} dt' \tag{26}$$

From Equation (25) and (26), even particle 2 can be viewed as a set of photons because it propagates at the speed of light, collectively it moves as a single particle with rest mass

$$m_2 = \frac{\mu_0 e^2 \dot{\beta}^2 \gamma^5}{6\pi c} dt' \tag{27}$$

With Equation (27), Equation (25) and (26) would become $\mathcal{E}_2 = m_2 \gamma c^2$ and $\boldsymbol{P}_2 = m_2 \gamma \boldsymbol{v}$ respectively. However, after particle 2 propagates through the entire field of particle 1, it becomes so diffuse that it could hardly be recognized as a single particle.

In section 3, the traditional Lorentz force is generalized to be the field force between EM fields. From Equation (19), the Lorentz force exerted on particle 1 by particle 2 is

$$\mathbf{F} = \varepsilon_0 \oiint_{\Sigma^-}^{\Sigma^+} (\mathbf{E}_2 + \boldsymbol{V}\mathrm{x}\mathbf{B}_2)\mathbf{E}_1 \cdot d\boldsymbol{S}$$

The surface vector of the inner boundary surface Σ^- is $-\hat{\boldsymbol{n}}$ and the surface vector of the outer boundary surface Σ^+ is $\hat{\boldsymbol{n}}$. The enclosed volume is a spherical shell in between. The integral on the inner boundary surface is

$$\boldsymbol{F}_{\Sigma^-} = \varepsilon_0 \oiint (\mathbf{E}_2 + \boldsymbol{V}\mathrm{x}\mathbf{B}_2)\mathbf{E}_1 \cdot d\boldsymbol{S}$$

From Equation (22), (23) and Figure 2(b)

$$\mathbf{E}_2 = \frac{e}{4\pi\varepsilon_0 c}\frac{\mathbf{n}\mathrm{x}[(\mathbf{n}-\boldsymbol{\beta})\mathrm{x}\dot{\boldsymbol{\beta}}]}{(1-\mathbf{n}\cdot\boldsymbol{\beta})^3 R} = \frac{e}{4\pi\varepsilon_0 c}\frac{\mathbf{n}\mathrm{x}[(\mathbf{n}-\beta\hat{\mathbf{z}})\mathrm{x}\dot{\beta}\hat{\mathbf{z}}]}{(1-\mathbf{n}\cdot\boldsymbol{\beta})^3 R} = \frac{e\dot{\beta}}{4\pi\varepsilon_0 c}\frac{(\mathbf{n}\cdot\hat{\mathbf{z}})\mathbf{n}-\hat{\mathbf{z}}}{(1-\mathbf{n}\cdot\boldsymbol{\beta})^3 R}$$
$$= \frac{e\dot{\beta}}{4\pi\varepsilon_0 c}\frac{\sin\theta'\cos\theta'\cos\varphi'\,\hat{\boldsymbol{x}} + \sin\theta'\cos\theta'\sin\varphi'\,\hat{\boldsymbol{y}} - (\sin\theta')^2\hat{\mathbf{z}}}{(1-\beta\cos\theta')^3 R}$$

$$\boldsymbol{V}\mathrm{x}\mathbf{B}_2 = \boldsymbol{V}\mathrm{x}\left(\frac{1}{c}\mathbf{n}\mathrm{x}\boldsymbol{E}_2\right) = \boldsymbol{\beta}\mathrm{x}(\mathbf{n}\mathrm{x}\boldsymbol{E}_2) = (\boldsymbol{\beta}\cdot\mathbf{E}_2)\mathbf{n} - (\boldsymbol{\beta}\cdot\mathbf{n})\mathbf{E}_2$$
$$= -\frac{e\beta\dot{\beta}}{4\pi\varepsilon_0 c}\frac{\sin\theta'\cos\varphi'\,\hat{\boldsymbol{x}} + \sin\theta'\sin\varphi'\,\hat{\boldsymbol{y}}}{(1-\beta\cos\theta')^3 R}$$

$$\mathbf{E}_2 + \boldsymbol{V}\mathrm{x}\mathbf{B}_2 = \frac{e\dot{\beta}}{4\pi\varepsilon_0 c}\frac{\sin\theta'(\cos\theta'-\beta)\cos\varphi'\,\hat{\boldsymbol{x}} + \sin\theta'(\cos\theta'-\beta)\sin\varphi'\,\hat{\boldsymbol{y}} - (\sin\theta')^2\hat{\mathbf{z}}}{(1-\beta\cos\theta')^3 R}$$

$$\mathbf{E}_1 \cdot d\boldsymbol{S} = \frac{e}{4\pi\varepsilon_0}\frac{(1-\beta^2)(\mathbf{n}-\beta\hat{\mathbf{z}})}{(1-\beta\cos\theta')^3 R^2}\cdot(-\mathbf{n})R^2\sin\theta'\,d\theta' d\varphi' = -\frac{e}{4\pi\varepsilon_0\gamma^2}\frac{\sin\theta'\,d\theta' d\varphi'}{(1-\beta\cos\theta')^2}$$

So

$$\boldsymbol{F}_{\Sigma^-} = \varepsilon_0 \oiint (\mathbf{E}_2 + \boldsymbol{V}\mathrm{x}\mathbf{B}_2)\mathbf{E}_1 \cdot d\boldsymbol{S} = \varepsilon_0\left(\frac{e\dot{\beta}}{4\pi\varepsilon_0 c}\right)\left(-\frac{e}{4\pi\varepsilon_0\gamma^2}\right)\int_0^{2\pi} d\varphi'$$
$$\int_0^{\pi}\frac{\{\sin\theta'(\cos\theta'-\beta)\cos\varphi'\,\hat{\boldsymbol{x}} + \sin\theta'(\cos\theta'-\beta)\sin\varphi'\,\hat{\boldsymbol{y}} - (\sin\theta')^2\hat{\mathbf{z}}\}\sin\theta'\,d\theta'}{(1-\beta\cos\theta')^5 R}$$
$$= \frac{e^2\dot{\beta}\hat{\mathbf{z}}}{8\pi\varepsilon_0 c\gamma^2}\int_0^{\pi}\frac{(\sin\theta')^3 d\theta'}{(1-\beta\cos\theta')^5 R}$$

Similarly

$$\boldsymbol{F}_{\Sigma^+} = -\frac{e^2\dot{\beta}\hat{\mathbf{z}}}{8\pi\varepsilon_0 c\gamma^2}\int_0^{\pi}\frac{(\sin\theta')^3 d\theta'}{(1-\beta\cos\theta')^5 (R+dR)}$$

So

$$\boldsymbol{F} = \boldsymbol{F}_{\Sigma^-} + \boldsymbol{F}_{\Sigma^+} = \frac{e^2\dot{\beta}\hat{\boldsymbol{z}}}{8\pi\varepsilon_0 c\gamma^2}\frac{dR}{R^2}\int_0^{\pi}\frac{(\sin\theta')^3 d\theta'}{(1-\beta\cos\theta')^5}$$
$$= \frac{e^2\dot{\boldsymbol{\beta}}}{8\pi\varepsilon_0 c\gamma^2}\frac{c(1-\beta\cos\theta')dt'}{R^2}\int_0^{\pi}\frac{(\sin\theta')^3 d\theta'}{(1-\beta\cos\theta')^5}$$
$$= \frac{e^2\dot{\boldsymbol{\beta}}}{8\pi\varepsilon_0\gamma^2}\frac{dt'}{R^2}\int_0^{\pi}\frac{(\sin\theta')^3 d\theta'}{(1-\beta\cos\theta')^4} = \frac{e^2\dot{\boldsymbol{\beta}}}{8\pi\varepsilon_0\gamma^2}\frac{dt'}{R^2}\frac{4}{3}\gamma^4 = \frac{e^2\gamma^2\dot{\boldsymbol{\beta}}}{6\pi\varepsilon_0}\frac{dt'}{R^2}$$

This is the momentum transfer rate from particle 2 to particle 1 at time $t > t'$. The total momentum gained by particle 1 when the ripple, particle 2, propagates through the entire field of particle 1 is

$$d\boldsymbol{P} = \int_{r_0/\gamma}^{\infty}\boldsymbol{F}\frac{dR}{c} = \int_{r_0/\gamma}^{\infty}\frac{e^2\gamma^2\dot{\boldsymbol{\beta}}}{6\pi\varepsilon_0}\frac{dt'}{R^2}\frac{dR}{c} = \frac{e^2\gamma^2\dot{\boldsymbol{\beta}}dt'}{6\pi\varepsilon_0 c}\frac{1}{r_0/\gamma} = \frac{e^2\gamma^3\dot{\boldsymbol{\beta}}}{6\pi\varepsilon_0 c r_0}dt' \tag{28}$$

Use Equation (5), Equation (28) becomes

$$\frac{d\boldsymbol{P}}{dt'} = \frac{4}{3}m_e c\gamma^3\dot{\boldsymbol{\beta}} \tag{29}$$

On the other hand, from Equation (16)

$$\frac{d\boldsymbol{P}}{dt'} = \frac{d(\frac{4}{3}m_e\gamma\boldsymbol{v})}{dt'} = \frac{4}{3}m_e c\left(\frac{d\gamma}{dt'}\boldsymbol{\beta} + \gamma\dot{\boldsymbol{\beta}}\right) = \frac{4}{3}m_e c\left(\beta^2\gamma^3\dot{\boldsymbol{\beta}} + \gamma\dot{\boldsymbol{\beta}}\right) = \frac{4}{3}m_e c\gamma^3\dot{\boldsymbol{\beta}}$$

This perfectly matches Equation (29). This exercise proves that when an electron is accelerated during the time interval $[t', t' + dt']$, the momentum of particle 1 is not changed right away during the same time interval. The momentum difference is only gained through the Lorentz force exerted by particle 2 when it propagates through the entire EM field of particle 1. The old assumption in classical EM theory that there is no interaction between particle 1 and particle 2 is not true.

However, we cannot simply apply the Newton's third law to particle 2 because the Lorentz force is non-symmetric. For example

$$\varepsilon_0 \oiint (\mathbf{E}_2 + \boldsymbol{V}\mathrm{x}\mathbf{B}_2)\mathbf{E}_1\cdot d\boldsymbol{S} \neq \varepsilon_0 \oiint (\mathbf{E}_1 + \boldsymbol{V}\mathrm{x}\mathbf{B}_1)\mathbf{E}_2\cdot d\boldsymbol{S}$$

So, what is the fate of particle 2? To answer this question, let us use the law of energy conservation. Assume that an electron is linearly accelerated in a constant electric field $\boldsymbol{E} = E\hat{\boldsymbol{z}}$ during the time interval $[t', t' + dt']$. At time $t = t'$ according to Equation (14), the energy of the electron is

$$\mathcal{E}_1 = m_e c^2\gamma\left(1 + \frac{\beta^2}{3}\right) \tag{30}$$

At time $t = t' + dt'$ the electron breaks into two: particle 1 and particle 2. The total energy is

$$\mathcal{E}_2 = U + \mho + m_e c^2\gamma\left(1 + \frac{\beta^2}{3}\right) + \frac{e^2\dot{\beta}^2\gamma^6}{6\pi\varepsilon_0 c}dt' \tag{31}$$

In Equation (31) the first term U is the potential energy of particle 1 in the external electric field $\boldsymbol{E}$. From Equation (29) it is derived as

$$U = -e\boldsymbol{E}\cdot d\boldsymbol{l} = -\boldsymbol{F}\cdot\boldsymbol{v}dt' = -\frac{d\boldsymbol{P}}{dt'}\cdot\boldsymbol{\beta}c dt' = -c\boldsymbol{\beta}\cdot d\boldsymbol{P} = -\frac{4}{3}m_e c^2\gamma^3\beta d\beta \tag{32}$$

The second term $\mho$ is the potential energy between particle 1 and particle 2 because there is a Lorentz force between them. The third term is the energy of particle 1 because its EM field is mostly unchanged at this time. The fourth term is the energy of particle 2 according to Equation (25). Combine Equation (31) and (32)

$$\mathcal{E}_2 = -\frac{4}{3}m_e c^2\gamma^3\beta d\beta + \mho + m_e c^2\gamma\left(1+\frac{\beta^2}{3}\right) + \frac{e^2\dot{\beta}^2\gamma^6}{6\pi\varepsilon_0 c}dt' \tag{33}$$

At time $t \gg t' + dt'$ after particle 2 propagates through the entire EM field of particle 1, the total energy is

$$\mathcal{E}_3 = -\frac{4}{3}m_e c^2\gamma^3\beta d\beta + m_e c^2\gamma\left(1+\frac{\beta^2}{3}\right) + d\mathcal{E} + d\epsilon \tag{34}$$

$$d\mathcal{E} = d\left(m_e c^2\gamma\left(1+\frac{\beta^2}{3}\right)\right) = \frac{1}{3}m_e c^2\gamma^3(5-\beta^2)\beta d\beta \tag{35}$$

$d\mathcal{E}$ is the energy change of particle 1 and $d\epsilon$ is the left-over energy of particle 2. The potential energy between particle 1 and particle 2 is fully released at this time. From the law of energy conservation

$$\mathcal{E}_1 \equiv \mathcal{E}_2 \equiv \mathcal{E}_3 \tag{36}$$

So

$$\begin{aligned} m_e c^2\gamma\left(1+\frac{\beta^2}{3}\right) &\equiv -\frac{4}{3}m_e c^2\gamma^3\beta d\beta + \mho + m_e c^2\gamma\left(1+\frac{\beta^2}{3}\right) + \frac{e^2\dot{\beta}^2\gamma^6}{6\pi\varepsilon_0 c}dt' \\ &\equiv -\frac{4}{3}m_e c^2\gamma^3\beta d\beta + m_e c^2\gamma\left(1+\frac{\beta^2}{3}\right) + \frac{1}{3}m_e c^2\gamma^3(5-\beta^2)\beta d\beta \\ &+ d\epsilon \end{aligned} \tag{37}$$

From Equation (37), it is easy to derive

$$\begin{cases} \mho = \dfrac{4}{3}m_e c^2\gamma^3\beta d\beta - \dfrac{e^2\gamma^6}{6\pi\varepsilon_0 c}\dot{\beta} d\beta \\ d\epsilon = -\dfrac{1}{3}m_e c^2\gamma\beta d\beta \end{cases} \tag{38}$$

So, the radiation power is

$$\frac{d\epsilon}{dt'} = -\frac{1}{3}m_e c^2\gamma\beta\dot{\beta} \tag{39}$$

From Equation (39), in the case of linear acceleration $\beta\dot{\beta} > 0$, $d\epsilon/dt' < 0$. This means that particle 2 or the radiation photons cannot escape. They are absorbed back by particle 1 when propagating through the EM field of particle 1.

However, in the case of linear deceleration $\beta\dot{\beta} < 0$, $d\epsilon/dt' > 0$. So, the positive radiation power is measurable. Furthermore, the radiation power is much stronger than the conventional EM theory predicts because in any normal circumstances

$$\frac{1}{3}m_e c^2\gamma\left|\beta\dot{\beta}\right| \gg \frac{e^2\dot{\beta}^2\gamma^6}{6\pi\varepsilon_0 c} \tag{40}$$

Therefore, in the X-ray tube the Bremsstrahlung radiation is so strong that the emitted X-ray photons have not only been observed but also widely used in many medical applications.

In QED, the concept of virtual photon is created to describe the EM force [9][10]. In the QED description, the electron changes its state by randomly emitting and absorbing virtual photons. In the discussion following Equation (29), it is demonstrated that particle 2 fulfills the role of virtual photon, it propagates at the speed of light and gives the momentum change needed by particle 1. In the case of linear acceleration, it is absorbed back by particle 1 when it fully propagates through the field of particle 1. In the case of linear deceleration, it becomes the measurable EM radiation. When particle 2 is just created, according to Equation (25) and (26), it appears to be a normal particle with mass in Equation (27) even it propagates at the speed of light. In the case of linear deceleration, even it becomes EM radiation, it is so dispersed after it spreads out that it could hardly be recognized as the particle when it was created.

To test the new theory presented, the following experiment is provided in Figure 3.

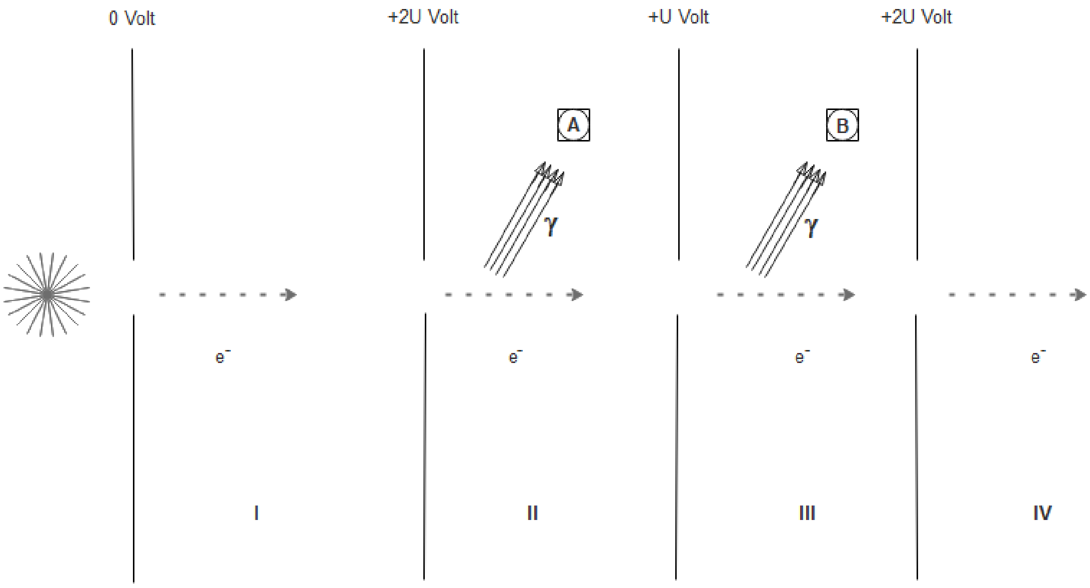

Fig. 3. Measuring EM radiation in acceleration and deceleration

Figure 3 is a modified X-ray tube [21]. In Figure 3, electrons are emitted by a hot source; they are accelerated in section I; decelerated in section II; re-accelerated in section III.

According to the classical EM radiation theory (i.e., Larmar formula) [14], an electron will radiate the same amount of energy in section II and section III. If we put two photon detectors A and B, they will detect the same amount of radiation energy.

However, according to Equation (39), there is hardly any radiation in section III, but much more radiation in section II. So, detector A will receive far more radiation energy than detector B.

We must say that the radiation measured by detector B is not completely zero. The reason is that even the tube is vacuumed, the temperature inside the tube is not at absolute zero degree. So, there exist the blackbody radiation photons inside the tube [22]. In section III, when the electron is accelerated, it will bump into these photons and transfer energy to them in a process called inverse Compton scattering [23] [24]. The detector B will receive these scattered photons. Nevertheless, the energy received by detector B will be much less than that by detector A.

People may argue that similar experiments have been done explicitly and implicitly in the past over and over, so the verdict is already in. In the appendix, we will describe the difference between this experiment and the past experiments and argue that the conclusion is yet to be determined.

Now let us draw some analogies between the results obtained in this section and QED. In QED, an electron randomly emits and absorbs photons. In Figure 4, we draw 3 Feynman diagrams.

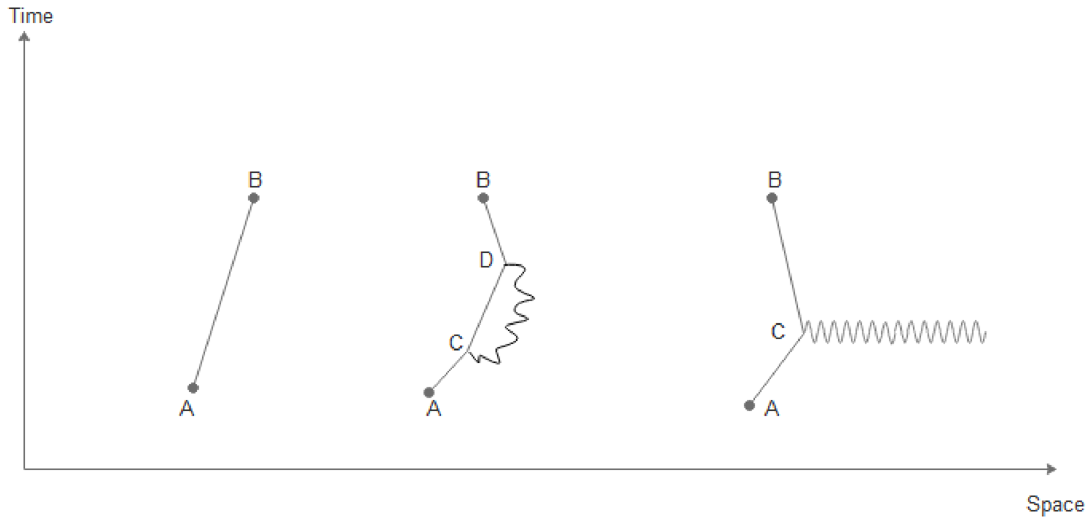

Fig. 4. Feynman diagrams

On the left diagram, the electron takes the direct path to go from point A to point B. In the middle diagram, the electron emits a virtual photon at point C, then absorbs the virtual photon at point D. So, its path from point A to point B is indirect. In QED, the vertex points C and D are instant points. This means that the electron instantly emits/absorbs the photon. The position of point C and D is random. This means that the distance between point C and D could be zero. In such case, the QED calculation on the electron's physical property such as self-energy will yield an infinity [9]. The infinite value problem is solved by Renormalization [9].

If we use the middle diagram to describe the electron's linear acceleration by treating the electron as particle 1 which is described by electron's Coulomb-like field and the virtual photon as particle 2 which is described by electron's radiating field, then we can gain some insight on how the virtual photon is emitted and absorbed.

At the vertex point C, the virtual photon is emitted during the time interval $[t', t' + dt']$ due to a jerk, acceleration by the Lorentz force from the external EM field. So, point C is not a mathematical abstract point but has a range. The time interval between point C and D corresponds to the time interval of $\left[t' + {}^{\frac{r_0}{\gamma}}\!/_{c}, t' + {}^{\frac{r_\infty}{\gamma}}\!/_{c}\right]$. Here r_0 is given by Equation (5) and r_∞ can be approximated by the electron's Compton radius in Equation (41) [14] due to the following reasons,

$$r_c = \frac{h}{m_e c} \tag{41}$$

1) Because $r_c > 100 r_0$, the energy of the electron's EM field inside r_c constitutes more than 99% of electron's mass.

2) According to the Compton scattering process, outside r_c the electron's reach is negligible.

From the earlier discussion, we know that during this time interval the virtual photon keeps interacting with the electron and provides the momentum change needed by the electron. So, the virtual photon is not instantly absorbed at point D but during the entire time interval between C and D. With this analogy, the vertex points C and D are not mathematical abstract points, the distance between C and D cannot be zero. So, we should never have the infinite value problem in the first place.

In technical terms, a virtual photon in QFT (or QED) is a planar wave. It is represented by the photon propagator in the 4-D frequency space. This photon propagator must be integrated in the entire frequency space. Unfortunately, this integral is an infinity, the so-called divergence problem. To overcome the divergence problem, an arbitrary cutoff frequency must be imposed in the integration boundary. This arbitrary cutoff frequency is cancelled out by the experimental results. This procedure is technically called Renormalization [10].

The infinite value problem not only exists in modern QED theory but also in classical EM theory. In classical EM theory, the electron is postulated as point of charge. The electron's EM field is infinite at the point of its location. So, the infinite value problem is not due to any physical meaning but due to the poor postulations: point of charge, instant emission and absorption of photon etc.

In our calculation, the virtual photon (particle 2) is a spherical wave instead of a planar wave; the virtual photon propagates from $\frac{r_0}{\gamma}$ to $\frac{r_\infty}{\gamma}$. Therefore, the virtual photon must be integrated in the frequency space. The cutoff frequency corresponds to the inner boundary $\frac{r_0}{\gamma}$ which is not at all an arbitrary number. So, we do not need Renormalization to fix anything. The cutoff frequency is expressed as

$$\Lambda = \frac{\gamma c}{r_0} = \frac{8\pi\varepsilon_0 m_e c^3 \gamma}{e^2} = \frac{2 m_e c^2 \gamma}{\hbar\alpha} \tag{42}$$

In Equation (42), $\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c}$ is the fine structure constant [14].

The right diagram in Figure 4 corresponds to the electron's linear deceleration process. In this case, the electron emits the photon, or photons if you think of particle 2 to be a set of photons. As described earlier, these photons are emitted during the time interval of $[t', t' + dt']$ and keep interacting with the electron during the time interval of $\left[t' + \frac{r_0}{\gamma}\Big/c, t' + \frac{r_\infty}{\gamma}\Big/c\right]$ by mopping up extra energy from the EM field of particle 1.

6. EM radiation by electron in circular motion

In Figure 2, assume that the electron is in circular motion. At time t' its velocity is $\boldsymbol{\beta} = \beta\hat{\boldsymbol{z}}$ and its acceleration is $\dot{\boldsymbol{\beta}} = \dot{\beta}\hat{\boldsymbol{x}}$. At time $t > t'$, the energy of particle 2 is in a spherical shell between two spherical surfaces Σ^+ and Σ^-,

$$d\mathcal{E} = \iiint \rho\, dV = \int_{R^-}^{R^+} R^2\, dR \int_0^{\pi} \sin\theta'\, d\theta' \int_0^{2\pi} d\varphi'\, \varepsilon_0 |\boldsymbol{E}_2|^2$$

$$= \int_{R^-}^{R^+} R^2\, dR \int_0^{\pi} \sin\theta'\, d\theta' \int_0^{2\pi} d\varphi'\, \varepsilon_0 \left| \frac{e}{4\pi\varepsilon_0 c} \frac{\mathbf{n}\text{x}[(\mathbf{n}-\boldsymbol{\beta})\text{x}\dot{\boldsymbol{\beta}}]}{(1-\mathbf{n}\cdot\boldsymbol{\beta})^3 R} \right|^2$$

$$= \int_{R^-}^{R^+} R^2\, dR \int_0^{\pi} \sin\theta'\, d\theta' \int_0^{2\pi} d\varphi'\, \varepsilon_0 \left(\frac{e}{4\pi\varepsilon_0 c}\right)^2 \left| \frac{\mathbf{n}\text{x}[(\mathbf{n}-\beta\hat{\boldsymbol{z}})\text{x}\dot{\beta}\hat{\boldsymbol{x}}]}{(1-\beta\cos\theta')^3 R} \right|^2$$

$$= \int_{R^-}^{R^+} R^2\, dR \int_0^{\pi} \sin\theta'\, d\theta' \int_0^{2\pi} d\varphi'\, \varepsilon_0 \left(\frac{e}{4\pi\varepsilon_0 c}\right)^2 \frac{\dot{\beta}^2[(1-\beta\cos\theta')^2 - (1-\beta^2)(\sin\theta'\cos\varphi')^2]}{(1-\beta\cos\theta')^6 R^2}$$

$$= \frac{e^2\dot{\beta}^2}{16\pi^2\varepsilon_0 c^2} \int_0^{2\pi} d\varphi' \int_0^{\pi} \frac{dR \sin\theta'\, [(1-\beta\cos\theta')^2 - (1-\beta^2)(\sin\theta'\cos\varphi')^2]}{(1-\beta\cos\theta')^6} d\theta'$$

Use $dR = c(1-\beta\, cos\,\theta')dt'$

$$d\mathcal{E} = \frac{e^2\dot{\beta}^2 dt'}{16\pi^2\varepsilon_0 c} \int_0^{2\pi} d\varphi' \int_0^{\pi} \frac{\sin\theta'\, [(1-\beta\cos\theta')^2 - (1-\beta^2)(\sin\theta'\cos\varphi')^2]}{(1-\beta\cos\theta')^5} d\theta'$$

$$= \frac{e^2\dot{\beta}^2 dt'}{8\pi\varepsilon_0 c} \left\{ \int_0^{\pi} \frac{\sin\theta'}{(1-\beta\, cos\,\theta')^3} d\theta' - \frac{(1-\beta^2)}{2} \int_0^{\pi} \frac{(\sin\theta')^3}{(1-\beta\, cos\,\theta')^5} d\theta' \right\}$$

$$= \frac{e^2\dot{\beta}^2 dt'}{8\pi\varepsilon_0 c} \left\{ \int_{-1}^{1} \frac{1}{(1-\beta x)^3} dx - \frac{1}{2\gamma^2} \int_{-1}^{1} \frac{(1-x^2)}{(1-\beta x)^5} dx \right\}$$

$$= \frac{e^2\dot{\beta}^2 dt'}{8\pi\varepsilon_0 c} \left\{ 2\gamma^4 - \frac{1}{2\gamma^2}\frac{4}{3}\gamma^6 \right\} = \frac{e^2\dot{\beta}^2 dt'}{8\pi\varepsilon_0 c}\frac{4}{3}\gamma^4$$

So

$$d\mathcal{E} = \frac{e^2\dot{\beta}^2\gamma^4}{6\pi\varepsilon_0 c} dt' \tag{43}$$

The momentum of particle 2 is

$$\boldsymbol{P} = \iiint \boldsymbol{Q}\, dV = \int_{R^-}^{R^+} R^2\, dR \int_0^{\pi} \sin\theta'\, d\theta' \int_0^{2\pi} d\varphi'\, \varepsilon_0 \boldsymbol{E}_2 \text{x} \boldsymbol{B}_2$$

$$= \int_{R^-}^{R^+} R^2\, dR \int_0^{\pi} \sin\theta'\, d\theta' \int_0^{2\pi} d\varphi'\, \varepsilon_0 \boldsymbol{E}_2 \text{x} \left(\frac{1}{c}\mathbf{n}\text{x}\boldsymbol{E}_2\right)$$

$$= \int_{R^-}^{R^+} R^2\, dR \int_0^{\pi} \sin\theta'\, d\theta' \int_0^{2\pi} d\varphi' \frac{\varepsilon_0}{c} |\boldsymbol{E}_2|^2 \mathbf{n}$$

$$= \int_{R^-}^{R^+} R^2\, dR \int_0^{\pi} \sin\theta'\, d\theta' \int_0^{2\pi} d\varphi' \frac{\varepsilon_0}{c} \left(\frac{e}{4\pi\varepsilon_0 c}\right)^2 \frac{\dot{\beta}^2[(1-\beta\cos\theta')^2 - (1-\beta^2)(\sin\theta'\cos\varphi')^2]}{(1-\beta\cos\theta')^6 R^2} (\cos\theta'\,\hat{\boldsymbol{z}}$$
$$+ \sin\theta'\cos\varphi'\,\hat{\boldsymbol{x}} + \sin\theta'\sin\varphi'\,\hat{\boldsymbol{y}})$$

$$= \frac{\mu_0 e^2\dot{\beta}^2\hat{\boldsymbol{z}}}{16\pi^2 c} \int_0^{2\pi} d\varphi' \int_0^{\pi} \frac{dR \sin\theta'\, [(1-\beta\cos\theta')^2 - (1-\beta^2)(\sin\theta'\cos\varphi')^2]\cos\theta'}{(1-\beta\cos\theta')^6} d\theta'$$

Use $dR = c(1-\beta\, cos\,\theta')dt'$

$$dP = \frac{\mu_0 e^2 \dot{\beta}^2 dt' \hat{\mathbf{z}}}{8\pi}\left\{\int_0^{\pi} \frac{\sin\theta' \cos\theta'}{(1-\beta\, cos\,\theta')^3} d\theta' - \frac{(1-\beta^2)}{2}\int_0^{\pi} \frac{(\sin\theta')^3 \cos\theta'}{(1-\beta\, cos\,\theta')^5} d\theta'\right\}$$

$$= \frac{\mu_0 e^2 \dot{\beta}^2 dt' \hat{\mathbf{z}}}{8\pi}\left\{\int_{-1}^{1} \frac{x}{(1-\beta x)^3} dx - \frac{1}{2\gamma^2}\int_{-1}^{1} \frac{(1-x^2)x}{(1-\beta x)^5} dx\right\}$$

$$= \frac{\mu_0 e^2 \dot{\beta}^2 dt' \hat{\mathbf{z}}}{8\pi}\left\{\int_{-1}^{1} \frac{x}{(1-\beta x)^3} dx - \frac{1}{2\gamma^2}\int_{-1}^{1} \frac{x}{(1-\beta x)^5} dx\right.$$

$$\left. + \frac{1}{2\gamma^2}\int_{-1}^{1} \frac{x^3}{(1-\beta x)^5} dx\right\}$$

$$= \frac{\mu_0 e^2 \dot{\beta}^2 dt' \hat{\mathbf{z}}}{8\pi}\left\{2\gamma^4\beta - \frac{\beta(\beta^2+5)}{3}\gamma^6 + \beta(\beta^2+1)\gamma^6\right\} = \frac{\mu_0 e^2 \dot{\beta}^2 dt' \hat{\mathbf{z}}}{8\pi}\frac{4}{3}\gamma^4\beta$$

So

$$dP = \frac{\mu_0 e^2 \boldsymbol{\beta} \dot{\beta}^2 \gamma^4}{6\pi} dt' \tag{44}$$

From Equation (43) and (44), even particle 2 can be viewed as a set of photons, collectively it moves as a single particle with rest mass

$$m_2 = \frac{\mu_0 e^2 \dot{\beta}^2 \gamma^3}{6\pi c} dt' \tag{45}$$

So, Equation (43) and (44) would take the form of $\mathcal{E}_2 = m_2\gamma c^2$ and $\boldsymbol{P}_2 = m_2\gamma\boldsymbol{v}$ respectively. The Lorentz force exerted on particle 1 by particle 2 is

$$\mathbf{F} = \varepsilon_0 \oiint_{\Sigma^-}^{\Sigma^+} (\mathbf{E}_2 + \boldsymbol{V}\mathrm{x}\mathbf{B}_2)\mathbf{E}_1 \cdot d\boldsymbol{S}$$

The integral on the inner boundary surface is

$$\boldsymbol{F}_{\Sigma^-} = \varepsilon_0 \oiint (\mathbf{E}_2 + \boldsymbol{V}\mathrm{x}\mathbf{B}_2)\mathbf{E}_1 \cdot d\boldsymbol{S}$$

From Equation (22), (23) and Figure 2(b)

$$\boldsymbol{E}_2 = \frac{e}{4\pi\varepsilon_0 c}\frac{\mathbf{n}\mathrm{x}[(\mathbf{n}-\boldsymbol{\beta})\mathrm{x}\dot{\boldsymbol{\beta}}]}{(1-\mathbf{n}\cdot\boldsymbol{\beta})^3 R} = \frac{e}{4\pi\varepsilon_0 c}\frac{(\mathbf{n}\cdot\dot{\boldsymbol{\beta}})(\mathbf{n}-\boldsymbol{\beta}) - (1-\mathbf{n}\cdot\boldsymbol{\beta})\dot{\boldsymbol{\beta}}}{(1-\mathbf{n}\cdot\boldsymbol{\beta})^3 R}$$

$$= \frac{e}{4\pi\varepsilon_0 c}\frac{(\dot{\beta}\sin\theta'\cos\varphi')[(\cos\theta'-\beta)\hat{\boldsymbol{z}} + \sin\theta'\cos\varphi'\,\hat{\boldsymbol{x}} + \sin\theta'\sin\varphi'\,\hat{\boldsymbol{y}}] - (1-\beta\cos\theta')\dot{\beta}\hat{\boldsymbol{x}}}{(1-\beta\cos\theta')^3 R}$$

$$= \frac{e\dot{\beta}}{4\pi\varepsilon_0 c}\frac{(\sin\theta'\cos\varphi')(\cos\theta'-\beta)\hat{\boldsymbol{z}} + (\sin\theta'\cos\varphi')^2\hat{\boldsymbol{x}} + (\sin\theta')^2\sin\varphi'\cos\varphi'\,\hat{\boldsymbol{y}} - (1-\beta\cos\theta')\hat{\boldsymbol{x}}}{(1-\beta\cos\theta')^3 R}$$

$$\boldsymbol{V}\mathrm{x}\mathbf{B}_2 = \boldsymbol{V}\mathrm{x}\left(\frac{1}{c}\mathbf{n}\mathrm{x}\boldsymbol{E}_2\right) = (\boldsymbol{\beta}\cdot\boldsymbol{E}_2)\mathbf{n} - (\boldsymbol{\beta}\cdot\mathbf{n})\boldsymbol{E}_2$$

$$= \frac{e\dot{\beta}}{4\pi\varepsilon_0 c}\frac{\beta(\sin\theta'\cos\varphi')(\cos\theta'-\beta)(\cos\theta'\,\hat{\boldsymbol{z}} + \sin\theta'\cos\varphi'\,\hat{\boldsymbol{x}} + \sin\theta'\sin\varphi'\,\hat{\boldsymbol{y}})}{(1-\beta\cos\theta')^3 R}$$

$$- \beta\cos\theta'\frac{e\dot{\beta}}{4\pi\varepsilon_0 c}\frac{(\sin\theta'\cos\varphi')(\cos\theta'-\beta)\hat{\boldsymbol{z}} + (\sin\theta'\cos\varphi')^2\hat{\boldsymbol{x}} + (\sin\theta')^2\sin\varphi'\cos\varphi'\,\hat{\boldsymbol{y}} - (1-\beta\cos\theta')\hat{\boldsymbol{x}}}{(1-\beta\cos\theta')^3 R}$$

$$= \frac{e\dot{\beta}\beta}{4\pi\varepsilon_0 c}\frac{[\cos\theta'(1-\beta\cos\theta') - \beta(\sin\theta'\cos\varphi')^2]\hat{\boldsymbol{x}} - \beta(\sin\theta')^2\sin\varphi'\cos\varphi'\,\hat{\boldsymbol{y}}}{(1-\beta\cos\theta')^3 R}$$

So

$\boldsymbol{E}_2 + \boldsymbol{V}\mathrm{x}\mathbf{B}_2$

$$= \frac{e\dot{\beta}}{4\pi\varepsilon_0 c}\frac{(\sin\theta'\cos\varphi')(\cos\theta'-\beta)\hat{\boldsymbol{z}} + (\sin\theta'\cos\varphi')^2\hat{\boldsymbol{x}} + (\sin\theta')^2\sin\varphi'\cos\varphi'\,\hat{\boldsymbol{y}} - (1-\beta\cos\theta')\hat{\boldsymbol{x}}}{(1-\beta\cos\theta')^3 R}$$

$$+ \frac{e\dot{\beta}\beta}{4\pi\varepsilon_0 c}\frac{[\cos\theta'(1-\beta\cos\theta') - \beta(\sin\theta'\cos\varphi')^2]\hat{\boldsymbol{x}} - \beta(\sin\theta')^2\sin\varphi'\cos\varphi'\,\hat{\boldsymbol{y}}}{(1-\beta\cos\theta')^3 R}$$

$$= \frac{e\dot{\beta}}{4\pi\varepsilon_0 c}\left\{\frac{[(1-\beta^2)(\sin\theta'\cos\varphi')^2 - (1-\beta\cos\theta')^2]}{(1-\beta\cos\theta')^3 R}\hat{\boldsymbol{x}} + \frac{(1-\beta^2)(\sin\theta')^2\sin\varphi'\cos\varphi'}{(1-\beta\cos\theta')^3 R}\hat{\boldsymbol{y}}\right.$$

$$\left. + \frac{(\sin\theta'\cos\varphi')(\cos\theta'-\beta)}{(1-\beta\cos\theta')^3 R}\hat{\boldsymbol{z}}\right\}$$

$$\mathbf{E}_1 \cdot d\boldsymbol{S} = \frac{e}{4\pi\varepsilon_0}\frac{(1-\beta^2)(\mathbf{n}-\beta\hat{\mathbf{z}})}{(1-\beta\cos\theta')^3 R^2}.(-\mathbf{n})R^2\sin\theta'\,d\theta'd\varphi' = -\frac{e}{4\pi\varepsilon_0\gamma^2}\frac{\sin\theta'\,d\theta'd\varphi'}{(1-\beta\cos\theta')^2}$$

So

$$\boldsymbol{F}_{\Sigma^-} = \varepsilon_0 \oiint (\mathbf{E}_2 + \boldsymbol{V}\mathrm{x}\mathbf{B}_2)\mathbf{E}_1\cdot d\boldsymbol{S} = \varepsilon_0\left(\frac{e\dot{\beta}}{4\pi\varepsilon_0 c}\right)\left(-\frac{e}{4\pi\varepsilon_0\gamma^2}\right)\int_0^{2\pi} d\varphi' \int_0^{\pi}\sin\theta'\,d\theta'$$

$$\frac{\{\sin\theta'(\cos\theta'-\beta)\cos\varphi'\,\hat{\boldsymbol{z}} + (1-\beta^2)(\sin\theta'\cos\varphi')^2\hat{\boldsymbol{x}} - (1-\beta\cos\theta')^2\hat{\boldsymbol{x}} + (1-\beta^2)(\sin\theta')^2\sin\varphi'\cos\varphi'\,\hat{\boldsymbol{y}}\}}{(1-\beta\cos\theta')^5 R}$$

$$= -\frac{e^2\dot{\beta}\hat{\boldsymbol{x}}}{16\pi^2\varepsilon_0 c\gamma^2 R}\left(\frac{1}{\gamma^2}\int_0^{\pi}\frac{(\sin\theta')^3 d\theta'}{(1-\beta\cos\theta')^5}\int_0^{2\pi}(\cos\varphi')^2 d\varphi' - \int_0^{\pi}\frac{\sin\theta'\,d\theta'}{(1-\beta\cos\theta')^3}\int_0^{2\pi}d\varphi'\right) =$$

$$= \frac{e^2\dot{\boldsymbol{\beta}}}{8\pi\varepsilon_0 c\gamma^2 R}\left(\int_0^{\pi}\frac{\sin\theta'\,d\theta'}{(1-\beta\cos\theta')^3} - \frac{1}{2\gamma^2}\int_0^{\pi}\frac{(\sin\theta')^3 d\theta'}{(1-\beta\cos\theta')^5}\right)$$

Similarly

$$\boldsymbol{F}_{\Sigma^+} = -\frac{e^2\dot{\boldsymbol{\beta}}}{8\pi\varepsilon_0 c\gamma^2(R+dR)}\left(\int_0^{\pi}\frac{\sin\theta'\,d\theta'}{(1-\beta\cos\theta')^3} - \frac{1}{2\gamma^2}\int_0^{\pi}\frac{(\sin\theta')^3 d\theta'}{(1-\beta\cos\theta')^5}\right)$$

So

$$\boldsymbol{F} = \boldsymbol{F}_{\Sigma^-} + \boldsymbol{F}_{\Sigma^+} = \frac{e^2\dot{\boldsymbol{\beta}}}{8\pi\varepsilon_0 c\gamma^2}\frac{dR}{R^2}\left(\int_0^{\pi}\frac{\sin\theta'\,d\theta'}{(1-\beta\cos\theta')^3} - \frac{1}{2\gamma^2}\int_0^{\pi}\frac{(\sin\theta')^3 d\theta'}{(1-\beta\cos\theta')^5}\right)$$

$$= \frac{e^2\dot{\boldsymbol{\beta}}}{8\pi\varepsilon_0 c\gamma^2}\frac{c(1-\beta\cos\theta')dt'}{R^2}\left(\int_0^{\pi}\frac{\sin\theta'\,d\theta'}{(1-\beta\cos\theta')^3} - \frac{1}{2\gamma^2}\int_0^{\pi}\frac{(\sin\theta')^3 d\theta'}{(1-\beta\cos\theta')^5}\right)$$

$$= \frac{e^2\dot{\boldsymbol{\beta}}dt'}{8\pi\varepsilon_0\gamma^2 R^2}\left(\int_0^{\pi}\frac{\sin\theta'\,d\theta'}{(1-\beta\cos\theta')^2} - \frac{1}{2\gamma^2}\int_0^{\pi}\frac{(\sin\theta')^3 d\theta'}{(1-\beta\cos\theta')^4}\right)$$

$$= \frac{e^2\dot{\boldsymbol{\beta}}dt'}{8\pi\varepsilon_0\gamma^2 R^2}\left(2\gamma^2 - \frac{1}{2\gamma^2}\frac{4}{3}\gamma^4\right) = \frac{e^2\dot{\boldsymbol{\beta}}}{8\pi\varepsilon_0\gamma^2}\frac{dt'}{R^2}\frac{4}{3}\gamma^2 = \frac{e^2\dot{\boldsymbol{\beta}}}{6\pi\varepsilon_0}\frac{dt'}{R^2}$$

This is the momentum transfer rate from particle 2 to particle 1 at time $t > t'$. The total momentum gained by particle 1 when the ripple, particle 2, propagates through the entire field of particle 1 is

$$d\boldsymbol{P} = \int_{r_0/\gamma}^{\infty}\boldsymbol{F}\frac{dR}{c} = \int_{r_0/\gamma}^{\infty}\frac{e^2\dot{\boldsymbol{\beta}}}{6\pi\varepsilon_0}\frac{dt'}{R^2}\frac{dR}{c} = \frac{e^2\dot{\boldsymbol{\beta}}dt'}{6\pi\varepsilon_0 c}\frac{1}{r_0/\gamma} = \frac{e^2\gamma\dot{\boldsymbol{\beta}}}{6\pi\varepsilon_0 c r_0}dt'$$

Use Equation (5), above equation is reduced to

$$\frac{d\boldsymbol{P}}{dt'} = \frac{4}{3}m_e c\gamma\dot{\boldsymbol{\beta}} \tag{46}$$

On the other hand, from Equation (16)

$$\frac{d\boldsymbol{P}}{dt'} = \frac{d(\frac{4}{3}m_e\gamma\boldsymbol{v})}{dt'} = \frac{4}{3}m_e c\left(\frac{d\gamma}{dt'}\boldsymbol{\beta} + \gamma\dot{\boldsymbol{\beta}}\right) = \frac{4}{3}m_e c[\gamma^3(\boldsymbol{\beta}\cdot\dot{\boldsymbol{\beta}})\boldsymbol{\beta} + \gamma\dot{\boldsymbol{\beta}}] = \frac{4}{3}m_e c\gamma\dot{\boldsymbol{\beta}}$$

This perfectly matches Equation (46). Once again, this exercise proves that when electron is accelerated during the time interval $[t', t' + dt']$, the momentum of particle 1 is not changed right away during the same time interval. The momentum difference is only gained through the Lorentz force exerted by particle 2 when it propagates through the EM field of particle 1. The old assumption in classical EM theory that there is no interaction between particle 1 and particle 2 is not true.

So, what is the fate of particle 2? To answer this question, let us use the law of energy conservation. Assume that an electron is perpendicularly accelerated in a constant magnetic field $\boldsymbol{B} = -B\hat{\boldsymbol{y}}$ during the time interval $[t', t' + dt']$. At time $t = t'$ according to Equation (14), the energy of the electron is

$$\mathcal{E}_1 = m_e c^2\gamma\left(1 + \frac{\beta^2}{3}\right) \tag{47}$$

At time $t = t' + dt'$ the electron breaks into two: particle 1 and particle 2. The total energy is

$$\mathcal{E}_2 = U + \mho + m_e c^2\gamma\left(1 + \frac{\beta^2}{3}\right) + \frac{e^2\dot{\beta}^2\gamma^4}{6\pi\varepsilon_0 c}dt' \tag{48}$$

In Equation (48) the first term U is the potential energy of particle 1 in the external magnetic field $\boldsymbol{B}$.

$$U = -\boldsymbol{F}\cdot d\boldsymbol{l} = -e\boldsymbol{V}\mathbf{x}\boldsymbol{B}\cdot\boldsymbol{V}dt' = 0 \tag{49}$$

The second term $\mho$ is the potential energy between particle 1 and particle 2 because there is a Lorentz force between them. The third term is the energy of particle 1 because its EM field is mostly unchanged at this time. The fourth term is the energy of particle 2 according to Equation (43). Combine Equation (48) and (49)

$$\mathcal{E}_2 = \mho + m_e c^2\gamma\left(1 + \frac{\beta^2}{3}\right) + \frac{e^2\dot{\beta}^2\gamma^4}{6\pi\varepsilon_0 c}dt' \tag{50}$$

At time $t \gg t' + dt'$ after particle 2 propagates through the entire EM field of particle 1, the total energy is

$$\mathcal{E}_3 = m_e c^2\gamma\left(1 + \frac{\beta^2}{3}\right) + d\mathcal{E} + d\epsilon \tag{51}$$

$$d\mathcal{E} = d\left(m_e c^2\gamma\left(1 + \frac{\beta^2}{3}\right)\right) = m_e c^2\left(1 + \frac{\beta^2}{3}\right)d\gamma + m_e c^2\gamma d\left(1 + \frac{\beta^2}{3}\right) = m_e c^2\left(1 + \frac{\beta^2}{3}\right)\gamma^3\boldsymbol{\beta}\cdot\dot{\boldsymbol{\beta}}dt' + \frac{2}{3}m_e c^2\gamma\boldsymbol{\beta}\cdot\dot{\boldsymbol{\beta}}dt' = 0 \tag{52}$$

$d\mathcal{E}$ is the energy change of particle 1 and $d\epsilon$ is the left-over energy of particle 2. The potential energy between particle 1 and particle 2 is fully released at this time. From the law of energy conservation

$$\mathcal{E}_1 \equiv \mathcal{E}_2 \equiv \mathcal{E}_3 \tag{53}$$

So

$$m_e c^2\gamma\left(1 + \frac{\beta^2}{3}\right) \equiv \mho + m_e c^2\gamma\left(1 + \frac{\beta^2}{3}\right) + \frac{e^2\dot{\beta}^2\gamma^4}{6\pi\varepsilon_0 c}dt' \equiv m_e c^2\gamma\left(1 + \frac{\beta^2}{3}\right) + d\epsilon \tag{54}$$

From Equation (54), it is easy to derive

$$\begin{cases} \mho = -\frac{e^2\dot{\beta}^2\gamma^4}{6\pi\varepsilon_0 c}dt' \\ d\epsilon = 0 \end{cases} \quad (55)$$

So, the radiation power is

$$\frac{d\epsilon}{dt'} = 0 \quad (56)$$

This is quite surprising because we do observe synchrotron radiation in the circular particle accelerator. So, what is the real source of those radiations?

At the end of section 5, we mention that in an X-ray tube (or a particle accelerator chamber), even the space is vacuumed, it still contains a lot of photons from the blackbody radiation because the temperature in the space is not at absolute zero degree. These photons will collide with the accelerated electron to slow it down. In the meantime, the collided photons will gain energy through the inverse Compton scattering process [23] [24]. They will appear to be the EM radiation by the accelerated electron. If this assumption is true, then increasing the temperature of the particle accelerator chamber will increase the radiation power of the accelerated electron.

People have used the technique of injecting photons to the particle accelerator chamber to increase the radiation power of synchrotron radiation [25]. This technique simply increases the chance of collision between photons and the accelerated electrons. Increasing the temperature of the particle accelerator chamber will achieve the same effect. According to the blackbody radiation, the photon density in the space is [22] [26],

$$n(T) = \int_0^\infty \frac{8\pi}{c^3}\frac{\nu^2}{e^{\frac{h\nu}{kT}} - 1}d\nu = \frac{8\pi}{c^3}\left(\frac{kT}{h}\right)^3 \int_0^\infty \frac{x^2}{e^x - 1}dx \propto T^3 \quad (57)$$

So, the synchrotron radiation power will be proportional to the power of 3 of the temperature in the particle accelerator chamber if our reasoning is correct. This can be easily tested by experiments.

In the study of certain Astronomical Gamma-ray bursts, the radiation is mostly attributed to the synchrotron radiation. However, from the conventional synchrotron radiation power which is given by Equation (43), the radiation energy could not account for all the observed Gamma-ray intensity. To compensate for the deficit, some radiation is attributed to the inverse Compton scattering by the background light [27]. However, according to our discussion, both radiations are caused by the collision between the accelerated electrons and the background photons.

The collision between the accelerated electron and the background photons is like the friction force on the moving electron. This is like the high school experiment to demonstrate the Newton's first law. For example, let an object glide freely on a smooth table surface. If the surface is not smooth enough, the friction force will slow down the object's speed. If the surface becomes smoother, then there will be lesser friction force, the object will move further. If the surface is completely smooth (free of friction), then the object will maintain its speed. Now let us combine Equation (39) and (56),

$$\frac{d\epsilon}{dt'} = -\frac{1}{3}m_e c^2\gamma\boldsymbol{\beta}\cdot\dot{\boldsymbol{\beta}} \quad (58)$$

This is the general formula of the radiation power by an electron moving in a constant electric field or magnetic field. From Equation (58), an electron only emits EM radiation in deceleration, the so-called Bremsstrahlung radiation. All the other classical EM radiation types by an electron are mainly caused by friction or resistance from the collision with photons or other particles in its path.

A good example to show this is as follow. In a supper-conductor ring, the persistent electric current can circulate forever. The electrons in the persistent electric current move in the ring without losing energy to radiation and heat due to zero resistance in the ring [28]. On the other hand, in a light bulb circuit the resistance is so high that the electrons' energy in the electric current is quickly radiated away as light and heat.

Before Quantum Mechanics was developed, one big problem facing the classical EM theory had been that it could not explain the hydrogen atom. According to the classical EM theory, when an electron moves around a proton nucleus in a hydrogen atom, it will lose radiation energy according to Equation (43), thus spiral toward the nucleus.

According to Equation (58), if the electron's classical orbit is a closed loop, then the average radiation power in one period is zero, Equation (59).

$$\langle\frac{d\epsilon}{dt'}\rangle = -\frac{1}{T}\int_0^T \frac{1}{3}m_e c^2\gamma\boldsymbol{\beta}\cdot\dot{\boldsymbol{\beta}}dt = -\frac{1}{T}\int_0^T \frac{1}{6}m_e c^2\frac{d(\beta^2)}{\sqrt{(1-\beta^2)}} = \frac{m_e c^2}{3T}\left(\frac{1}{\gamma(T)}-\frac{1}{\gamma(0)}\right) = 0 \quad (59)$$

The electron emits radiation in deceleration and absorbs it back in acceleration, the net radiation power is zero. The radiation behaves as virtual photon which could not escape.

One may argue that the electron's classical orbit is an outdated concept because in Quantum Mechanics the electron can only be described in probability wave due to the Complementarity principle [29] [30]. However, in the recent single photon double slit experiment [31] one can detect which slit the photon passes through and observe the interference pattern at the same time by using weak measurement [32]. This implies that we can observe the particle and wave phenomena simultaneously. In our first paper [12], we explain the particle (electron or photon) as a local EM field and its wavefunction as non-local pseudo-EM wave which is formed by the particle's physical EM field and its non-physical image replicas. So, in principle we should be able to simultaneously see the particle's trajectory path and interference pattern by weak measurement [32].

To test the new theory, the following experiment is proposed in Figure 5.

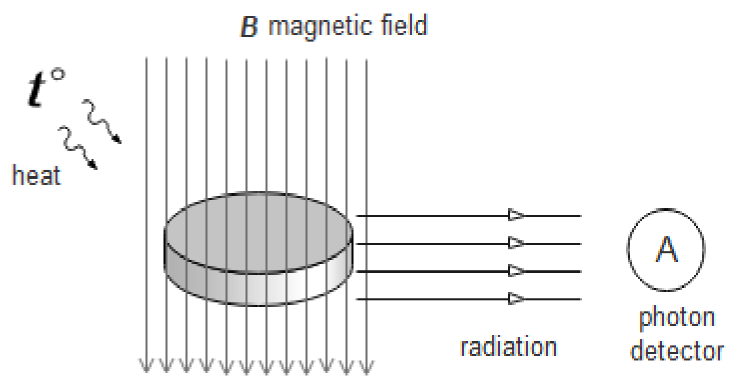

Fig. 5 Measuring EM radiation by electron in circular motion

Put a vacuum chamber in a constant magnetic field; an electron in the chamber is doing circular motion; a photon detector is measuring the electron's radiation.

According to the classical radiation theory, the electron will lose some energy in each turn by radiating some photons. So, the electron's circular path will spiral toward the center.

According to the new theory, if we cool down the ambient temperature inside the chamber, then there will be less friction between the circulating electron and the blackbody radiation photons, there will be less radiation. Then the electron will circulate more turns. If the new theory is correct, then the radiation detected by detector A will have a correlation with the temperature in the chamber. Otherwise, the classical radiation theory prevails.

7. Electron under the gravity force

The famous paradox on a charge particle free-falling under gravity is described as follow. When a charge particle and a neutron particle both free-fall in a gravity field, General Relativity states that both particles fall at the same acceleration and speed. But classical EM theory states that the accelerated charge particle emits radiation, thus falls slower than the neutron particle.

To explain this paradox, we need to notice the difference between gravity force and Lorentz force. Gravity force uniformly applies to any mass. If an electron is treated as its own EM field, then its mass is distributed in its EM field. When an electron free-falls under gravity, every part of its mass experiences the same acceleration, thus uniformly falls at the same speed. There is no ripple and dislocation in its EM field as oppose to Figure 1. Thus, there is no radiation. It is the ripple which may contribute to the EM radiation. On the other hand, in classical EM theory the Lorentz force acts on the charge of the electron, which is at the center of its EM field. The acceleration by Lorentz force is non-uniform. It starts from the center and propagates through the EM field of the electron like a ripple in Figure 1. This ripple travels at the speed of light and may carry away the radiation energy.

As a similar example, when a spring free-falls under gravity, its length will not change because every part of the spring experiences the same acceleration and uniformly falls at the same speed. But when someone drags one end of the spring and runs in acceleration, the spring will not only accelerate, but its length will also oscillate as well because the acceleration applied to the spring is non-uniform and travels from one end to the other.

The paragraphs in this section had been posted by the author on the popular wiki page “Paradox of radiation of charged particles in a gravitational field” [34] in 2017 before being removed by the page owner in 2018.

8. Conclusion

In physics matter exists in two forms: particle and field. By treating an electron as its own EM field and its location being at the center of mass of its field, it is explained why an electron will not radiate when free-falling in a gravity field. This solves the paradox on the radiation by a charge particle under gravity. So, classical EM theory and General Relativity are compatible.

By generalizing the Lorentz force on an electron in an external EM field as the field force between the electron’s EM field and the external EM field, the classical EM radiation theory is re-visited. It is demonstrated that when an electron accelerates in a constant EM field, the force exerted on the electron’s Coulomb-like field by the electron’s radiating field provides the exact momentum change needed by the electron’s Coulomb-like field. Thus, the radiating EM field of an accelerated electron plays the role of virtual photon in the modern QED theory.

The radiation power of an accelerated electron is re-calculated. Unlike classical EM theory predicts, a linearly accelerated electron emits no radiation if its collision with the blackbody radiation photons is ignored; a linearly decelerated electron emits much stronger radiation. The proposed experiment will expose the asymmetry of the radiation power by an electron in acceleration and deceleration.

The general radiation power by an electron in a constant EM field is given by Equation (58). From this formula, only the Bremsstrahlung radiation is a valid radiation type among all the classical EM radiation types by an electron. The other radiation types by an electron are mainly caused by resistance from the collision with background photons or other particles in the electron's moving path.

By using Equation (58), it is proved that the net radiation power by an electron in the hydrogen atom is zero if the electron's classical orbit is a closed loop. This invalidates one big reason for physicists to abandon classical physics a century ago when they first studied the hydrogen atom.

In QED, a virtual photon is represented by a planar wave and its propagator must be integrated over the entire frequency space. In the integral, an arbitrary cutoff frequency must be imposed to avoid the divergence [10]. We demonstrate that the virtual photon is represented by a spherical wave and an analytical form of the cutoff frequency is given by Equation (42).

Renormalization is the procedure to circumvent the infinite value problem which plagues both Quantum Mechanics and QFT [7][10]. There are two types of calculation which lead to an infinity: 1) integrate over the entire wavefunction or field quantum; 2) use the simplified mathematically abstract conditions such as point of charge, instant emission and absorption of photon etc. In our first paper, we indicate that the particle's wavefunction is formed by the particle's physical field and its non-physical image replicas [12]. When integrate over the entire wavefunction, the same physical value will be counted repeatedly. To avoid this problem, if the interference is negligible, then when we calculate a physical property of the particle, we can only count the contribution from the particle's physical field while discarding the contribution from its image replicas [12]. In this paper we demonstrate that when the simplified mathematically abstract condition is replaced by the physically realistic field condition and field process, the infinite value problem can be completely avoided. Without infinity, the artificial Renormalization is not needed.

By using the classical approach in the first paper [12] and the current paper, we find the missing links between classical physics and quantum physics. As consequence, the two mysteries: Quantum Collapse and Renormalization, which modern physics could not answer, are explained.

Appendix: Comparison with the past experiments

In the past, people have used the Rontgen tube to measure the X-ray radiation by the accelerated electrons [21]. The experiment setup is like section I in Figure 3 except that the positive electrode plate has no hole in the center. The accelerated electrons in the tube will hit the metal electrode plate to cause strong Bremsstrahlung radiation.

The problem of these experiments is that the photon detector cannot differentiate between the radiation photons by the accelerated electrons when moving in the tube and the radiation photons emitted when the electrons hit the metal electrode plate.

In the experiment in Figure 3, we drill a hole in the center of each metal electrode plate to avoid the direct collision between electrons and the metal electrode plates. The metal electrode plates also serve as screen shields which prevent the EM radiation from different sections to mix so that the photon detector A only measures radiation generated in section II and the photon detector B only measures radiation generated in section III.

In the past, people have used the particle accelerator to measure the EM radiation by accelerated electrons. However, in the particle accelerator the electron's speed is very close to the speed of light. So, the electron's radiation cone is very close to the electron's moving direction $\theta \approx {}^{1}/_{\gamma}$ [14]. To measure the electron's radiation, the electron's moving path must be redirected so that we can measure its radiation along the tangential to its trajectory. The radiation in this measurement is called synchrotron radiation. This setup totally defeats our purpose of measuring the radiation by a linearly accelerated or decelerated electron. Its path is not linear anymore.

Another difference in the modern particle accelerator such as linear accelerator or synchrotron accelerator is that the electrons are normally accelerated by the electric field of the RF wave in the RF cavity [33]. This time-varying electric field is very different from our experimental setup in Figure 3 where we use the constant static electric field to accelerate the electrons.

It is still an open debate on the subject of "If electron radiates when uniformly accelerated". There are two camps on this subject. One camp led by Feynman claims that there is no radiation [16]. Another camp (S. Parrott) claims that there is radiation [17] [18]. There is a whole book to cover this subject [19].

In Feynman's argument "Since we have inherited a prejudice that an accelerating charge should radiate, whereas we do not expect a charge lying in a gravitational field to radiate. This is, however, not due to a mistake in our statement of equivalence but to the fact that the rule of the power radiated by an accelerating charge (Larmar formula), has led us astray." [16]. According to the Lorentz-Dirac equation on a one-dimensional, non-relativistic moving electron, the radiation power is

$$\frac{d\mathcal{E}}{dt} = -\frac{e^2\beta\ddot{\beta}}{6\pi\varepsilon_0 c} = \frac{e^2\dot{\beta}^2}{6\pi\varepsilon_0 c} - \frac{e^2}{6\pi\varepsilon_0 c}\frac{d}{dt}\left(\beta\dot{\beta}\right) \tag{60}$$

According to Equation (60), if the electron is uniformly accelerated $\ddot{\beta} = 0$, such as when it is in a constant static electric field, then there is no radiation. However, if the electron is in cyclic or bounded motion, then the time average of the second term on the right-hand side of the equation will become zero, Equation (60) will reduce to the conventional Larmar equation [16].

So, there will be three possible outcomes from the experiment in Figure 3. If Feynman's argument is correct, then both detector A and detector B will measure zero radiation. If the conventional Larmar formula is correct, then both detector A and detector B will measure the same amount of non-zero radiation. If our theory is correct, then detector A will measure much stronger radiation than detector B. In any case, this experiment is worth trying.

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