Quantum Speculations 3 (2021) 1 - 31

Original Paper

Quantum Wavefunction Explained by the Sampling Theory

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Received: 8 September 2020 / Accepted: 27 December 2020 / Published online: 31 December 2020

Abstract: A new interpretation for Quantum Mechanics, which is based on the sampling theory, is presented. In the new interpretation, a particle is treated as a 3-D continuous signal in the coordinate space. Because of the limited resolution in the reciprocal space, which is known as the frequency space, its spectrum can only be represented by a discrete signal which is a sampled version of the continuous spectral signal. From the sampling theory, the particle will have infinite number of image replicas in the coordinate space. It is these image replicas that form the particle's wavefunction. Based on the new interpretation, a photon is a local Electro-Magnetic (EM) excitation pulse, the photon wavefunction is a non-local monochrome pseudo EM wave which is formed by the photon's excitation signal and its image replicas. An electron is its EM field, the electron wavefunction is a non-local polychrome pseudo EM wave which is formed by the electron's EM field and its image replicas. The properties of the electron wavefunction are calculated. These properties are used in a proposed experiment which can distinguish the new interpretation from the existing probability interpretation. Based on the new interpretation, it is demonstrated that the quantum effect is a relativistic effect. Quantum Mechanics and Special Relativity are compatible and expose two different aspects of space time. Both Quantum Coherence and Quantum Collapse are naturally explained based on the new interpretation.

Keywords: Quantum coherence; Quantum collapse; Sampling theory; Quantum interpretation

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1. Introduction

In the early 20th century, it is observed that photon and electron exhibit both particle phenomenon and wave phenomenon [1] [2] [3] [4] [5] [6]. Quantum Mechanics was established to describe such particle and wave duality of the elementary particles [7].

Since its establishment, Quantum Mechanics has been characterized as: mathematically beautiful, extremely accurate when comparing to all the conducted experiments, difficult to understand. The reason that it is hard to understand is mostly because of its probability interpretation which is commonly known as Copenhagen interpretation [7]. Although this interpretation has been widely accepted in the physics community, it has troubled some greatest minds. Erwin Schrödinger used the Schrödinger cat paradox [8] to show his frustration. Albert Einstein once said "God does not play dice with the universe" [9].

One reason that the probability interpretation being hard to understand is that it cannot explain Quantum Collapse, the instant collapse of the particle's wavefunction upon measurement, even though it is essential to the measurement problem and the entanglement problem.

According to Copenhagen interpretation, when a particle is not measured, it is in different places at any moment. The probability of it being at any place is described by a wavefunction. Upon measurement, the wavefunction collapses instantly, and the particle is only at the place where it is detected.

Unlike the classical wave such as the water wave which dissipates as it propagates, the particle's wavefunction is like a pattern of repetition. At the exact wavelength multiples, the phase of the particle's wavefunction is precisely synchronized no matter how extended the wavefunction is in space time. This property is called Quantum Coherence for a single particle and Quantum Correlation for a pair of entangled particles. The pattern of repetition of the particle's wavefunction, no matter how extended in space, will instantly collapse when the particle is measured. This phenomenon troubled Einstein so much that he called it a "spooky action at a distance".

To expose this problem, in 1935 Albert Einstein et al postulated the EPR paradox [10]. In this thought experiment, two entangled particles are separated by a long distance. Measuring one particle will instantly affect the measuring result of the other particle. At the time, there were only two possible explanations. The first is of course the "spooky action at a distance". The second is that the two particles carry the information with them through certain hidden variables so that the two entangled particles have a classical correlation. EPR preferred the second explanation [10].

Based on EPR's paper, in 1964 John Stewart Bell postulated the Bell Inequality theorem [11]. In the theorem, Bell demonstrated that the measuring result from the hidden variable solution and that from Quantum Mechanics are distinguishable.

To make things worse, Bell Inequality has been proved to be violated by multiple experiments [12] [13]. These experiments eliminate any hope for the hidden variable solution to EPR paradox. This leaves the "spooky action at a distance" as the only explanation. The violation of Bell Inequality indicates that the Quantum Correlation between the entangled particles cannot be explained by the classical statistical correlation.

The term Quantum Collapse was re-formulated as instant collapse of quantum field in Quantum Field Theory (QFT) by Julian Schwinger [14]. In QFT, the particle's non-local coherent wavefunction is treated as a physical field quantum. Its instant collapse is treated as a real physical process. For example, when an electron moves in a cloud chamber, its coherent wavefunction is instantly collapsed and re-created when the electron interacts with the gas molecules. Unfortunately, the instant collapse process of the physical field quantum is not explained by QFT [14]. This undoubtedly makes Quantum Collapse even more mysterious. Today it remains one unexplained mystery in physics.

Historically, there have been three major interpretations on matter wave or the quantum wavefunction. The first is probability wave or Copenhagen interpretation [7]. The second is pilot wave or De Broglie – Bohm theory [15]. The third is field quantum in QFT [14]. In the first two interpretations, only the point-like particle is treated as physical, and the wavefunction is treated as non-physical. In the third interpretation, the complete wavefunction is treated as a physical unit.

When people use the first interpretation to explain the famous single photon double slit experiment, they claim that the photon passes through both slits when not observed, but only passes through one slit when observed. However, this claim is shattered by a 2011 experiment which uses the weak measurement to detect which slit the photon passed through and observe the interference pattern at the same time [16]. If the photon only passes through one slit, then with whom does it interfere?

To use the second interpretation to explain the same experiment, the photon only passes through one slit, but its pilot wave can sense both slits and cause the interference pattern. This will mysteriously give the particle the remote sensing ability [17]. But it cannot explain why such remote sensing ability is turned off when a particle moves in a cloud chamber where the interference pattern is completely missing and only the particle's classical trajectory can be observed. Nevertheless, the meaning of the pilot wave is not given by the De Broglie – Bohm theory. Is it real? What is it made of?

The third interpretation has no trouble to explain the same experiment because the physical field quantum is non-local, and it passes through both slits [14]. However, if you put a photon detector at each slit, there is only one click when the physical field quantum passes through both slits [14]. To think of the complete wavefunction as one physical unit is mindboggling. For example, when a photon travels from the Sun to Earth, its non-local wavefunction can instantly reach us, but we must wait for 8.3 minutes to feel its energy due to the speed of light.

Although there had been heated debates on the interpretation of quantum wavefunction among the founders of Quantum Mechanics, people became complacent on this subject. "Shut up and calculate" has become the common attitude.

However, this trend has changed recently. In 2017, Steven Weinberg wrote an article "The Trouble with Quantum Mechanics" to summarize the growing unsatisfaction with Copenhagen interpretation and other existing interpretations on Quantum Mechanics [18].

The motivation of this paper is to explain Quantum Coherence and Quantum Collapse. To achieve this goal, we must look at the particle's wavefunction from a new perspective. In the existing framework, there has not been much success to understand these phenomena in almost a century. How do we explain the wavefunction or pattern of repetition without referring to probability?

In the field of Signal Processing, a well-known sampling theory [19] [21] serves as the link between Continuous Signal Processing (CSP) and Discrete Signal Processing (DSP). This theory, which was first derived by mathematician Claude Shannon, has been widely proved in both theory and practice. It successfully explains the image replicas and their interference when using a discrete signal to approximate a continuous signal. For example, the commonly known Moiré pattern in digital image and fidelity distortion in digital audio are perfectly explained by this theory. Although little referenced in physics, this theory has become a cornerstone in modern digital communication. Can this theory also serve as the link between Classical Mechanics and Quantum Mechanics?

In physics, matter exists in two forms: particle and field. In the book "The Evolution of Physics" [20], when Albert Einstein discussed the line between particle and field, he stated a hypothesis that a particle is composed of field and the particle's location is at the center of mass of its field. This idea will be exploited in the rest of the paper. An electron is simply treated as the center of mass of its EM field. It is described by a 3-D continuous signal in the coordinate space.

2. Sampling theory

Let us briefly introduce the sampling theory. In the field of Signal Processing [21], the Continuous Signal Processing (CSP) describes a continuous signal $f(x)$ in the coordinate space and its spectrum $F(k)$ in the frequency space. It is given by the Fourier transform and the inverse Fourier transform,

$$F(k) = \int_{-\infty}^{\infty} f(x) e^{ikx} \, dx \qquad (1)$$

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(k) e^{-ikx} \, dk \qquad (2)$$

Both $f(x)$ and $F(k)$ are continuous functions. The mapping between $f(x)$ and $F(k)$ is one-to-one as illustrated in Figure 1 and Figure 2,

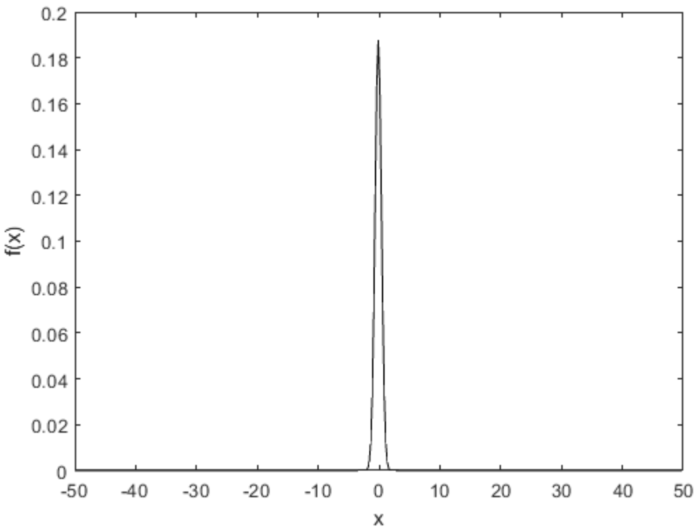


Fig. 1. Continuous signal f(x) in coordinate space

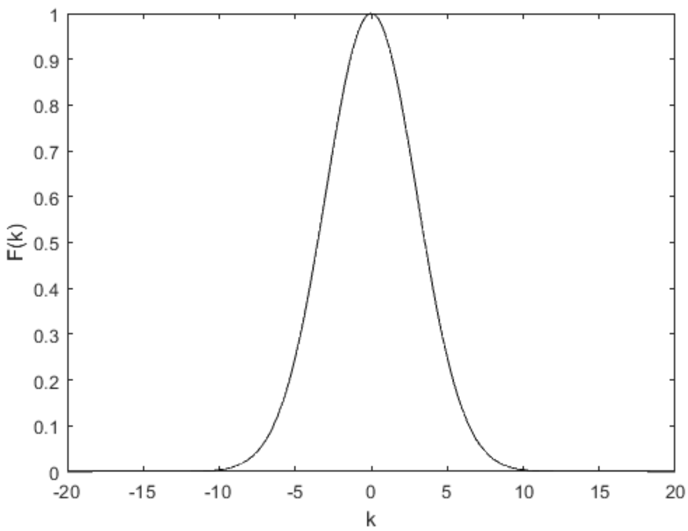


Fig. 2. Continuous spectrum F(k) in frequency space

If the frequency space is not continuous and has a limited resolution, then the spectrum must be represented by a discrete signal $F(k_n)$, where $k_n = nk_s$, $n = 0, \pm 1, \pm 2 \cdots$

This is a sampled version of the continuous signal $F(k)$, with the sampling frequency k_s. Corresponding to this discrete spectrum, the signal in the coordinate space then becomes a series of image replica of $f(x)$, separated by a distance $x_s = \frac{2\pi}{k_s}$ [19]. This is shown in Figure 3 and Figure 4.

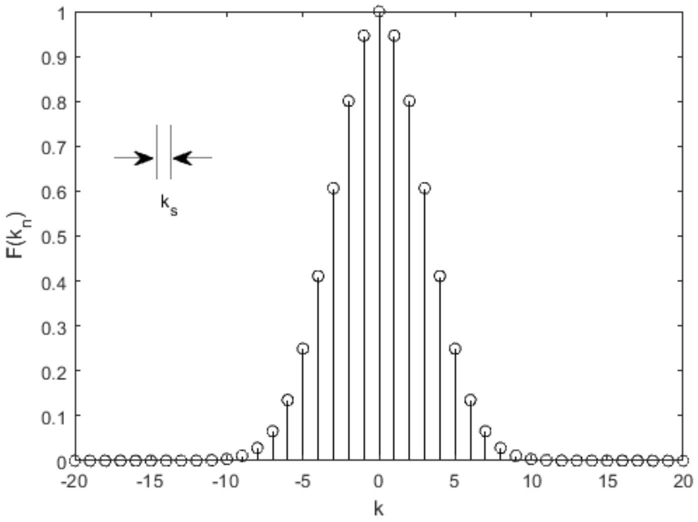


Fig. 3. Discrete spectrum $F(k_n)$ in frequency space

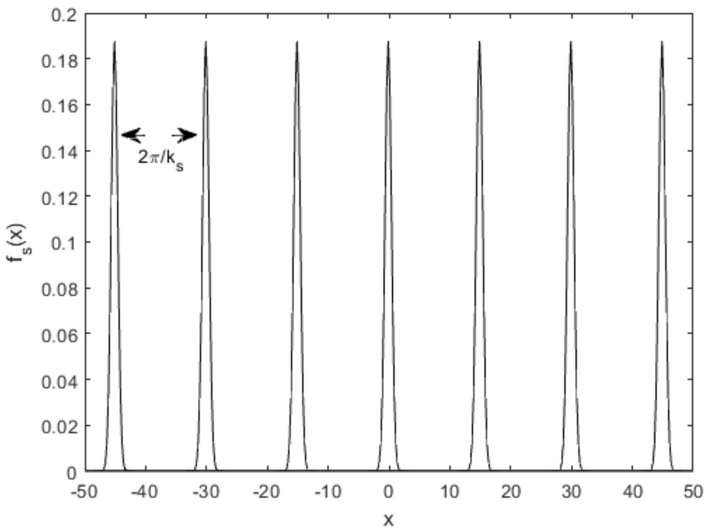


Fig. 4. Continuous signal f(x) and its image replicas in coordinate space

As shown in Fig. 4, the signal $f(x)$ and its replicas form a periodic continuous function as

$$f_s(x) = \sum_{\ell=-\infty}^{\infty} f(x - \ell x_s) \qquad (3)$$

As the sampling frequency k_s in the frequency space becomes larger (coarse sampling), the distance between the image replicas $x_s = \frac{2\pi}{k_s}$ in the coordinate space becomes smaller. You could even see image overlap when the sampling is too coarse. On the other hand, when the sampling frequency becomes smaller (fine sampling), the distance between the image replicas in the coordinate space becomes larger. At the extreme case when the sampling frequency becomes infinitesimally small, the discrete spectral signal converges to the continuous spectral signal, the image replicas in the coordinate space disappear.

It should be noted that sampling is a common practice to describe Nature. Mathematicians use all the real numbers to form a 1-D continuum, then use the sampling frequency of 1/1 to sample it to form all the integers. If he decides to describe the number in 3 decimal precision, then he will use the sampling frequency of 1000/1 to sample that 1-D continuum to form a discrete set of decimals.

Physicists use sampling all the time. For example, they use a 1-D continuum to describe the time dimension. But in practice, they only use the discrete values in year, day, second, or micro-second, etc. to describe time. These numbers are all the sampled version of the 1-D time continuum. Interestingly, the different levels of these discrete values are corresponding to certain periodic movements in Nature such as celestial object rotation, mechanical clock movement, crystal oscillation, or electron's periodic movement in an atom, etc. These periodic movements form the patterns of repetition with different periods. The sampling theory is the mathematical transformation which provides the mapping between the pattern of repetition and discreteness.

So, the sampling theory is not a mere trick invented by mathematicians, but a fundamental law imposed by Nature. Shannon is just the lucky one who discovered it.

If we want to use this pattern of repetition or image replicas in the (x, y, z) coordinate space to explain the electron wavefunction in Quantum Mechanics, then we must assume that the 3-D frequency space (k_x, k_y, k_z) is discrete in nature.

If this hypothesis is correct, then what is the sampling frequency in the 3-D frequency space (k_x, k_y, k_z) ? Before answering this question, let us visit another old hypothesis on particle and field.

Where is the line between particle and field? Is particle the concentration of field? Is a particle simply the center of mass of its field?

3. Particle and field

In classical EM [22], an electron's electric field is described by Gauss's law

$$\oiint \boldsymbol{E} \cdot d\boldsymbol{S} = {}^{e}/_{\varepsilon_0} \qquad (4)$$

where $\boldsymbol{E}$ is the electric field, e is the electron's charge.

In the first inertial coordinate system (CS), an electron rests at the origin $(x, y, z) = (0,0,0)$.

From spherical symmetry it is easy to calculate that

$$\boldsymbol{E} = \frac{e}{4\pi\varepsilon_0} \frac{\hat{\boldsymbol{r}}}{r^2} \qquad (5)$$

The energy density of the electric field is

$$\rho_E = \frac{\varepsilon_0}{2} E^2 \qquad (6)$$

If we assume that Equation (5) only holds true at $r \geq r_0$, where r_0 denotes the radius of a very small spherical surface which serves as a boundary, inside which $\boldsymbol{E} = 0$, then the total energy of the electric field

$$\mathcal{E}_E = \iiint \rho_E \, dV = \int_{r_0}^{\infty} r^2 \, dr \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\varphi \frac{\varepsilon_0}{2} \left(\frac{e}{4\pi\varepsilon_0 r^2}\right)^2 = \frac{e^2}{8\pi\varepsilon_0 r_0}$$

Let us equate it to the energy of electron's mass m_e, thus

$$m_e c^2 = \frac{e^2}{8\pi\varepsilon_0 r_0} \qquad (7)$$

Now consider a second CS moving at a constant speed $-v\hat{\mathbf{z}}$ with respect to the first CS, where $\hat{\mathbf{z}}$ is the unit vector of z-axis. In the second CS, the electron moves at a constant speed $v\hat{\mathbf{z}}$.

With Lorentz transform, the electron's electric field and magnetic field are [22]

$$\mathbf{E} = \frac{e}{4\pi\varepsilon_0} \frac{1-\beta^2}{[1-(\beta\sin\theta)^2]^{3/2}} \frac{\hat{\mathbf{r}}}{r^2} \qquad (8)$$

$$\mathbf{B} = \frac{\mu_0 e c}{4\pi} \frac{\beta(1-\beta^2)\sin\theta}{[1-(\beta\sin\theta)^2]^{3/2}} \frac{\hat{\boldsymbol{\varphi}}}{r^2} \qquad (9)$$

Equation (8) and Equation (9) are written in the spherical coordinates (r, θ, φ).

The energy density of the electric field and magnetic field are thus

$$\rho_E = \frac{\varepsilon_0}{2} E^2 = \frac{\varepsilon_0}{2} \left(\frac{e}{4\pi\varepsilon_0}\right)^2 \frac{(1-\beta^2)^2}{[1-(\beta\sin\theta)^2]^3} \frac{1}{r^4} \qquad (10)$$

$$\rho_B = \frac{1}{2\mu_0} B^2 = \frac{1}{2\mu_0} \left(\frac{\mu_0 e c}{4\pi}\right)^2 \frac{[\beta(1-\beta^2)\sin\theta]^2}{[1-(\beta\sin\theta)^2]^3} \frac{1}{r^4} \qquad (11)$$

The boundary, a spherical surface

$$r_s = r_0 \qquad (12)$$

in the first CS, then becomes an elliptical surface in the second CS due to the relativistic shrinking in z-axis. The surface is thus

$$r_s = \frac{r_0}{\gamma[1-(\beta\sin\theta)^2]^{1/2}} \qquad (13)$$

Here $\beta = \frac{v}{c}$ and $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ with c being the speed of light.

The total energy of the electric field is

$$\mathcal{E}_E = \iiint \rho_E \, dV = \int_{r_s}^{\infty} r^2 \, dr \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\varphi \frac{\varepsilon_0}{2} \left(\frac{e}{4\pi\varepsilon_0}\right)^2 \frac{(1-\beta^2)^2}{[1-(\beta\sin\theta)^2]^3} \frac{1}{r^4}$$

$$= \frac{e^2(1-\beta^2)^2}{16\pi\varepsilon_0} \int_0^{\pi} \frac{\sin\theta}{r_s[1-(\beta\sin\theta)^2]^3} d\theta$$

$$= \frac{e^2(1-\beta^2)^2\gamma}{16\pi\varepsilon_0 r_0} \int_0^{\pi} \frac{\sin\theta}{[1-(\beta\sin\theta)^2]^{5/2}} d\theta$$

Use

$$\int_0^{\pi} \frac{\sin\theta}{[1-(\beta\sin\theta)^2]^{5/2}} d\theta = \int_{-1}^{1} \frac{dx}{(1-\beta^2)^{\frac{5}{2}}\left(1+\frac{\beta^2 x^2}{1-\beta^2}\right)^{5/2}} = 2\gamma^4\left(1-\frac{\beta^2}{3}\right)$$

Then

$$\mathcal{E}_E = \frac{e^2\gamma}{8\pi\varepsilon_0 r_0}\left(1-\frac{\beta^2}{3}\right) \qquad (14)$$

Similarly, the total energy of the magnetic field is

$$\mathcal{E}_B = \iiint \rho_B\, dV = \int_{r_s}^{\infty} r^2\, dr \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\varphi \frac{1}{2\mu_0}\left(\frac{\mu_0 e c}{4\pi}\right)^2 \frac{[\beta(1-\beta^2)\sin\theta]^2}{[1-(\beta\sin\theta)^2]^3}\frac{1}{r^4}$$
$$= \frac{\mu_0 e^2 c^2 \gamma}{8\pi r_0}\left(\frac{2\beta^2}{3}\right) \qquad (15)$$

Using $c^2 = \frac{1}{\varepsilon_0 \mu_0}$ and Equation (7), Equation (14) and Equation (15) are reduced to

$$\mathcal{E}_E = m_e c^2 \gamma \left(1 - \frac{\beta^2}{3}\right) \qquad (16)$$

$$\mathcal{E}_B = m_e c^2 \gamma \left(\frac{2\beta^2}{3}\right) \qquad (17)$$

An interesting observation can be made here. The total energy of the moving electron not only has the traditional component $m_e c^2 \gamma$ but also has an oscillating component in the form of

$$\begin{pmatrix}\mathcal{E}_E \\ \mathcal{E}_B\end{pmatrix} = \begin{pmatrix} -m_e c^2 \gamma \frac{\beta^2}{3} \\ m_e c^2 \gamma \frac{2\beta^2}{3}\end{pmatrix} \qquad (18)$$

In this newly discovered oscillating component, the electric field and magnetic field are coupled. They are alternating into each other and propagating at the same speed of the traditional electron particle $m_e c^2 \gamma$. The ratio between the electric field energy and the magnetic field energy in the coupled field is

$$\frac{(-m_e c^2 \gamma \frac{\beta^2}{3})}{(m_e c^2 \gamma \frac{2\beta^2}{3})} = -\frac{1}{2} \qquad (19)$$

Interestingly this ratio is a Lorentz invariant and equals to the electron's spin.

The newly discovered component in Equation (18) does not contradict Special Relativity which only predicts the normal component $m_e c^2 \gamma$. Unlike a classical particle, electron emits EM radiation when it is in deceleration. The component in Equation (18) plays an important role in the electron's EM radiation. However, the EM radiation by an electron is a distinctive topic which deserves a separate paper to cover.

4. Continuous frequency analysis of electron's EM field

To discover the discrete nature of electron's EM field in the frequency space, let us calculate the Fourier transform of the energy density of the electric field of an electron in the second CS where it is moving at a constant speed $v\hat{\boldsymbol{z}}$

$$\Psi_E(\boldsymbol{k}) = \iiint \rho_E \exp(i\boldsymbol{k}\cdot\boldsymbol{r})\, dV$$

The energy density ρ_E is given by Equation (10). In the spherical coordinates (r, θ, φ)

$$\boldsymbol{r} = r\cos\theta\,\hat{\boldsymbol{z}} + r\sin\theta\cos\varphi\,\hat{\boldsymbol{x}} + r\sin\theta\sin\varphi\,\hat{\boldsymbol{y}}$$
$$\boldsymbol{k} = k\cos\vartheta\,\hat{\boldsymbol{z}} + k\sin\vartheta\,\hat{\boldsymbol{y}}$$

Due to symmetry, we only need to consider $\boldsymbol{k}$ in the y-z plane. Thus

$$\Psi_E(\boldsymbol{k}) = \int_{r_s}^{\infty} r^2\, dr \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\varphi \frac{\varepsilon_0}{2}\left(\frac{e}{4\pi\varepsilon_0}\right)^2 \frac{(1-\beta^2)^2}{[1-(\beta\sin\theta)^2]^3}\frac{1}{r^4}\exp[ikr(\cos\vartheta\cos\theta + \sin\vartheta\sin\theta\sin\varphi)]$$

$$\Psi_E(\boldsymbol{k}) = \begin{cases} \dfrac{e^2}{16\pi\varepsilon_0}(1-\beta^2)^2 \displaystyle\int_0^\pi \frac{\sin\theta}{[1-(\beta\sin\theta)^2]^3}d\theta \int_{r_s}^\infty \frac{\exp(ikr\cos\theta)}{r^2}dr, & \vartheta = 0 \\ 0, \quad others & \\ \dfrac{e^2}{16\pi\varepsilon_0}(1-\beta^2)^2 \displaystyle\int_0^\pi \frac{\sin\theta}{[1-(\beta\sin\theta)^2]^3}d\theta \int_{r_s}^\infty \frac{\exp(-ikr\cos\theta)}{r^2}dr, & \vartheta = \pi \end{cases}$$

r_s is the boundary surface given by Equation (13). After laborious calculation,

$$\Psi_E(\boldsymbol{k}) = \begin{cases} \Phi_E(k), & (k_x, k_y, k_z) = (0,0,k) \\ 0, & (k_x, k_y, k_z) = others \\ \Phi_E(k), & (k_x, k_y, k_z) = (0,0,-k) \end{cases}$$

$$\Phi_E(k) = m_e c^2\gamma\left\{\frac{\sin x}{x} + x\left[h(x) - \frac{\pi}{4}\right]\right\}$$
$$- m_e c^2\gamma\beta^2\left\{\frac{\sin x}{x} - \frac{2}{x^2}\left[\frac{\sin x}{x} - \cos x\right] + x\left[g(x) - \frac{\pi}{8}\right]\right\}, \qquad x = \frac{kr_0}{\gamma} \quad (20)$$

$$h(x) = \sum_{n=0}^\infty \frac{(-1)^n x^{2n+1}}{(2n+3)(2n+1)(2n+1)!} \quad (21)$$

$$g(x) = \sum_{n=0}^\infty \frac{(-1)^n x^{2n+1}}{(2n+5)(2n+1)(2n+1)!} \quad (22)$$

At the boundary cases,

$$h(x) = \begin{cases} \dfrac{x}{3} + O(x^3), & x \ll 1 \\ \dfrac{\pi}{4} - \dfrac{\sin x}{x^2} + \dfrac{\cos x}{x^3} + \dfrac{3\sin x}{x^4} + O\left(\dfrac{1}{x^5}\right), & x \gg 1 \end{cases} \quad (23)$$

$$g(x) = \begin{cases} \dfrac{x}{5} + O(x^3), & x \ll 1 \\ \dfrac{\pi}{8} - \dfrac{\sin x}{x^2} - \dfrac{\cos x}{x^3} + \dfrac{2\sin x}{x^4} + O\left(\dfrac{1}{x^5}\right), & x \gg 1 \end{cases} \quad (24)$$

Thus, at the boundary cases,

$$\Phi_E(k) = \begin{cases} m_e c^2\gamma\left[1 - \dfrac{\pi}{4}x + O(x^2)\right] - m_e c^2\gamma\beta^2\left[\dfrac{1}{3} - \dfrac{\pi}{8}x + O(x^2)\right], & x = \dfrac{kr_0}{\gamma} \ll 1 \\ m_e c^2\gamma\left[\dfrac{\cos x}{x^2} + O\left(\dfrac{1}{x^3}\right)\right] - m_e c^2\gamma\beta^2\left[\dfrac{\cos x}{x^2} + O\left(\dfrac{1}{x^3}\right)\right], & x = \dfrac{kr_0}{\gamma} \gg 1 \end{cases} \quad (25)$$

Here we can make a few observations:

- The energy density $\rho_E(x,y,z)$ which is a 3-D continuous function in (x,y,z) coordinate space is reduced to a 1-D function $\Phi_E(k)$ in (k_x,k_y,k_z) frequency space. A particle looks like a string in the frequency space.

- At low frequency when $\frac{kr_0}{\gamma} \ll 1$, the spectrum $\Phi_E(k)$ oscillates as $\frac{\sin x}{x}$ function, where $x = \frac{kr_0}{\gamma}$.
- $\Phi_E(0) = m_e c^2 \gamma \left(1 - \frac{\beta^2}{3}\right)$. This equals to the electric field energy given by Equation (16).
- At high frequency when $\frac{kr_0}{\gamma} \gg 1$, the spectrum $\Phi_E(k)$ converges to zero at the speed of $O\left(\frac{1}{k^2}\right)$.
- The spectrum $\Phi_E(k)$ behaves like a low-pass filter in the frequency space.

Similarly, we can calculate the spectrum for the magnetic field

$$\Psi_B(\boldsymbol{k}) = \iiint \rho_B \exp(i\boldsymbol{k}\cdot\boldsymbol{r})\, dV$$

$$\Psi_B(\boldsymbol{k}) = \int_{r_s}^{\infty} r^2\, dr \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\varphi \frac{1}{2\mu_0} \left(\frac{\mu_0 ec}{4\pi}\right)^2 \frac{[\beta(1-\beta^2)\sin\theta]^2}{[1-(\beta\sin\theta)^2]^3} \frac{1}{r^4} \exp[ikr(\cos\vartheta\cos\theta + \sin\vartheta\sin\theta\sin\varphi)]$$

$$\Psi_B(\boldsymbol{k}) = \begin{cases} \frac{\mu_0 e^2 c^2}{16\pi}\beta^2(1-\beta^2)^2 \int_0^{\pi} \frac{(\sin\theta)^3}{[1-(\beta\sin\theta)^2]^3} d\theta \int_{r_s}^{\infty} \frac{\exp(ikr\cos\theta)}{r^2} dr, & \vartheta = 0 \\ 0, \quad 0 < \vartheta < \pi & \\ \frac{\mu_0 e^2 c^2}{16\pi}\beta^2(1-\beta^2)^2 \int_0^{\pi} \frac{(\sin\theta)^3}{[1-(\beta\sin\theta)^2]^3} d\theta \int_{r_s}^{\infty} \frac{\exp(-ikr\cos\theta)}{r^2} dr, & \vartheta = \pi \end{cases}$$

$$\Psi_B(\boldsymbol{k}) = \begin{cases} \Phi_B(k), & (k_x, k_y, k_z) = (0,0,k) \\ 0, & (k_x, k_y, k_z) = others \\ \Phi_B(k), & (k_x, k_y, k_z) = (0,0,-k) \end{cases}$$

$$\Phi_B(k) = m_e c^2 \gamma \beta^2 \left\{\frac{2}{x^2}\left[\frac{\sin x}{x} - \cos x\right] + x\left[h(x) - g(x) - \frac{\pi}{8}\right]\right\}, \qquad x = \frac{kr_0}{\gamma} \tag{26}$$

At boundary conditions,

$$\Phi_B(k) = \begin{cases} m_e c^2 \gamma \beta^2 \left[\frac{2}{3} - \frac{\pi}{8}x + O(x^2)\right], & x = \frac{kr_0}{\gamma} \ll 1 \\ m_e c^2 \gamma \beta^2 \left[\frac{3\sin x}{x^3} + O\left(\frac{1}{x^4}\right)\right], & x = \frac{kr_0}{\gamma} \gg 1 \end{cases} \tag{27}$$

5. Discrete signal processing on electron's EM field

In Equation (20) and Equation (26), if the frequency k can only take discrete values as

$$k = nk_s, n = 0, \pm1, \pm2 \cdots$$

With the sampling frequency k_s satisfying

$$\frac{k_s r_0}{\gamma} = Kv, \qquad v \text{ is the velosity and } K \text{ is a constant}$$

Then from the sampling theory, the electron in (x, y, z) coordinate space will have a series of image replicas separated by distance

$$\lambda = \frac{2\pi}{k_s} = \frac{2\pi r_0}{K\gamma v} = \frac{2\pi r_0}{K\gamma c\beta} \qquad (28)$$

According to Equation (3), the electron wavefunction can be written as

$$\Psi(x, y, z, t) = \sum_{\ell=-\infty}^{\infty} f\left(x, y, (z - vt) - \ell\frac{2\pi}{k_s}\right) \qquad (29)$$

This is a moving periodic continuous function along z-dimension with $f(x, y, z)$ being represented by a complex EM vector field which is also called Riemann-Silberstein (RS) vector field [23],

$$f(x, y, z) = \sqrt{\frac{\varepsilon_0}{2}}\boldsymbol{E} + \frac{i}{\sqrt{2\mu_0}}\boldsymbol{B} \qquad (30)$$

E and ***B*** are described by Equation (8) and Equation (9). RS field has the property of

$$f^*(x, y, z) \cdot f(x, y, z) = \frac{\varepsilon_0}{2}\ E^2 + \frac{1}{2\mu_0}\ B^2 = \rho_E + \rho_B \qquad (31)$$

This is the energy density of the EM field. According to Fourier analysis, a periodic continuous function can be expanded as a Fourier series [24] [25]. So, Equation (29) can be written as

$$\Psi(x, y, z, t) = \sum_{\ell=-\infty}^{\infty} f\left(x, y, (z - vt) - \ell\frac{2\pi}{k_s}\right) = \sum_{n=1}^{\infty} A_n(x, y, k_s) exp\big(ink_s(z - vt)\big) \quad (32)$$

In Equation (32), we only take the positive frequency Fourier components which correspond to the time moving forward.

To take advantage of the cylindrical symmetry, we re-write Equation (30) and Equation (32) in the cylindrical coordinates (R, ϕ, z), $\quad R = \sqrt{x^2 + y^2}$

$$f(R, \phi, z) = \begin{pmatrix} \sqrt{\frac{\varepsilon_0}{2}} E_R + \frac{i}{\sqrt{2\mu_0}} B_R \\ \sqrt{\frac{\varepsilon_0}{2}} E_\phi + \frac{i}{\sqrt{2\mu_0}} B_\phi \\ \sqrt{\frac{\varepsilon_0}{2}} E_z + \frac{i}{\sqrt{2\mu_0}} B_z \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{\varepsilon_0}{2}} E_R(R, z) \\ \frac{i}{\sqrt{2\mu_0}} B_\phi(R, z) \\ \sqrt{\frac{\varepsilon_0}{2}} E_z(R, z) \end{pmatrix} \qquad (33)$$

$$\Psi(R, \phi, z, t) = \sum_{n=1}^{\infty} A_n(R, k_s) exp\big(ink_s(z - vt)\big) \qquad (34)$$

The Fourier coefficients are [25]

$$A_n(R,k_s) = \frac{k_s}{2\pi}\int_{-\frac{\pi}{k_s}}^{\frac{\pi}{k_s}}\left[\sum_{\ell=-\infty}^{\infty} f\left(R,\phi,z-\ell\frac{2\pi}{k_s}\right)\right]exp(-ink_s z)dz$$
$$= \frac{k_s}{2\pi}\int_{-\frac{\pi}{k_s}}^{\frac{\pi}{k_s}} f(R,\phi,z)exp(-ink_s z)dz$$
$$+ \frac{k_s}{2\pi}\int_{-\frac{\pi}{k_s}}^{\frac{\pi}{k_s}}\left[\sum_{\ell=\pm 1} f\left(R,\phi,z-\ell\frac{2\pi}{k_s}\right)\right]exp(-ink_s z)dz$$
$$+ \frac{k_s}{2\pi}\int_{-\frac{\pi}{k_s}}^{\frac{\pi}{k_s}}\left[\sum_{\ell=\pm 2} f\left(R,\phi,z-\ell\frac{2\pi}{k_s}\right)\right]exp(-ink_s z)dz + \cdots \qquad (35)$$

According to Figure 4, if the sampling frequency is small enough, then the distance between the image replicas is large enough, then we can ignore the interference between the image replicas. This means that we can approximate Equation (35) by dropping the terms of $|\ell| \geq 1$, then

$$A_n(R,k_s) \cong \frac{k_s}{2\pi}\int_{-\frac{\pi}{k_s}}^{\frac{\pi}{k_s}} f(R,\phi,z)exp(-ink_s z)dz$$
$$= \frac{k_s}{2\pi}\int_{-\frac{\pi}{k_s}}^{\frac{\pi}{k_s}} \begin{pmatrix} \sqrt{\frac{\varepsilon_0}{2}}E_R(R,z) \\ \frac{i}{\sqrt{2\mu_0}}B_\phi(R,z) \\ \sqrt{\frac{\varepsilon_0}{2}}E_z(R,z) \end{pmatrix} exp(-ink_s z)dz \qquad (36)$$

By using Equation (28) and Equation (7)

$$k_s(z-vt) = \frac{K\gamma c\beta}{r_0}(z-c\beta t) = K\frac{8\pi\varepsilon_0 c^2}{e^2}(m_e\gamma vz - \beta^2 m_e c^2\gamma t) = K\frac{8\pi\varepsilon_0 c^2}{e^2}(pz-\beta^2 Et)$$

Substituting this term with that in Equation (34)

$$\Psi(R,\phi,z,t) = \sum_{n=1}^{\infty} A_n(R,k_s)exp\left(inK\frac{8\pi\varepsilon_0 c^2}{e^2}(pz-\beta^2 Et)\right) \qquad (37)$$

In Quantum Mechanics, a photon wavefunction in free coordinate space is expressed as a planar wave

$$\Psi(x,y,z,t) = A\ \ exp\left[\frac{2\pi i}{h}(pz-Et)\right] \qquad (38)$$

Compare Equation (37) and Equation (38), we could deduce

$$K = \frac{e^2}{8\pi\varepsilon_0\hbar c^2} = \frac{1}{2c}\frac{e^2}{4\pi\varepsilon_0\hbar c} = \frac{\alpha}{2c} \qquad (39)$$

In Equation (39), $\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c}$ is the fine structure constant [22].

Combine Equation (28) and Equation (39), the sampling frequency in (k_x, k_y, k_z) frequency space is

$$k_s = \frac{\alpha\beta\gamma}{2r_0} \qquad (40)$$

In Equation (40), α is the fine structure constant, $\beta = \frac{v}{c}, \gamma = \frac{1}{\sqrt{1-\beta^2}}$ and r_0 is the radius of the particle. Combine Equation (37) and Equation (39)

$$\Psi(R, \phi, z, t) = \sum_{n=1}^{\infty} A_n(R, k_s) exp\left(2\pi i \frac{n}{h}(pz - \beta^2 Et)\right) \quad (41)$$

Equation (41) is more realistic when representing a free moving particle than a planar wavefunction such as Equation (38) since we know that a free moving particle travels in a beam rather than across an infinite plane. The planar wavefunction is just a simplified mathematical abstraction.

From Equation (8), (9), (13), (36) and (40), we can derive the Fourier coefficients as follow

$$A_{n,R}(R, k_s) = \frac{k_s}{2\pi} \int_{-\frac{\pi}{k_s}}^{\frac{\pi}{k_s}} \sqrt{\frac{\varepsilon_0}{2}} E_R(R, z) exp(-ink_s z) dz$$

$$= \frac{k_s}{2\pi} \sqrt{\frac{\varepsilon_0}{2}} \int_{-\frac{\pi}{k_s}}^{\frac{\pi}{k_s}} \frac{e}{4\pi\varepsilon_0} \frac{1-\beta^2}{[1-(\beta \sin\theta)^2]^{3/2}} \frac{\sin\theta}{r^2} [\cos(nk_s z) - i \sin(nk_s z)] dz$$

$$= \frac{k_s}{2\pi} \sqrt{\frac{\varepsilon_0}{2}} \int_{-\frac{\pi}{k_s}}^{\frac{\pi}{k_s}} \frac{e}{4\pi\varepsilon_0} \frac{(1-\beta^2)R}{[z^2 + (1-\beta^2)R^2]^{3/2}} [\cos(nk_s z) - i \sin(nk_s z)] dz$$

$$= \frac{k_s}{\pi} \sqrt{\frac{\varepsilon_0}{2}} \int_{z_0}^{\frac{\pi}{k_s}} \frac{e}{4\pi\varepsilon_0} \frac{(1-\beta^2)R \cos(nk_s z)}{[z^2 + (1-\beta^2)R^2]^{3/2}} dz$$

$$= \frac{k_s}{\pi} \sqrt{\frac{\varepsilon_0}{2}} \int_{k_s z_0}^{\pi} \frac{e}{4\pi\varepsilon_0} \frac{(1-\beta^2)R \cos(nx)}{[\left(\frac{x}{k_s}\right)^2 + (1-\beta^2)R^2]^{3/2}} \frac{dx}{k_s}$$

$$= \frac{e(1-\beta^2)R{k_s}^3}{4\pi^2\sqrt{2\varepsilon_0}} \int_{k_s z_0}^{\pi} \frac{\cos(nx)}{[x^2 + (1-\beta^2)R^2{k_s}^2]^{3/2}} dx \quad (42)$$

It is difficult to derive an analytical form of Equation (42), but we can do so at the two boundary cases. When $\frac{R}{r_0} \gg 1$, Equation (42) is reduced to

$$
\begin{aligned}
&A_{n,R}(R,k_s)\\
&=\frac{e(1-\beta^2)R{k_s}^3}{4\pi^2\sqrt{2\varepsilon_0}}\int_0^\pi \frac{\cos(nx)}{[x^2+(1-\beta^2)R^2{k_s}^2]^{3/2}}dx\\
&=\frac{e(1-\beta^2)R{k_s}^3}{4\pi^2\sqrt{2\varepsilon_0}[(1-\beta^2)^{\frac{1}{2}}Rk_s]^3}\int_0^\pi \frac{\cos(nx)}{[1+\left(\frac{\gamma x}{Rk_s}\right)^2]^{3/2}}dx\\
&\cong\frac{e\gamma}{4\pi^2\sqrt{2\varepsilon_0}R^2}\int_0^\pi\left[1-\frac{3}{2}\left(\frac{\gamma x}{Rk_s}\right)^2\right]\cos(nx)\,dx\\
&=\frac{e\gamma}{4\pi^2\sqrt{2\varepsilon_0}R^2}\left(-\frac{3}{2}\right)\left(\frac{\gamma}{Rk_s}\right)^2\int_0^\pi x^2\cos(nx)\,dx\\
&=\frac{e\gamma}{4\pi^2\sqrt{2\varepsilon_0}R^2}\left(-\frac{3}{2}\right)\left(\frac{\gamma}{Rk_s}\right)^2\frac{2\pi}{n^2}(-1)^n\\
&=\frac{(-1)^{n+1}}{n^2}\frac{3e\gamma}{\pi\sqrt{2\varepsilon_0}\alpha^2\beta^2{r_0}^2}\left(\frac{r_0}{R}\right)^4 \qquad (43)
\end{aligned}
$$

When $\frac{R}{r_0}\ll 1$, Equation (42) is reduced to

$$
\begin{aligned}
&A_{n,R}(R,k_s)\\
&=\frac{e(1-\beta^2)R{k_s}^3}{4\pi^2\sqrt{2\varepsilon_0}}\int_{\frac{k_s r_0}{\gamma}}^\pi \frac{\cos(nx)}{[x^2+(1-\beta^2)R^2{k_s}^2]^{3/2}}dx\\
&\cong\frac{e(1-\beta^2)R{k_s}^3}{4\pi^2\sqrt{2\varepsilon_0}}\int_{\frac{k_s r_0}{\gamma}}^\pi\frac{\cos(nx)}{x^3}dx=\frac{e\gamma}{4\pi^2\varepsilon_0{r_0}^2}\left(\frac{R}{r_0}\right)\left(\frac{\alpha\beta}{2}\right)^3\int_{\frac{\alpha\beta}{2}}^\pi\frac{\cos(nx)}{x^3}dx\\
&=\frac{e\gamma}{4\pi^2\sqrt{2\varepsilon_0}{r_0}^2}\left(\frac{R}{r_0}\right)\left(\frac{\alpha\beta}{2}\right)^3\int_{\frac{\alpha\beta}{2}}^\pi\frac{1-\frac{(nx)^2}{2!}+\frac{(nx)^4}{4!}+\cdots}{x^3}dx\\
&=\frac{e\gamma}{4\pi^2\sqrt{2\varepsilon_0}{r_0}^2}\left(\frac{R}{r_0}\right)\left(\frac{\alpha\beta}{2}\right)^3\left[C+\frac{1}{2}\left(\frac{\alpha\beta}{2}\right)^{-2}+\frac{n^2}{2}ln\left(\frac{\alpha\beta}{2}\right)-\frac{n^4}{48}\left(\frac{\alpha\beta}{2}\right)^2+\cdots\right]\\
&\cong\frac{e\gamma}{8\pi^2\sqrt{2\varepsilon_0}{r_0}^2}\left(\frac{\alpha\beta}{2}\right)\left(\frac{R}{r_0}\right) \qquad (44)
\end{aligned}
$$

Combine Equation (43) and (44)

$$
A_{n,R}(R,k_s)=\begin{cases}\frac{e\gamma}{8\pi^2\sqrt{2\varepsilon_0}{r_0}^2}\left(\frac{\alpha\beta}{2}\right)\left(\frac{R}{r_0}\right), & \frac{R}{r_0}\ll 1\\ \frac{(-1)^{n+1}}{n^2}\frac{3e\gamma}{\pi\sqrt{2\varepsilon_0}\alpha^2\beta^2{r_0}^2}\left(\frac{r_0}{R}\right)^4, & \frac{R}{r_0}\gg 1\end{cases} \qquad (45)
$$

Similarly, we can derive the Fourier coefficient $A_{n,\phi}(R,k_s)$ and $A_{n,z}(R,k_s)$ as

$$A_{n,\phi}(R,k_s)=\frac{k_s}{2\pi}\int_{-\frac{\pi}{k_s}}^{\frac{\pi}{k_s}}\frac{i}{\sqrt{2\mu_0}}B_\phi(R,z)exp(-ink_sz)dz$$

$$=\frac{k_s}{2\pi}\frac{i}{\sqrt{2\mu_0}}\int_{-\frac{\pi}{k_s}}^{\frac{\pi}{k_s}}\frac{\mu_0 ec}{4\pi}\frac{\beta(1-\beta^2)\sin\theta}{[1-(\beta\sin\theta)^2]^{3/2}}\frac{1}{r^2}[\cos(nk_sz)-i\sin(nk_sz)]dz$$

$$=i\sqrt{\frac{\mu_0}{2}}\frac{ec\beta(1-\beta^2)k_s}{4\pi^2}\int_{z_0}^{\frac{\pi}{k_s}}\frac{R\cos(nk_sz)}{[z^2+(1-\beta^2)R^2]^{3/2}}dz$$

$$=i\sqrt{\frac{\mu_0}{2}}\frac{ec\beta(1-\beta^2)R{k_s}^3}{4\pi^2}\int_{k_sz_0}^{\pi}\frac{\cos(nx)}{[x^2+(1-\beta^2)R^2{k_s}^2]^{3/2}}dx \qquad (46)$$

$$A_{n,\phi}(R,k_s)=\begin{cases}\dfrac{ie\gamma\beta}{8\pi^2\sqrt{2\varepsilon_0}{r_0}^2}\left(\dfrac{\alpha\beta}{2}\right)\left(\dfrac{R}{r_0}\right), & \dfrac{R}{r_0}\ll 1\\ \dfrac{(-1)^{n+1}}{n^2}\dfrac{3ie\gamma}{\pi\sqrt{2\varepsilon_0}\alpha^2\beta{r_0}^2}\left(\dfrac{r_0}{R}\right)^4, & \dfrac{R}{r_0}\gg 1\end{cases} \qquad (47)$$

$$A_{n,z}(R,k_s)=\frac{k_s}{2\pi}\int_{-\frac{\pi}{k_s}}^{\frac{\pi}{k_s}}\sqrt{\frac{\varepsilon_0}{2}}E_z(R,z)exp(-ink_sz)dz$$

$$=\frac{k_s}{2\pi}\sqrt{\frac{\varepsilon_0}{2}}\int_{-\frac{\pi}{k_s}}^{\frac{\pi}{k_s}}\frac{e}{4\pi\varepsilon_0}\frac{1-\beta^2}{[1-(\beta\sin\theta)^2]^{3/2}}\frac{\cos\theta}{r^2}[\cos(nk_sz)-i\sin(nk_sz)]dz$$

$$=-i\frac{e(1-\beta^2)k_s}{4\pi^2\sqrt{2\varepsilon_0}}\int_{z_0}^{\frac{\pi}{k_s}}\frac{z\sin(nk_sz)}{[z^2+(1-\beta^2)R^2]^{3/2}}dz$$

$$=-i\frac{e(1-\beta^2){k_s}^2}{4\pi^2\sqrt{2\varepsilon_0}}\int_{k_sz_0}^{\pi}\frac{x\sin(nx)}{[x^2+(1-\beta^2)R^2{k_s}^2]^{3/2}}dx \qquad (48)$$

$$A_{n,z}(R,k_s)=\begin{cases}\dfrac{ien}{4\pi^2\sqrt{2\varepsilon_0}{r_0}^2}\left(\dfrac{\alpha\beta}{2}\right)^2 ln\left(\dfrac{\alpha\beta}{2}\right), & \dfrac{R}{r_0}\ll 1\\ \dfrac{(-1)^n}{n}\dfrac{ie}{2\pi\sqrt{2\varepsilon_0}\alpha\beta{r_0}^2}\left(\dfrac{r_0}{R}\right)^3, & \dfrac{R}{r_0}\gg 1\end{cases} \qquad (49)$$

From Equation (41), (45), (47) and (49), the electron wavefunction is composed of a transverse wave $\begin{pmatrix}A_{n,R}(R,k_s)\\A_{n,\phi}(R,k_s)\end{pmatrix}$ and a longitudinal wave $A_{n,z}(R,k_s)$. The wavefunction not only has the baseband frequency which corresponds to the De Broglie wavelength λ, but also has higher frequency components which correspond to the integer divided De Broglie wavelength of $\frac{\lambda}{n}$. When observed in the normal laboratory condition where $\frac{R}{r_0}\gg 1$, the intensities of the high frequency terms in the transverse wave decrease as $\frac{1}{n^4}$ and the intensities of the high frequency terms in the longitudinal wave decrease as $\frac{1}{n^2}$.

Compared with Equation (41), the wavefunction of a free moving electron in the traditional Quantum Mechanics becomes a special case which only takes the baseband frequency $n = 1$ and treats A_1 as a constant scaler.

We can also make the following observations,

- The spectrum of electron's EM field is a 1-D discrete signal in (k_x, k_y, k_z) frequency space described by the following function in which $\Phi_E(k), \Phi_B(k)$ are described by Equation (20) and Equation (26)
    - $\begin{pmatrix}\Phi_E(nk_s)\\ \Phi_B(nk_s)\end{pmatrix}, \quad n = 0, \pm 1, \pm 2 \cdots$
- The quantum effect, a wavefunction Equation (41) in (x, y, z) coordinate space, is due to the cause that the electron's spectrum in (k_x, k_y, k_z) frequency space is a discrete signal.
- From Equation (40), the sampling frequency is larger for smaller particle. The quantum effect is more severe for smaller particle than bigger particle.
- From Equation (40), the sampling frequency is larger for higher speed particle. So, quantum effect for high speed particle is more severe than that of low speed particle.

Even all the previous calculations are based on electron, they should apply to all leptons equally well. Among the 3 leptons: tau, muon and electron, the only difference is the mass. Based on Equation (7), the heavier particle has smaller radius, thus muon should be smaller than electron. This seems counter-intuitive, but it could easily explain the "proton radius puzzle" [26]. In that experiment, researchers used muon to replace electron to form a heavier hydrogen atom. By measuring the spectroscopy of the atom, they could deduce the proton's radius. In theory this radius should be the same as that measured by using electron as probe because both electron and muon are treated as a singular point in physics. But the results show that the proton radius measured by muon is smaller. A similar experiment on muonic deuterium confirms that the finding is not isolated [27]. From Equation (7), the heavier muon is smaller, so it is more approximate to a singular point than electron is. Using it as probe, muon is more accurate than electron.

If above reasoning is true, then using the heaviest lepton tau to measure the proton radius would result in an even smaller value.

To test the validity of the new theory presented in this paper, the following experiment is proposed.

Use Equation (8), (9) and (40), Equation (45), (47) and (49) can be written in the following lab friendly form. At the normal lab condition $\frac{R}{r_0} \gg 1$,

$$A_{n,R} = \frac{(-1)^{n+1}}{n^2}\frac{3\gamma^2}{4\pi^2}\sqrt{\frac{\varepsilon_0}{2}}E_0\left(\frac{\lambda}{r_0}\right)^2\left(\frac{r_0}{R}\right)^4 \qquad (50)$$

$$A_{n,\phi} = \frac{(-1)^{n+1}}{n^2}\frac{3i\gamma^2}{4\pi^2}\frac{B_0}{\sqrt{2\mu_0}}\left(\frac{\lambda}{r_0}\right)^2\left(\frac{r_0}{R}\right)^4 \qquad (51)$$

$$A_{n,z} = \frac{(-1)^n}{n} \frac{i}{2\pi} \sqrt{\frac{\varepsilon_0}{2}} E_0 \left(\frac{\lambda}{r_0}\right) \left(\frac{r_0}{R}\right)^3 \qquad (52)$$

In Equation (50), (51) and (52),

$$E_0 = \frac{e\gamma}{4\pi\varepsilon_0 {r_0}^2} \qquad (53)$$

$$B_0 = \frac{\mu_0 ec\beta\gamma}{4\pi {r_0}^2} \qquad (54)$$

$$\lambda = \frac{2\pi}{k_s} = \frac{h}{p} \qquad (55)$$

From Equation (50), (51), (52), (53) and (54), the intensity ratio of the n^{th} component to the base component is

$$\frac{\mathrm{I}(n)}{\mathrm{I}(1)} = \frac{|A_{n,R}|^2 + |A_{n,\phi}|^2 + |A_{n,z}|^2}{|A_{1,R}|^2 + |A_{1,\phi}|^2 + |A_{1,z}|^2} = \frac{1}{n^2} \frac{1 + \frac{1}{n^2}(1+\beta^2)\left(\frac{3\gamma^2}{2\pi}\frac{\lambda}{R}\right)^2}{1 + (1+\beta^2)\left(\frac{3\gamma^2}{2\pi}\frac{\lambda}{R}\right)^2} \qquad (56)$$

People like to use the surface of crystalline as the diffraction grating to study the spectrum of X-ray and electron beam [5] [28] [29]. In the famous 1928 Davisson-Germer experiment, a monochrome electron beam, in which electrons having the fixed momentum as in Equation (55), was shot to the surface of nickel crystalline. By detecting the diffraction peak at the De Broglie wavelength, they proved the electron's wave property [5].

According to Equation (56), the electron wave not only has the De Broglie wavelength but also integer divided De Broglie wavelength. If we use a photograph plate to capture the diffraction patterns of a monochrome X-ray or electron beam, then between two major bright rings the electron's photograph will be less dark than the X-ray's photograph because there are minor peaks between major peaks in the electron's photograph. If we use a high-resolution electron detector to record the diffraction patterns, then these minor peaks should be detectable. An experiment setup which is like the Davisson-Germer experiment is depicted in Figure 5.

If the incident electron beam is perpendicular to the nickel surface, according to the diffraction formula [5], the m^{th} order diffraction peak from the n^{th} component of the electron wave is described by Equation (57),

$$m\frac{\lambda}{n} = D\sin\theta_{n,m} \qquad (57)$$

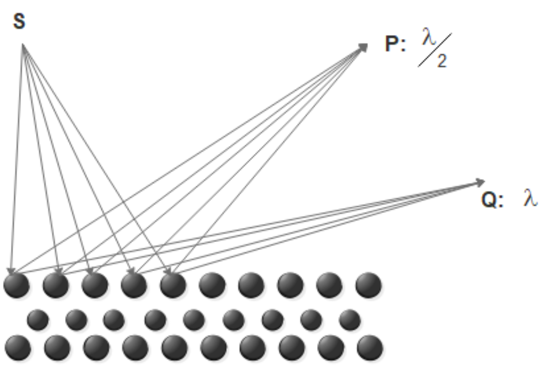
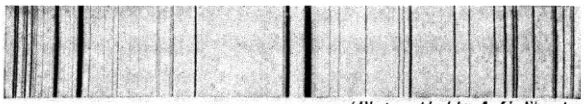


Fig. 5. Electron beam is diffracted by the nickel crystalline surface

For example, between the two major peaks $\theta_{1,1}$and $\theta_{1,2}$ from the De Broglie wavelength, there is a minor peak $\theta_{2,3}$ from the half of De Broglie wavelength. The relative intensity of the minor peak to the major peak is described by Equation (56) and the order of the peak.

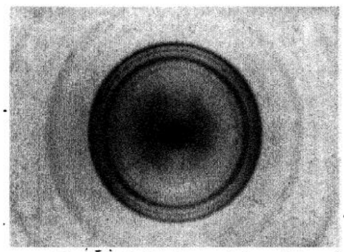
It is well known that the diffraction pattern of electron wave has more features than the diffraction pattern of X-ray. The photograph plates in the next page is directly copied from Einstein's book [20]. People commonly attribute the rich features in the electron diffraction photograph plate to the strong interaction between electron and atom [30]. However, can these patterns be simply explained by the sub-band signals (higher frequency components) of the electron wave? Only a careful experiment and data analysis can tell the truth. Before we discuss the difference between electron wave and photon wave, let us first study the photon wave.

PLATE III



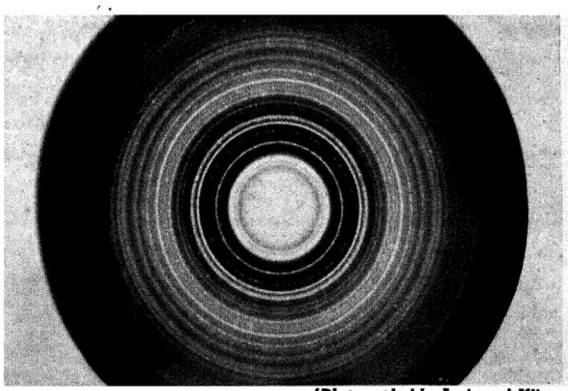
(*Photographed by A. G. Shenstone*)

Spectral lines



(*Photographed by Lastowiecki and Gregor*)

Diffraction of X-rays



(*Photographed by Loria and Klinger*)

Diffraction of electronic waves

6. Photon wavefunction

Since 1995, some researchers have realized that the photon wavefunction is EM wave of the complex vector field described by Equation (30) rather than the originally thought probability wave [23] [31] [33] [34]. Raymer and Smith demonstrate that when the photon wavefunction is described by Equation (30), the relativistic massless spin-1 Dirac equation [32] is equivalent to the Maxwell equations [33].

Although these researchers derived the correct photon wavefunction, they failed to solve the photon's local and non-local dilemma which is the manifest of the photon's particle and wave duality. For example, when a photon is emitted by an atom, it is created locally in space time. But when it travels, it propagates as a non-local monochrome wave. This dilemma has puzzled physicists since the Newton's era. Einstein wrote in 1951: "All these fifty years of pondering have not brought me any closer to answering the question, what are light quanta?"

People have tried to use the non-monochromatic wavelet to address the local issue for photon [34], but this approach contradicts the observed atomic spectroscopy in which an emitted photon is shown as a distinct spectral line in the spectrum.

It is very easy to solve this dilemma by the sampling theory. However, the formulas derived in section 5 for electron are not suitable for photon. For example, the sampling frequency in Equation (40) is meaningless for photon.

Because photon has the property of $\omega = c|\boldsymbol{k}|$, we can simplify the derivation of the photon's wave property in the 1-D time domain. In the time domain, when a photon is emitted, it is created as an excitation pulse which is an EM oscillation in a very short time interval. If we define the pulse width to be Δt, and the excitation frequency to be $\omega_0 = \frac{2\pi}{\Delta t}$, then the pulse is described by Equation (58).

$$f(t) = \begin{cases} Ae^{-i\omega_0 t}, & 0 \le t \le \Delta t \\ 0, & others \end{cases} \qquad (58)$$

The continuous spectrum of the pulse is

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{i\omega t}\,dt = A\int_{0}^{\Delta t} e^{i(\omega-\omega_0)t}\,dt = Ae^{i(\omega-\omega_0)\frac{\Delta t}{2}}\int_{-\frac{\Delta t}{2}}^{\frac{\Delta t}{2}} e^{i(\omega-\omega_0)t'}\,dt'$$

$$= A\Delta t e^{i(\omega-\omega_0)\frac{\Delta t}{2}}\frac{\sin\left((\omega-\omega_0)\frac{\Delta t}{2}\right)}{(\omega-\omega_0)\frac{\Delta t}{2}} \qquad (59)$$

Equation (59) is a modified $Sinc$ function centered at ω_0. If the frequency domain is discrete with the sampling frequency $\omega_s = \omega_0$, then the photon's discrete spectrum is

$$F(n\omega_s) = A\Delta t e^{i(n\omega_0-\omega_0)\frac{\Delta t}{2}}\frac{\sin\left((n\omega_0-\omega_0)\frac{\Delta t}{2}\right)}{(n\omega_0-\omega_0)\frac{\Delta t}{2}} = A\Delta t(-1)^{n-1}\frac{\sin\left((n-1)\pi\right)}{(n-1)\pi},$$

$$n = 0, \pm 1, \pm 2 \cdots \qquad (60)$$

Equation (60) is zero everywhere except at $n = 1$, so it is equivalent to the delta function

$$F(\omega) = A\Delta t\delta(\omega-\omega_0) \qquad (61)$$

One can immediately recognize that this is the spectrum of a monochrome wave. Apply the inverse Fourier transform,

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{-i\omega t}\,d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} A\Delta t\delta(\omega-\omega_0)e^{-i\omega t}\,d\omega = \frac{A\Delta t}{2\pi}e^{-i\omega_0 t} \qquad (62)$$

Through sampling, a localized EM excitation Equation (58) becomes a non-local monochrome wave Equation (62). This non-local wave is formed by the local excitation signal and its image replicas.

From the above derivation, the photon's frequency is related to the excitation pulse width $\nu = \frac{1}{\Delta t}$ and the sampling frequency happens to be the same.

$$\nu_s = \frac{1}{\Delta t} \qquad (63)$$

People may argue that the sampling theory and Fourier transform are mere mathematical tricks and have no physical meaning. I would like to view both as mathematical transformations which map different physical descriptions. For example, a 3-D rotation transformation is a mapping between the physical descriptions by two observers whose viewing angle is different.

Particle physicists have no trouble to describe the particle collision process in either the coordinate space or the frequency space. For example, in the frequency space, a virtual photon is represented by the photon propagator which is a function of the photon's 4-D frequency vector (or the 4-D momentum vector in natural unit) [35]. Fourier transform is the mapping between the physical descriptions from these two observing spaces.

Physicists do not have any trouble to describe physics in either the continuous term or the discrete term. For example, the EM field can be described as the continuous field in classical EM or a swarm of discrete photons in modern QED. The sampling theory is the mathematical transformation which maps the continuous description in coordinate space to the discrete description in frequency space. This transformation was discovered and formulated by the famous mathematician Claude Shannon [19]. It is worth mentioning that besides the sampling theory, Shannan also formulated the physical concept "entropy" in the rigorous mathematical language and introduced its usage to the modern information technology.

It would not be a surprise if we think of the Lorentz transform, which maps the descriptions of two inertial CS, to be a physical law imposed by Nature instead of a mathematical trick invented by Lorentz and Einstein. Similarly, the sampling theory and Fourier transform, which are the mapping between the continuous coordinate space and the discrete frequency space, are physical laws imposed by Nature instead of a mere mathematical trick invented by Shannon. These people are not inventors, they are discoverers who found certain mathematical transformations imposed by Nature.

Physics can be described in any observing space. A mathematical transformation is the mapping between the description from a pair of observing spaces. It is very popular for contemporary physicists to study different transformation groups to see the symmetries of Nature. Rotation group and Lorentz group are two good examples. The sampling theory should fall into this category because it maps the description from the continuous coordinate space and the description from the discrete frequency space, both of which are legitimate observing space to describe physics.

To derive the photon wavefunction in the 4-D space time, for an easy description without losing much generality, let us consider a special case in which a right-handed circular-polarized photon travels in the z direction so that $\boldsymbol{k} = k\hat{\boldsymbol{z}}$. The photon wavefunction is written in $\boldsymbol{E}$ field and $\boldsymbol{B}$ field separately,

$$\Psi_E(x,y,z,t) = \sqrt{\frac{\varepsilon_0}{2}}\boldsymbol{E}e^{i(kz-\omega t)} = \sqrt{\frac{\varepsilon_0}{2}}\frac{E_0}{\sqrt{2}}\begin{pmatrix}1\\-i\\0\end{pmatrix}e^{i(kz-\omega t)} = \frac{\sqrt{\varepsilon_0}}{2}E_0\begin{pmatrix}1\\-i\\0\end{pmatrix}e^{i(kz-\omega t)} \qquad (64)$$

$$\Psi_B(x,y,z,t) = \frac{i}{\sqrt{2\mu_0}}\boldsymbol{B}e^{i(kz-\omega t)} = \frac{i}{\sqrt{2\mu_0}}\frac{\boldsymbol{k}}{\omega}\times\boldsymbol{E}e^{i(kz-\omega t)} = \frac{i}{\sqrt{2\mu_0}}\frac{\hat{\boldsymbol{z}}}{c}\times\boldsymbol{E}e^{i(kz-\omega t)}$$

$$= i\sqrt{\frac{\varepsilon_0}{2}}\hat{\boldsymbol{z}}\times\frac{E_0}{\sqrt{2}}\begin{pmatrix}1\\-i\\0\end{pmatrix}e^{i(kz-\omega t)} = \frac{\sqrt{\varepsilon_0}}{2}E_0\begin{pmatrix}-1\\i\\0\end{pmatrix}e^{i(kz-\omega t)}$$

$$= -\Psi_E(x,y,z,t) \qquad (65)$$

From the earlier discussion, the photon wavefunction is formed by the photon's EM signal and its image replicas. When we calculate the photon's energy, we should only calculate the contribution from its own signal without counting its image replicas. Otherwise we would encounter the infinite value problem in Quantum Mechanics or in QFT and must use re-normalization to obtain a meaningful physical value.

In the z-dimension, the photon's EM signal extends $c\Delta t = \frac{c}{\nu} = \lambda$ length. In the x-y dimensions, the photon's EM signal apparently could not extend in the entire x-y plane because we know that a photon travels in a beam instead of crossing an infinite plane. Let us assume that in the x-y plane, Equation (64) and (65) are only valid within a circle $\sqrt{x^2+y^2} < R,\ R = \frac{\lambda}{2\pi}$. This is a reasonable assumption because we know that the cross-section of a laser beam is much smaller than that of a microwave beam. So, the photon's energy is confined in a cylinder and equals to [22]

$$\mathcal{E} = \int_0^\lambda dz\int_0^{2\pi} d\phi\int_0^R rdr\left(\frac{\varepsilon_0}{2}\boldsymbol{E}^*\cdot\boldsymbol{E} + \frac{1}{2\mu_0}\boldsymbol{B}^*\cdot\boldsymbol{B}\right) = \varepsilon_0{E_0}^2\pi R^2\lambda = \frac{\lambda^3}{4\pi}\varepsilon_0{E_0}^2 = h\nu \qquad (66)$$

From Equation (66), it is derived

$$\sqrt{\frac{\varepsilon_0}{2}}E_0 = \frac{2\pi}{\lambda^2}\sqrt{\hbar c} \qquad (67)$$

Combine Equation (64) or (65) and (67), the right-handed circular-polarized photon wavefunction becomes

$$\Psi_E(x,y,z,t) = \frac{2\pi}{\lambda^2}\sqrt{\frac{\hbar c}{2}}\begin{pmatrix}1\\-i\\0\end{pmatrix}e^{i(kz-\omega t)},\qquad \begin{matrix}\sqrt{x^2+y^2} < \dfrac{\lambda}{2\pi}\\ \omega = ck\end{matrix} \qquad (68)$$

$$\Psi_B(x,y,z,t) = -\Psi_E(x,y,z,t) \qquad (69)$$

The photon's momentum equals to [22]

$$\boldsymbol{P} = \int_0^\lambda dz\int_0^{2\pi} d\phi\int_0^R rdr\,\varepsilon_0\boldsymbol{E}^*\times\boldsymbol{B} = \int_0^\lambda dz\int_0^{2\pi} d\phi\int_0^R rdr\,\varepsilon_0\boldsymbol{E}^*\times\left(\frac{\hat{\boldsymbol{z}}}{c}\times\boldsymbol{E}\right) = \varepsilon_0{E_0}^2\frac{\hat{\boldsymbol{z}}}{c}\pi R^2\lambda$$

$$= \left(\frac{2\pi}{\lambda^2}\sqrt{2\hbar c}\right)^2\frac{\hat{\boldsymbol{z}}}{c}\pi\left(\frac{\lambda}{2\pi}\right)^2\lambda = \frac{h}{\lambda}\hat{\boldsymbol{z}} \qquad (70)$$

The photon's angular momentum equals to [36], in which $\boldsymbol{A}$ is the vector potential of the EM field,

$$\boldsymbol{L} = \int_0^\lambda dz\int_0^{2\pi} d\phi\int_0^R rdr\,\varepsilon_0\boldsymbol{E}^*\times\boldsymbol{A} = \int_0^\lambda dz\int_0^{2\pi} d\phi\int_0^R rdr\frac{\varepsilon_0}{i\omega}\boldsymbol{E}^*\times\boldsymbol{E} = -\varepsilon_0{E_0}^2\frac{\hat{\boldsymbol{z}}}{\omega}\pi R^2\lambda$$

$$= -\left(\frac{2\pi}{\lambda^2}\sqrt{2\hbar c}\right)^2\frac{\hat{\boldsymbol{z}}}{\omega}\pi\left(\frac{\lambda}{2\pi}\right)^2\lambda = -\hbar\hat{\boldsymbol{z}} \qquad (71)$$

So, the classical right-handed circular-polarized photon is a spin -1 photon. Similarly, the classical left-handed circular-polarized photon is a spin 1 photon and its wavefunction are

$$\Psi_E(x,y,z,t) = -\Psi_B(x,y,z,t) = \frac{2\pi}{\lambda^2}\sqrt{\frac{\hbar c}{2}}\begin{pmatrix}1\\ i\\ 0\end{pmatrix} e^{i(kz-\omega t)}, \qquad \begin{matrix}\sqrt{x^2+y^2} < \dfrac{\lambda}{2\pi} \\ \omega = ck\end{matrix} \tag{72}$$

Let us compare Equation (72) and the planar wavefunction Equation (38) which is the photon's wavefunction described by Quantum Mechanics. Both functions are valid solution to the photon's field equation: the Maxwell equations or the relativistic massless spin-1 Dirac equation [32].

Even Equation (72) is composed of the localized photon's EM field (one specific period in the z direction) and its image replicas, mathematically it is in-distinguishable from the continuous harmonic solution which is extended in space time.

Equation (38) is uniform in the entire x-y plane and Equation (72) is confined within a circle in the x-y plane. The reason for Equation (38) being uniform in the entire x-y plane is due to the uncertainty principle in Quantum Mechanics [7]. Because the photon travels in the z direction, $p_x = p_y = 0$, thus $\Delta p_x = \Delta p_y = 0$. From the uncertainty principle, $\Delta p_x \Delta x \geq \frac{\hbar}{2}$ and $\Delta p_y \Delta y \geq \frac{\hbar}{2}$, thus the photon must be uniformly distributed in the entire x-y plane. But this is not true because we know that the photon travels in a beam. Let us say a photon travels from a distant star to our observing telescope, its traveling direction can be precisely determined. Let us denote it as the z direction. If the photon is uniformly distributed in the entire x-y plane, then there is no way that our telescope can form the star's image. So, Equation (72) is more realistic than Equation (38) to represent the photon wave.

Now let us compare the electron wavefunction found in section 5 and the photon wavefunction found in this section.

Both the electron wave and the photon wave are pseudo EM waves. For the photon wave, because the field divergence $\nabla \cdot \boldsymbol{E} = 0$ and $\nabla \cdot \boldsymbol{B} = 0$, it is a transverse wave. For the electron wave, it is composed of a transverse wave and a longitudinal wave. The photon wave is monochrome with a single wavelength. The electron wave is polychrome with De Broglie wavelength and integer divided De Broglie wavelength. The intensity of the shorter wavelength terms in the electron wave drop significantly as the wavelength becomes shorter. The photon wave propagates at the speed of light. The electron wave propagates at the electron's moving speed.

When people think about particle and field, they normally view a particle to be a tiny object within a boundary surface and field is in dispersion without a boundary. But there is no reason we must think this way. If we think in the opposite way, a particle in dispersion and field with a boundary, then the distinction between particle and field becomes very blur.

In this study we treat a particle as a blob of EM field. Electron is its EM field with an inner boundary surface, Equation (13). Photon is the oscillating EM field within a cylinder, Equation (72).

To treat a particle as field is nothing new. In QFT, a particle is simply a field quantum which is a wavefunction in the extended space [14]. The wavefunction is a unit solution of the partial differential field equation which describes the dynamics of the field. In QFT, the entire wavefunction is treated as a physical field quantum.

However, in our treatment we only consider the localized particle's field as physical and the complete wavefunction is formed by the particle's field and its non-physical image replicas. Among all the image replicas, it is very difficult to point which image corresponds to the physical energy carrying particle without measurement. This can easily lead people to think that the particle moves randomly and must be described by the probability theory.

We call the photon wavefunction the pseudo EM wave because even it is described as the monochrome EM wave, only one period in the moving direction is physical. On the other hand, the classical monochrome EM wave or the field quantum in QED is physical along the entire moving direction. As pointed out in section 1, it is very difficult to treat the entire wavefunction as physical because when a photon travels from the Sun to Earth, even its wavefunction can instantly reach us, we still need to wait 8.3 minutes for the physical packet to arrive. Same argument applies to the electron wavefunction which is a polychrome pseudo EM wave.

It is very interesting to note that although the image replicas of the particle's field are non-physical, when overlapped they will cause interference. This effect is called signal aliasing in DSP [21]. A good example is the Moiré pattern in digital images. When people digitize a conventional photo to produce a digital copy, according to the famous Nyquist theorem in DSP [21], if the sampling frequency is less than twice of the maximum frequency in the image, then the Moiré pattern will appear in the produced digital image. This pattern is caused by the interference of the overlapping between the spectrum of the image and its replicas.

Single photon double slit experiment is probably the most popular experiment in Quantum Mechanics. It is described by almost every textbook of Quantum Mechanics. However, none of the book provides any reasonable explanation of the experiment. The following description is taken from Feynman's book [37]. An explanation is provided thereafter.

In Figure 6, two tiny holes (at A and B) in a screen that is between a source S and a detector D let nearly the same amount of light through (in this case 1%) when one or the other hole is open. When both holes are open, interference occurs; the detector clicks from zero to 4% of the time, depending on the separation of A and B, shown in Figure 8 (a). [37]

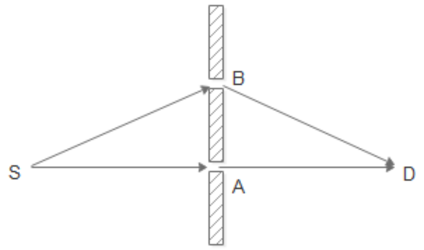


Fig. 6. Single photon double hole without detector in the path

In Figure 7, when special detectors are put in at A and B to tell which way the light went when both holes are open, the experiment has been changed. Because a photon always goes through one hole or the other (when you are checking the holes), there are two distinguishable final conditions: 1) the detectors at A and D go off, and 2) the detectors at B and D go off. The probability of either event happening is about 1%. The probabilities of the two events are added in the normal way, which accounts for a 2% probability that the detector at D goes off, shown in Figure 8 (b). [37]

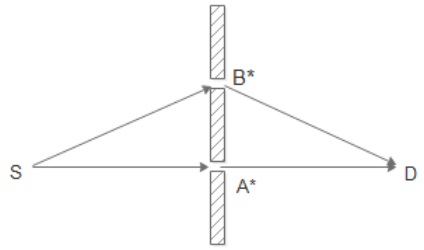


Fig. 7. Single photon double hole with detectors in the path

In Figure 8, when there are no detectors at A or B, there is interference: the amount of light varies from zero to 4% (a). When there are detectors at A and B that are 100% reliable, there is no interference: the amount of light reaching D is a constant 2% (b). When the detectors at A and B are not 100% reliable (i.e., when sometimes there is nothing left in A or in B that can be detected), there are now three possible final conditions: A and D go off, B and D go off, and D goes off alone. The final curve is thus a mixture, made up of contributions from each possible final condition. When the detectors at A and B are less reliable, there is more interference present. Thus, the detectors in case (c) are less reliable than in case (d). [37]

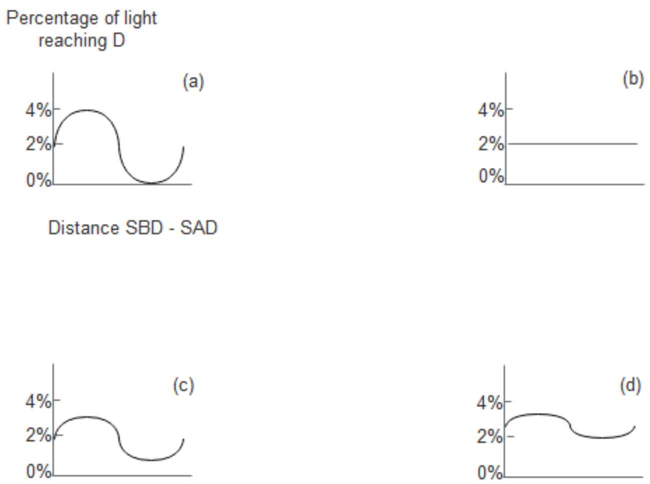


Fig. 8. Percentage of light reaching detector D

Here is an easy explanation to the above strange photon behaviors based on the new theory presented in this paper. A photon can only go through one hole whether you observe it or not because it cannot be divided. However, its image replicas can certainly go through the other hole and interfere with the photon at detector D. Because the phases of the photon and its image replicas are precisely synchronized if no detection occurs, the so-called Quantum Coherence, the interference pattern in (a) is observed.

When the detector at A or B registered a detection, the interaction between the photon and the detector caused the Quantum Coherence suddenly to collapse, the so-called Quantum Collapse. Without phase synchronization, there is no interference, pattern (b) is observed.

Absence of interference pattern is not directly caused by measurement but because of lack of Quantum Coherence, or Quantum Collapse which is caused by measurement. Without Quantum

Collapse, interference pattern still appears. Altering the detector's reliability simply changes the percentage rate of Quantum Collapse, thus produces patterns in (c) or (d).

In Feynman's description, there are two types of act from the particle. The first type, such as when the photon passes through one hole without a detection, Quantum Coherence is preserved. The second type, such as when the photon interacts with the detector, Quantum Collapse happens. Both types of act happen constantly in Nature. For example, when an electron moves in a static EM field such as inside a hydrogen atom, Quantum Coherence is preserved so that the electron always interferes with its image replicas. This causes the electron can only occupy certain "orbits" because if the electron wavefunction does not form a standing wave then the interference will cancel out the entire wavefunction. A vivid picture in the book "Fearful Symmetry" shows a French prince watching the electron wave going around an atomic nucleus: The wavelength is quantized because the electron wave has to catch its tail after going around [38]. When an electron moves in a cloud chamber, Quantum Collapse constantly happens when it collides with a gas molecule. This prevents the interference from happening so you can only observe the electron's classical trajectory path.

If we can replace the detectors in Figure 7 with weak measurement [39] [40], the measurement is so weak that a detection won't destroy the Quantum Coherence of the photon, then we should be able to know which hole it passed through and in the meantime observe the interference pattern. Such experiment was indeed carried out in 2011 [16] and the experiment result perfectly matches our "prediction". Recently a group of Yale researchers even went a step further to use the weak measurement to predict and reverse a quantum jump [41].

These experiments clearly demonstrate that the particle's behavior is not random. Thus, the wavefunction cannot be interpreted as the probability wave. To accommodate the new experiments, in 2017 Yakir Aharonov et al proposed a new interpretation on Quantum Mechanics [17]. They chose Heisenberg's operator representation over Schrödinger's wave representation on Quantum Mechanics even the two had been proved to be equivalent. Their explanation is "instead of a quantum wave passing through both slits, we have a localized particle with nonlocal interactions with the other slit. Key to this explanation is dynamical nonlocality, which naturally appears in the Heisenberg picture as nonlocal equations of motion" [17]. In another word, "the particle has both a definite location and a nonlocal modular momentum that can sense the presence of the other slit and therefore, create interference" [17]. However, such particle's remote-sensing capability is as spooky as the probability wave.

When scrutinize closely, there are two types of physical process in Quantum Collapse. The first type, a particle is annihilated and created such as when a photon is absorbed and emitted by an atom. The second type, a particle transits from one coherent state to another coherent state such as when a photon is scattered by an electron in the Compton scattering process [42]. QFT treats both types in the same way: particle annihilation and creation [35]. In the transitional period, there is no phase synchronization between the particle and its image replicas, or no Quantum Coherence, so we cannot observe any interference pattern. Thus, Quantum Collapse always happens locally in space and time. In the rest of the paper, we take the QFT's notion on Quantum Collapse: a particle is annihilated.

7. Quantum Mechanics and Special Relativity

In section 5, in the second CS, the moving electron exhibits a cylindrical wave Equation (41) due to the discrete nature in its frequency space. However, the second CS was chosen quite arbitrarily. We could have chosen the second CS moving in any other direction and in any speed relative to the first CS, and the wave will then travel in that chosen direction. This means that the quantum effect is a relativistic effect.

More than a century ago before Special Relativity had been accepted, people used to have the following assumptions about space and time,

1) The space and time are absolute and independent.
2) The space and time are continuous.

In Special Relativity, Albert Einstein shows that 1) is not true. The classical transform must be replaced by Lorentz transform. Classical Mechanics only becomes valid when the speed is close to zero.

In this paper it is demonstrated that 2) is not true either. The space time (x, y, z, t) is not continuous in a way that its frequency counter-part (k_x, k_y, k_z, ω) is discrete in nature. It only becomes continuous when the speed is close to zero as shown by Equation (40). The quantum effect just reveals another relativistic aspect of space time.

From Lorentz transform and the sampling frequency of $k_s = \frac{\alpha\beta\gamma}{2r_0}$ for a massive particle, it is noted that only the spatial dimension of the object's moving direction and time dimension are affected by the relativistic effect and the quantum effect.

Conventional thinking on the coordinate space is that it is a reference frame in which a particle occupies and travels through. A new thinking introduced in this paper is that the coordinate space is the reciprocal space of the frequency space (or energy space) and a particle in the coordinate space is the projected image from the frequency space.

In Classical Mechanics, the frequency space is treated as continuous due to the scale of the object being considered. The projected image of the object in the coordinate space is unique and the object's traveling path is deterministic. In Quantum Mechanics, the frequency space is discrete due to the scale of the particle being considered. The projected images of the particle in the coordinate space is infinite and the particle travels as a propagating wave. The infinite number of images are all aliases to the same particle and are precisely synchronized. The particle's wavefunction can instantly collapse without information being passed through distance. Information can never travel faster than the speed of light.

According to Equation (40), at slow speed the sampling frequency is so small that the particle's wave phenomenon can be safely ignored. Classical Mechanics is a good approximation to describe the particle's motion.

According to Equation (63), for the static EM field or the slowly time-varying EM field, the sampling frequency is so small that the EM field can be safely treated as continuous field. Classical EM theory is a good approximation to describe the EM field. This is the reason why in the blackbody radiation the Rayleigh-Jeans law is a good approximation at the low frequencies [22].

When Einstein postulated the EPR paradox, one of the intentions is to prove that Quantum Coherence and Quantum Correlation are not random and cannot be explained by the probability theory. The violation of Bell Inequality theorem indeed proves that the Quantum Correlation cannot be explained by the statistical correlation. We follow their leads to further prove that the particle's coherent wavefunction is a relativistic effect instead of a statistical effect.

If a particle (electron or photon) is treated as a blob of EM field, then its physical properties (mass, charge, and spin) are just the aggregate properties of this blob of field. In Quantum Mechanics or QFT, a particle is represented by these abstract properties, its state representation is a vector in the mathematically abstract Hilbert space: a wavefunction in Quantum Mechanics or a field quantum in QFT. The interpretation of such abstract representation becomes very difficult. On the other hand, if a particle is treated as a blob of EM field, its wavefunction is formed by this physical field and the non-physical image replicas of this field, then the interpretation of such representation becomes easy to understand.

In the physical representation, the state dynamics of a particle (electron or photon) is described by the classical Maxwell equations [22]. In the abstract representation, the state dynamics of the particle is described by the relativistic Dirac equation [32]. Even these two descriptions look so different, they are mathematically equivalent as some studies have shown [23] [31] [33] [34].

Among the three popular interpretations on matter wave or quantum wavefunction in technical term, which are described in section 1, the field quantum in QFT has the most resemblance to the pseudo EM wave which is our new interpretation.

QFT describes the field quantum in both the continuous 4-D coordinate space and the continuous 4-D frequency space. A free field quantum is represented by a planar wave of complex vector for both electron and photon. In QFT, the complete wavefunction is treated as physical [14][35]. This makes it very hard to explain Quantum Collapse. How does the physical giant which is extended in the entire Universe to collapse instantly?

The pseudo EM wave is composed of one physical copy (particle's physical field) and infinite number of non-physical copies (image replicas of the particle's field). Because both the physical copy and the non-physical copy interfere in the same way, the pseudo EM wave and field quantum behave the same in terms of propagation and interference.

If we cut the field quantum into multiple segments each of which is one wavelength long, treat only one segment as physical and all the other segments as non-physical aliases, then collapsing the physical copy will collapse them all. The collapsing process is not instant but in a finite time interval of $\Delta t = \frac{\lambda}{c} = \frac{1}{\nu}$. All these operations will naturally emerge if we treat the frequency space as the discrete space due to the sampling theory. If so, Quantum Collapse can be easily explained, the pseudo EM wave and field quantum will be the same.

8. Conclusion and discussion

By using electron, it is demonstrated that an elementary particle is not a singular point but a distribution function of its EM field. The particle can be described in either the coordinate space or the frequency space. Because the frequency space has limited resolution, a particle can only be described as a 1-D discrete signal in the frequency space. This 1-D discrete signal when projected onto the coordinate space becomes a 3-D continuous wavefunction. The link between the 1-D discrete signal in the frequency space and the 3-D continuous wavefunction in the coordinate space is the sampling theory.

The elementary particle electron is the center of mass of its EM field. The electron wavefunction is pseudo EM wave formed by the electron's EM field and its image replicas.

A photon wave is a monochrome pseudo EM wave formed by the photon's EM excitation signal and the image replicas of the excitation signal due to the sampling in the frequency space.

The quantum effect, particle and wave duality, reveals that the coordinate space is not continuous in nature. Along the particle's moving dimension, the metric not only shrinks but also becomes discrete. When explained by the sampling theory, Quantum Mechanics is not only compatible with Special Relativity, together they reveal two important aspects of space time. The nature of space time is not only governed by Lorentz transform, its discrete nature is also revealed by Equation (40) for a massive particle and Equation (63) for a massless particle.

Quantum Coherence, the phase synchronization of a particle's wavefunction in the extended space, is because of the infinite number of image replicas of the same particle. These images are all aliases to the same particle and are precisely synchronized in motion.

Quantum Collapse, the instant collapse of the particle's wavefunction upon measurement, is because when the particle is absorbed by the detector, all its image replicas disappear with its own image synchronously. These images are all aliases to the same particle and cease to exist precisely at the same time.

Quantum Entanglement, a pair of entangled particles having a Quantum Correlation, is the manifest of Quantum Coherence and Quantum Collapse in the case of two entangled particles.

When people first developed Quantum Mechanics, they discovered two phenomena: a particle's physical properties such as energy and momentum can only have discrete values; a particle's appearance exhibits a pattern of repetition. Through the sampling theory, a link between the two phenomena is provided. In this paper it is demonstrated that these two phenomena are in fact two manifests of the same phenomenon, one in the frequency space (discrete) and one in the coordinate space (pattern repetition).

Quantum Coherence and Quantum Collapse are also the same phenomenon in two different phases of a particle's life. Quantum Coherence is the synchronization in motion of all the image aliases of the same particle. Quantum Collapse is the synchronization in death of all the image aliases of the same particle.

Finally, we should point out that the study presented in this paper only aims to provide a reasonable explanation to the behavior of Quantum Mechanics, mathematically speaking it is equivalent to Quantum Mechanics. In the coordinate space, the particle's continuous wavefunction propagates and interferes in the same way as that described by Quantum Mechanics. If the frequency space is viewed as an eigenspace, then a photon's state is a single state which is represented by one sample and an electron's state is a superposition state which is represented by a series of samples.

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