

Original Paper

The Reality of Quantum-Events and the Existence of a Cosmological Constant

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Abstract: Collapse theories of quantum mechanics assume that quantum-events in space-time are real phenomena. In the causal-set picture events spread stochastically, and in the transactional picture they are additionally accompanied by the exchange of gauge bosons. We base on elements of these pictures and use the emitted gauge-bosons as clocks. We show that, in case of exchanged photons, the synchronization of the corresponding clocks in a gravitational field is governed by Einstein's equations including a cosmological term Λ .

Keywords: Quantum-event; general relativity; Einstein equations; cosmological constant

1. Introduction

Recent years have seen the development of theories of quantum-events [1 – 5], which take the collapse of the state vector for a real phenomenon. The theory of causal-sets models events as a stochastic process [4,5], while the transactional approach additionally bases on interactions and connects an event to the exchange of four-momentum by gauge bosons [1,2]. It is beyond our means to honor all of the work done in the field, and we will use elements of the two above-mentioned theories. In [6] it is shown that, if the exchanged bosons are photons, it is possible to use them as clocks in order to locally measure the duration of events. The synchronization of these light-clocks in a gravitational field leads to a dynamic space-time metric, governed by the Einstein equations. The result makes use of the energy-exchange, i.e. of the zero-component of the transferred four-momentum. In [7] the cosmological constant Λ is brought into connection with fluctuations in the growth-process of causal-sets and in [8] parallels are drawn to the transfer of 3-momentum in transactions. In this paper we aim to show that the correct equations, which govern the synchronization of local light-clocks under full account of the exchanged four-momentum, are indeed Einstein's equations with a cosmological term.

The article is structured as follows: in paragraph 2 we briefly introduce the interpretative background and the basic facts in [6], needed to formulate the main result, which we give in paragraph 3. In paragraph 4 we summarize, and the appendices contain related information about Lorentz manifolds and local gravity.

2. Background and Principles

2.1. Transactions and Light Clocks

In the transactional interpretation [1] a quantum state $|\psi\rangle$ is launched as an "offer-wave" by an emitter and gets possible responses by "confirmation-waves" $\langle\psi_i|$, $i \in I$, which are the projections of $\langle\psi|$ onto absorbers. The selection of a particular confirmation $i_0 \in I$ leads to a "transaction" which is the actualization of emission and absorption as real events in space-time. The specific probability for a particular transaction $i_0 \in I$ is $|\langle\psi_{i_0}|\psi\rangle|^2$. The relativistic transactional interpretation [2,3] adds to this the explanation, why offer-waves (and confirmation-waves) are actually created. Relativistic interactions can be thought of as the mutual exchange of virtual bosons, creating possibilities in a pre space-time process. Transactions in turn, are triggered by the exchange of real bosons. The amplitude for emission or absorption of real bosons is the coupling amplitude between the fields. "Space-time" thus becomes the connected set of emission-and absorption events corresponding to actualized transactions, which define, by the four momentum of the exchanged boson, a time-like (or null) space-time interval whose end points are the emission and absorption events. It is here, where the transactional view touches causal-set theory, in which events spread in space-time by a stochastic Poisson-process. Boson-exchange, understood as a decay-process, is of Poissonian nature..¹ We will in the sequel focus on the electromagnetic force and the related exchange of photons. Our mathematical result does not depend upon the details of either interpretation. They just form a background, which allows an understanding of what is physically going on.

It takes a (closed and isolated) quantum system, represented by a vector in Hilbert space, $|\psi_0\rangle \in H_{\mathbb{C}}$, with average energy \bar{E} and lowest energy-value E_0 , minimally a time of

$$\bar{\Delta t} = \frac{h}{4(\bar{E} - E_0)}, \quad (1)$$

in order to unitarily evolve to an orthogonal state $|\psi_1\rangle$, $\langle\psi_0|\psi_1\rangle = 0$, ($h \stackrel{\text{def}}{=} \text{Planck's constant}$) [9]. We can use such a system as a clock² with period $\bar{\Delta t}$. Special interest will lie on the case, where a system is emitting (or absorbing) a photon of energy $E = h\nu$. The corresponding light-clock has period (1)

$$\bar{\Delta t} = \frac{1}{4\nu}. \quad (2)$$

We will encounter the situation, where there is not a single photon but many over a range of frequencies in thermal equilibrium (radiation), and where the energy is given by a temperature T . For oscillators with $\bar{E}_\nu \approx k_B T$ ($h\nu \ll k_B T$, $k_B \stackrel{\text{def}}{=} \text{Boltzmann constant}$) we get a corresponding clock with period

$$\bar{\Delta t} = \frac{h}{4k_B T}. \quad (3)$$

¹ The transactional interpretation thinks of space-time slightly different than the causal-set-approach does. This has no impact on our mathematical result.

² We actually use it as the "core" of a clock.

We call the special light-clock (3) a thermal clock.

2.2. Minkowski Space-Time

For any photon in vacuum the ratio between its energy E and its 3-momentum $p = |\vec{p}|$ is a constant, namely the speed of light c

$$\frac{E}{p} = c. \quad (4)$$

Equation (4) is a quantum-identity and, if expressed in space-time, must hold in every inertial reference frame. If we write energy and momentum in space-time coordinates, we get $E = h\Delta\nu = \frac{h}{\Delta t}$ and, by the de Broglie-relation, $p = \frac{h}{\Delta x}$. Therefore (4) takes the form

$$\frac{E}{p} = \frac{\Delta x}{\Delta t} = c. \quad (5)$$

Since equation (5) must hold in every inertial reference frame $\bar{x} = (t, x) \in \mathbb{R}^4$, it constrains the metric resulting in Minkowski space-time \mathbb{M}^4 with linear isometries $O(1,3)$, the Lorentz transformations. As indicated in paragraph 2.1, we take the ontological standpoint that quantum-systems spontaneously break the unitary time-evolution through the exchange of real bosons and thus become manifest in space-time. This is what we call "events" or synonymously "actualizations". The kind of bosons depends upon the force in action. So we can think of space and time as distinct attributes of matter, represented by a four-dimensional continuum, which adopts its metric structure by the "sprinkling" of matter through events.

The concept of a thermal clock (3) unfolds its power, if we consider multiple events of interacting quantum-systems. Multiple events manifest themselves in space-time by acceleration. In \mathbb{M}^4 physical systems of constant acceleration κ in x -direction, say, can be expressed in Rindler-coordinates. This happens by choosing a co-moving coordinate system, defined in the wedge limited by $|x| = t$, and given by the transformations

$$x = \varrho \cosh(\kappa\vartheta), \quad t = \varrho \sinh(\kappa\vartheta), \quad \varrho \geq 0, -\infty < \vartheta < \infty. \quad (6)$$

The corresponding line-element is

$$ds^2 = \left(\frac{\kappa\varrho}{c^2}\right)^2 c^2 d\vartheta^2 - d\varrho^2 - dy^2 - dz^2. \quad (7)$$

Contrary to velocity, acceleration is not purely perspectival and cannot be transformed away by a Lorentz transformation. But there is a local inertial reference-frame at $t = 0$, where the system is instantaneously at rest. Assume, that in this reference-frame there is a thermal bath of temperature T , and we want to gauge proper time by a corresponding thermal clock. By (3) and (7) we get

$$d\tau = \frac{ds}{\Delta t} = \frac{4}{h} k_B T \frac{c^2}{\kappa} d\vartheta. \quad (8)$$

We now want to synchronize³ (8) with a quantum-clock, defined by a matter-wave with rest mass m_0 , frequency $\omega = 2\pi\nu$ and corresponding acceleration κ_ω . In its respective oscillatory rest-frame and for $m_0 \ll \frac{\hbar\omega}{c^2}$, the matter-clock measures time in units of (8)

$$d\tau_\omega = \frac{4}{\hbar} E_\omega \frac{c^2}{\kappa_\omega} d\vartheta. \quad (9)$$

By the de Broglie-relation there holds with $k = |\vec{k}|$ denoting the wave vector

$$E_\omega^2 = \hbar^2 \omega^2 = c^2 \hbar^2 k^2 + m_0^2 c^4. \quad (10)$$

Further with $u_\omega = \frac{\omega}{k}$ and $v_\omega = c^2 \frac{k}{\omega}$ denoting the phase-and group velocity, respectively, we have

$$\kappa_\omega = 2\pi u_\omega \omega. \quad (11)$$

By (11) equation (9) turns into

$$d\tau_\omega = \frac{4}{\hbar} \hbar \omega \frac{c^2 k}{2\pi \omega^2} d\vartheta = \frac{c^2 k}{\pi^2 \omega} d\vartheta = \frac{v_\omega}{\pi^2}. \quad (12)$$

If we synchronize the two clocks, $d\tau = d\tau_\omega$, we therefore get

$$\frac{4}{\hbar} k_B T \frac{c^2}{\kappa} = \frac{v_\omega}{\pi^2}. \quad (13)$$

For the temperature T this implies

$$T_{k,\omega} = \frac{\hbar \kappa v_\omega}{2\pi k_B c^2}. \quad (14)$$

Expression (14) is a generalized Davies-Unruh temperature. If we choose a massless wave ($m_0 = 0$), then we are in the situation $u_\omega = v_\omega = c$ and (14) turns into the familiar Davies-Unruh formula [10,11]

$$T_k = \frac{\hbar \kappa}{2\pi k_B c}. \quad (15)$$

3. Gravitation

Let us now consider gravitational acceleration, $\kappa = g_R$, in the Newtonian limit at distance R of a mass M at relative rest. With G denoting the gravitational constant we have

$$g_R = \frac{GM}{R^2}. \quad (16)$$

³ By the term "synchronization" we just understand equality of periods.

A derivation of the gravitational acceleration g_R , which fits well into the context of our work, is explained in [12], where gravitation can be interpreted as an emerging entropic force, resulting from events (see appendix B).

3.1. Energy-density

Let a test-system at small distance R be actualized by exchanging photons with M and feel the acceleration g_R . The energy-emission by the photons appears in the local rest-frame of the system as an emission from a heat bath in the environment. The temperature is T_{g_R} (15), since the period of the corresponding thermal clock should be synchronized with the one of the corresponding light-clock (9). This amounts by (13) to the equation

$$\frac{4}{h} k_B T_{g_R} \frac{c}{g_R} = \frac{1}{\pi^2}. \quad (17)$$

With $E = Mc^2$, $l_P = \sqrt{\frac{G\hbar}{c^3}}$ (Planck length), and $A_R = 4\pi R^2$ we derive from (17)

$$k_B T_{g_R} A_R = 4l_P^2 E. \quad (18)$$

By using (15), we arrive at

$$g_R A_R = \frac{4\pi G}{c^2} E. \quad (19)$$

In the sequel we will continue to work in the local inertial coordinate-chart around the origin (M), and develop Einstein's equations for the $oo(tt)$ -component. This will suffice to reveal the structure of the equations. The general equations can be derived, for instance, by assuming to work in an asymptotically flat manifold with a global time-like Killing field and by using generalized expressions for the terms in (19), as done in [12] (see appendix A). With $V_R(t)$ denoting the volume of a small ball of test-systems at radius $R(t)$ around the origin, with $R(0) = R$, $\dot{R}(0) = 0$ and $\ddot{R}(0) = g_R$, we can re-write (19) as [13,14]

$$\left. \frac{d^2}{dt^2} \right|_{t=0} V_R = \frac{4\pi G}{c^2} E. \quad (20)$$

If we introduce the energy-momentum tensor T_{ab} with zero-component $T_{00} = \frac{E}{V_R}$, denoting the energy density in V_R , and use the local properties of the Ricci tensor R_{ab} , we have at the origin [13,14]

$$\left. \frac{\ddot{V}_R}{V_R} \right|_{t=0} \xrightarrow{R \rightarrow 0} c^2 R_{00}, \quad (21)$$

and (20) turns into

$$R_{00} = \frac{4\pi G}{c^4} T_{00}. \quad (22)$$

3.2. Momentum-flow

In the transactional picture there is a transfer of four-momentum through photons coinciding with an event. Let us call this momentum-transfer "event-radiation". In order to synchronize local light-clocks (22) we have so far only made use of the energy (zero)-component of event-radiation (9,17). From the 3-momentum there arises a pressure, which defines the Laue-scalar

$$T = \sum_{i=1}^3 T_{ii} = \sum_{i=1}^3 \frac{F_i}{A_i} = \sum_{i=1}^3 \frac{1}{A_i} \frac{dp_i}{dt}. \quad (23)$$

This quantity also contributes to the energy (mass) density (19), (22). Let $N_R(t)$ be the number of actualizations within volume V_R at time t . We have with $x_0 = ct$, $\tilde{N}_R(x_0) = N_R\left(\frac{x_0}{c}\right)$ and the de Broglie-relation $p = \frac{h}{R}$

$$T = 3 \frac{dN_R(t)}{dt A_R} \cdot \frac{h}{R} = 3 \frac{c \cdot h}{3} \cdot \frac{d\tilde{N}_R(x_0)}{dx_0 V_R} = c \cdot h \cdot \frac{d\lambda(x_0)}{dx_0}. \quad (24)$$

The function $\lambda(x_0) = \frac{\tilde{N}_R(x_0)}{V_R}$ denotes the number of events per 3-volume at time t . If we set $\Lambda(x_0) = \frac{d\lambda(x_0)}{dx_0}$, then $\Lambda(x_0)$ is the change-rate of actualizations per 3-volume. We assumed that $\lambda(x_0)$ is constant over 3-space (i.e. in particular independent of R), which amounts to the homogeneity and isotropy of space with respect to actualizations.

We have also tacitly assumed that $N_R(t), (\lambda(x_0))$, is a differentiable function in t . This is an assumption, which cannot hold in the quantum-realm, since events represent discrete sets and are not deterministic, but obey a random-process. The only known Lorentz-invariant stochastic law for the spreading of events in \mathbb{M}^4 , such that $N_R \sim V_R$, is a Poisson-process with constant average (photon) transaction-rate ϱ_γ . The homogeneity and isotropy of space-time are thus an immediate consequence [15]. Hence, in the above terminology we have for the averages (expectation values) and $\Delta x_0 > 0^4$

$$\bar{\lambda}(x_0 + \Delta x_0) = \bar{\lambda}(x_0) + \varrho_\gamma \cdot \Delta x_0. \quad (25)$$

So by (25) we can define in analogy to (24)

$$\bar{T}_\gamma = 3 \frac{c \cdot h}{3} \cdot \frac{\Delta \bar{\lambda}(x_0)}{\Delta x_0} = c \cdot h \cdot \varrho_\gamma. \quad (26)$$

If we set $T = (T_{00} - \bar{T}_\gamma)$ we can complete the right hand side of (22) to

$$\frac{4\pi G}{c^4} T_{00} \rightarrow \frac{8\pi G}{c^4} \left(T_{00} - \frac{1}{2} T \delta_{00} \right). \quad (27)$$

We may alternatively shift the added amount to the left of (22), where we have by (26)

⁴ We can expect that there is a lower bound $0 < ct_0 \leq \Delta x_0$.

$$\frac{4\pi G}{c^4} \bar{T}_\gamma = \frac{4\pi G h}{c^3} \varrho_\gamma = 8\pi^2 l_P^2 \varrho_\gamma. \quad (28)$$

Therefore, with

$$\Lambda = 8\pi^2 l_P^2 \varrho_\gamma, \quad (29)$$

the synchronization-equation takes the form

$$R_{00} - \Lambda \delta_{00} = \frac{4\pi G}{c^4} T_{00}. \quad (30)$$

Note that Λ has the dimension of $\frac{1}{length^2}$. If matter-energy does not only stem from a static mass M , but from more complicated systems of relatively moving bodies, which also exercise pressure T , we finally get our main result by repeating the procedure in (27)

$$R_{00} - \Lambda \delta_{00} = \frac{8\pi G}{c^4} \left(T_{00} - \frac{1}{2} T \delta_{00} \right). \quad (31)$$

Under the assumption of known transformation rules, the full Einstein equations are equivalent to the fact that (31) holds in every local inertial coordinate system around every point in space-time [14].

4. Summary

To derive equation (31) we have used three ideas. The first one is, that quantum-events are real actualizations of quantum-systems in space-time and are accompanied by the transfer of four-momentum through gauge bosons, so called event-radiation. The number of events follows a Poisson-process, and the type of bosons depends on the respective force in action [1 – 5]. The second idea is that quantum-systems can serve as clocks (1), and that the rhythm of actualizations, induced by electromagnetic forces, is best measured by the light-clocks, naturally given by the transferred photons. The third idea is that clock-periods from the perspective of unequally accelerated systems need to be synchronized, in order to define the same rhythm of time. If the acceleration is of gravitational origin⁵, then the full synchronization-equation turns out to be (31). The dynamic and expanding space-time of general relativity is hence a consequence of event-radiation and of a fixed "yardstick", namely the locally constant speed of light c , implicit in the light-clocks, used to measure time under gravitational acceleration. There is in particular no direct connection to the energy of the quantum-vacuum. Under the assumption of the existence of the constant Λ , it is further possible to derive the MOND⁶ corrections of gravity [7,8], [16,17].

Our result was derived under the assumption of a constant cosmological term Λ (i.e. ϱ_γ). It is well possible that the value of Λ is in fact varying with cosmic time and only appears to be constant over the time periods, which we can possibly oversee. This allows the connection to the Hubble "constant" $\Lambda \sim H^2$, which seems to hold, given the empirical data and the theoretical models at our disposal today [7].

⁵ As mentioned in paragraph 3, gravitational acceleration itself can be thought to result from events (appendix B).

⁶ Modified Newtonian Dynamics.

Appendix A

For completeness sake we sketch the derivation of (31) directly from covariant quantities of a Lorentz manifold. We follow the exposition in [12]. Let there be an asymptotically flat Lorentz-manifold (\mathcal{M}, g) with global time-like Killing field ξ^a and with normalized redshift-factor $e^{\varphi(\xi)}$, where $\varphi(\xi) = \frac{1}{2} \ln(-\xi^a \xi_a)$ is a generalization of Newton's potential with $\varphi(\xi) = 0$ at spatial infinity. In such a space-time the notion of "staying in place" is well defined and means following an orbit of the Killing field ξ^a with acceleration $a^b = -\nabla^b \varphi = e^{-2\varphi} \xi^a \nabla_a \xi^b$. The direct generalization of equation (19) is then the definition of the Komar-mass inside a topological two-sphere \mathcal{S} [18]

$$\frac{4\pi G}{c^2} E = \int_{\mathcal{S}} e^{\varphi} \nabla \varphi dA. \quad \text{A1}$$

A1 can be re-expressed by use of the Killing equation $\nabla_a \xi_b - \nabla_b \xi_a = 0$ and Stokes theorem [17]

$$\frac{8\pi G}{c^2} E = - \int_{\mathcal{S}} dx^a \wedge dx^b \varepsilon_{abcd} \nabla^c \xi^d = c^2 \int_V R_{ab} n^a \xi^b dV. \quad \text{A2}$$

In A2 V denotes the 3-volume enclosed by \mathcal{S} and n^a the unit-normal. Equation (31) hence takes the form

$$\int_V (R_{ab} n^a \xi^b - \Lambda g_{ab} n^a \xi^b) dV = \frac{8\pi G}{c^4} \int_V \left(T_{ab} n^a \xi^b - \frac{1}{2} T g_{ab} n^a \xi^b \right) dV. \quad \text{A3}$$

Since there are many ways to enclose a mass M by a surface \mathcal{S} , the normal n^a can be varied. In order to vary ξ^b one can look at arbitrary small space-time regions, which look approximately like Minkowski space and chose approximate Killing vectors. Under the assumption that, if matter crosses the screen, then the Komar-integral A1 changes by exactly its mass m , A3 holds for every such Killing vector and region. Hence A3 holds for the integrand and we have

$$R_{ab} - \Lambda g_{ab} = \frac{8\pi G}{c^4} \left(T_{ab} - \frac{1}{2} T g_{ab} \right). \quad \text{A4}$$

The approach is similar to [19], where light-like vectors k^a, k^b are considered in place of n^a, ξ^b . A general exposition with detailed calculations can also be found in [20].

Appendix B

The open point is, where the Newtonian acceleration $g_R = \frac{GM}{R^2}$ in (16) stems from. There is a derivation of g_R in [12], which fits very well into our model of an emerging empirical space-time. If a particle with mass m and some "central" mass M come into being by a transaction, at relative rest and distance R of each other in locally flat space-time, there

is a reduction of entropy in quantum space-time ΔS , which must be equalized. We can assume this information to be one elementary bit connected to the existence or non-existence of the particle. We assume a holographic principle, namely that the information contained in a ball of radius R around M is actualized on its surface, and that a bit of information is part of the surface-information, once it is at a distance of its Compton length $\lambda = \frac{\hbar}{mc}$ from the surface [12,21].⁷ This holds, because structureless particles can reasonably be supposed to have the size of their Compton-length. Further assume that entropy changes linearly with the distance to the surface

$$\Delta S = 2\pi k_B \frac{mc}{\hbar} \Delta x. \quad \text{B1}$$

For the energy on the surface, corresponding to the total energy within the ball, we have

$$E = Mc^2 = \frac{1}{2} k_B NT. \quad \text{B2}$$

The number T is the surface-temperature and N denotes the number of bits on the surface, for which we get by the holographic principle and the Planck-length $l_P = \sqrt{\frac{G\hbar}{c^3}}$

$$N = \frac{A_R}{l_P^2} = \frac{4\pi R^2 c^3}{G\hbar}. \quad \text{B3}$$

By B2 and B3 we get for the surface-temperature T

$$T = \frac{MG\hbar}{2\pi R^2 k_B c}. \quad \text{B4}$$

For the total energy-change on the surface we have the entropic-force equation

$$\Delta S \cdot T = F \cdot \Delta x. \quad \text{B5}$$

By plugging B1 and B4 into B5, we arrive at

$$F = G \frac{Mm}{R^2} = mg_R. \quad \text{B6}$$

Therefore, one can think of local gravitational acceleration as a kind of "osmotic pressure" towards the other emerging parts of space-time. Local gravity thus becomes a consequence of light-induced events, the second law and a holographic principle.

⁷ By symmetry we could also choose m to be the "central" mass.

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