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Original Paper

On Time and Space from an Elementary Perspective

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Abstract: We investigate the role of time and space from the perspective of the basic notions of "change" and "identity". We formally analyze these notions and find primitive aspects of time and space, which we then rediscover in a realist model of quantum physics and in relativity. By our analysis of time and space we gain a coherent perspective on the relationship between both theories.

Keywords: Space; Time; Quantum Physics; Special Relativity; General Relativity

1. Introduction

In the fifth century BC the school of Elea, with its proponent Parmenides and other renowned thinkers such as Zeno, contested the reality of change. At around the same time Heraklitus taught that, on the contrary, change was the essence of things. There has been an ongoing tension between the two views until today. The conflict is immediately connected to the problem of the nature of time, which is inextricably intertwined with the notion of "change" [1]. The debate about the nature of time and space became prominent in modern natural philosophy by the dispute between Newton and Leibniz, plaid a key role in the philosophy of Kant and has continued in various forms to the present day. Some excerpts of this tradition can be found in e.g. [2 − 5]. Especially the two main theories of modern physics, quantum mechanics and relativity, have inspired renewed efforts to understand the nature of time and space and their relationship, not least because of the fact that the two theories treat the notions differently and demand, each in its own way, a radically changed understanding of reality. It is probably fair to say that in spite of the much higher precision of the questions, which can be asked on the topic today, final answers have remained elusive. In this paper we want to develop the thinking about time and space by starting from two basic intuitions: namely that physical systems undergo change, and that they keep their identity while changing.[1] We try to formalize these facts, in order to work with precise definitions and assumptions. The analysis will lead us to a primitive notion of time and space and give first hints as to what they signify. In a second step we will rediscover these primitive notions in quantum physics and relativity and analyze, what these theories further add to the structure of space and time and how they fit into a coherent picture of the world.

[1] The preservation of identity is already implied in the meaning of the word "change". In its absence it would be "creation".

2. Change and Identity

In what follows, we base on two fundamental intuitions, namely that physical systems undergo change and still can be identified as being the same. We further work with the idea that physical systems can be uniquely defined by a set of properties.[2] This premise immediately leads to the old tension: if its defining properties do change, how can a system then remain the same? Or by citing a famous example attributed to Zeno: if the arrow is defined by a fixed position at any moment, how can it move qua "the same" arrow?

2.1. Time

Let there be a set of physical systems $\Sigma = \{\sigma_i\}_{i\epsilon I}$ and a set $\mathcal{M}$ of properties of these physical systems, with $M\epsilon\mathcal{M}, M = \prod_{k=1}^{K} M_k$. Further assume that there is a property M, such that an element $\bar{p}\epsilon M, \bar{p} = \{p_k\}_{k\epsilon K}, p_k \epsilon M_k$, unambiguously defines a system σ_i and vice versa. Unique definability means that the map $P: \Sigma \to M, \sigma_i \longmapsto \bar{p}(\sigma_i)$ is one to one

$$(i = j) \Leftrightarrow \bar{p}(\sigma_i) = \bar{p}(\sigma_j). \tag{1}$$

If all the components $p_k \epsilon M_k$ of the property can change, leading to a map $U: M \to M$, $\bar{p}(\sigma_i) \overset{U}{\to} \bar{p}'(\sigma_i)$, and the identity of the system is nevertheless preserved, then we have in contradiction to (1)

$$(i = j) \wedge \bar{p}(\sigma_i) \neq \bar{p}'(\sigma_j). \tag{2}$$

We can resolve this contradiction by introducing a parameter $\lambda\epsilon\Lambda$, such that (1) holds for every λ

$$(i = j) \Leftrightarrow \ \bar{p}(\sigma_i, \lambda) = \bar{p}(\sigma_j, \lambda), \tag{3}$$

and in addition

$$\bar{p}(\sigma_i, \lambda_k) \neq \bar{p}'(\sigma_j, \lambda_l) \wedge (i = j) \Longrightarrow (k \neq l). \tag{4}$$

The parameter λ is what we call "time". It is clear that "time" cannot have the status of a property, otherwise we would be back to square one. It is a logical parameter, which must exist in its own right. What is it then, that preserves identity? For an element $(\sigma_i, \lambda_0)\epsilon\Sigma \times \Lambda$ and corresponding $\bar{p}(\sigma_i, \lambda_0)\epsilon M$, we define a "path" to be a map

$$U: \Lambda \times M \to M, \lambda \mapsto U\big(\lambda, \bar{p}(\sigma_i, \lambda_0)\big) = \bar{q}_U(\sigma_i, \lambda), \tag{5}$$
$$U(\lambda_0, .) = \mathbb{I}.$$

Because of (3), $U(\lambda): M \to$ M must be one to one for each $\lambda\epsilon\Lambda$ and the "paths" belonging to different "initial" elements $(\sigma_i, \lambda_0), \big(\sigma_j, \lambda_0\big), i \neq j$, must be disjoint. The identity of a system is hence defined by the unique "path", on which it lies.

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[2] We will show this in relevant cases.

At this very abstract level neither the set Λ, nor Σ or M, are further specified. There is a priori no structure, like an order-relation, a metric or a direction on Λ. Neither is there a "law", which we call (by slight abuse of meaning) evolution-law[3], that would determine the "correct" U. In fact, "time-points" $\lambda \epsilon \Lambda$ do not need to be universal and might have a dependence on the systems $\lambda = \lambda(\sigma_i)$. There is nothing dynamical at all in this notion of "change" or "time". The physical system is actually the unique "path" in its entirety; hence we are back in the world of Parmenides. To go further from here, we will have to specify (mathematical) models (Σ, M, Λ, U) of nature.

2.2. Space

In the last paragraph we have assumed that there is a uniquely defining property $M \epsilon \mathcal{M}$ of a physical system, i.e. $\sigma_i \leftrightarrow \overline{p}\,(\sigma_i) \epsilon M$. The question is, of course, whether such a property exists. The general considerations in the last paragraph do not resolve this issue and we have to search for an answer in a concrete model of nature, where Σ, M, Λ and U are specified. Let us preliminarily take classical physics, which is close to our intuitions, and where Σ consists of collections of point-masses and (vector-) fields. In classical physics there are a number of properties, which are uniquely defined by a physical system, like its energy, momentum, charge etc. But different systems can have the same momentum or charge or energy. There is one property, however, which defines a system uniquely: namely its position in space $\vec{x} \epsilon \mathbb{R}^3$ at time $t \epsilon \mathbb{R}$. Classical physics makes the choice $M = \mathbb{R}^3$ and $\Lambda = \mathbb{R}$. The unique definability works for both types of physical systems, particles and fields. No two different particles can at the same time $t \epsilon \mathbb{R}$ be at the same point in space $\vec{x} \epsilon \mathbb{R}^3$, nor can two classical fields coincide at all points in space without being identical. We think that it is the fact of unique definability, which singles out space and "paths" in space $U\colon \mathbb{R} \to \mathbb{R}^3, t \mapsto \vec{x}_U(\sigma_i, t)$, as the primary object in classical physics. Regarding the evolution-law U we know that, while the paths $\vec{x}_U(\sigma_i, t)$ satisfy differential equations, these equations in turn can be derived from variation principles, which span over all of time and leave no asymmetry on the time-axis. We will analyze the role of space in quantum physics in paragraph 3.5. To get there, we have to first look at the model of (non-relativistic) quantum physics in light of the framework (Σ, M, Λ, U), which we have introduced in paragraph 2.1.

3. Quantum Physics

3.1 Properties

The ansatz of the mathematical theory of (non-relativistic) quantum physics is to represent a measurable quantity of a physical system, called an observable, as a self-adjoint operator $A \epsilon L(H_{\mathbb{C}})$ in the space of linear operators over a state space $H_{\mathbb{C}}$, which carries the structure of a complex Hilbert space. The states $|c\rangle \epsilon H_{\mathbb{C}}$ represent the physical systems Σ.[4] The values, which a measurable quantity can assume in an experiment, are the corresponding eigenvalues $\lambda \epsilon \sigma_A \subset \mathbb{R}$ of A. Quantum theory assigns probabilities, which can be observed by experimentalists in repeated experiments on identically prepared systems, to these eigenvalues. To do this, states are represented as unit-vectors $|c\rangle \in, H_{\mathbb{C}}, \langle c|c\rangle = 1$. They can be linearly expanded on the basis of orthonormal eigenstates of A, $\{|e_k^A\rangle\}_{k \epsilon K_A} \subset H_{\mathbb{C}}$, $\langle e_i^A | e_j^A \rangle = \delta_{ij}$, $|c\rangle = \sum_{k \epsilon K_A} c_k |e_k^A\rangle, c_k \epsilon \mathbb{C}$.

[3] The term "evolution" has a dynamical connotation.
[4] We just consider pure states, which are actually "rays" $e^{i\theta}|c\rangle, \theta \epsilon \mathbb{R}$.

The index-range K_A can be identified with σ_A, hence for the cardinality we have $|K_A| = |\sigma_A|$. A probability pr_k is then assigned to the measurement of eigenvalue $\lambda_k \epsilon \sigma_A$ by

$$pr_k = c_k c_k^* = |c_k|^2.^5 \tag{6}$$

This assignment is also known as the "Born-rule" [6]. Within our framework we identify a property of a system $|c\rangle \epsilon H_{\mathbb{C}}$ with the amplitudes $\bar{c} = \{c_k\}_{k\epsilon K_A} \epsilon S^{K_A} \subset \mathbb{C}^{K_A}$ of its expansion in an eigenbasis of an observable A. So the set of properties are the unit spheres $\mathcal{M} = \{M_A\}_{A\epsilon L(H_{\mathbb{C}})}, M_A = S^{K_A}$. If there are observables $A, B \epsilon L(H_{\mathbb{C}})$ with zero commutator, $[A\ B] = 0$, then $M_A = M_B$. The spectrum σ_A therefore serves two purposes: it is the definition-range of a property on the one hand, and the range of empirical measurement-results on the other. Under the realist assumption that $\bar{c} \epsilon S^{K_A}$ represents a real physical quantity of the respective system and a reasonable assumption on forming composite systems, a theorem by Pusey, Barrett and Rudolph [7] assures, that the assignment of $\bar{c} \epsilon S^{K_A}$ to a physical system is unique. Of course, there might still be several systems having the same property, like in classical physics.

3.2 Time and paths

Quantum physics makes the choice $\Lambda = \mathbb{R}$, like classical physics. This reflects our experience, that change "comes in a row", is continuous and that there is a metric, expressing temporal distance, i.e. duration. We are now in a position to define continuous paths through M_A by a set of linear maps $\{U(t)\}_{t\epsilon\mathbb{R}} \subset L(\mathbb{C}^{K_A})$. From paragraph 2.1 we know, that for each $t\epsilon\mathbb{R}$ the linear map $U(t): \mathbb{C}^{K_A} \to \mathbb{C}^{K_A}$ must be one to one. Because of (6) it must preserve length and is hence unitary. By choice of $H \epsilon L(H_{\mathbb{C}})$, with $H^* = H$ [6], we can write $(h = 1)$ $U(t) = e^{-iHt}$. The operator H represents the energy of the system. For physical reasons H is bounded from below[7], which allows to show that time is not an observable $T \epsilon L(H_{\mathbb{C}})$ and hence not the source of a property [8]. This is in total agreement with our findings in paragraph 2. Given $H \epsilon L(H_{\mathbb{C}})$ and a $\bar{c}_0 = \bar{c}(0) \epsilon S^{K_A}$, the evolution-law is simply

$$\bar{c}(t) = e^{-iHt} \bar{c}(0), t\epsilon\mathbb{R}. \tag{7}$$

Evolution (7) is time-symmetric in the sense that $\bar{c}(0) = U^{-1}(t)\bar{c}(t) = U(-t)\bar{c}(t)$.[8] The differential form of (7) is the Schrödinger- equation

$$\begin{gathered} i\partial_t \bar{c}(t) = H\bar{c}(t) \\ \bar{c}(0) = c_0. \end{gathered} \tag{8}$$

3.3 Measurement

At this point a challenge arises, because the empirical results of experiments, i.e. measurement-interactions in a lab, are not the properties $\bar{c} \epsilon S^{K_A}$, but the eigenvalues $\lambda \epsilon \sigma_A$, and they are single-valued, i.e. measurements have unique results. Our experience hence demands the existence of an alternative type of paths to the ones defined by (7), i.e. an alternative evolution-law. Let $\bar{\delta}_j \epsilon S^{K_A}, j\epsilon I$, be the property, defined by $\left(\bar{\delta}_j\right)_k = \begin{cases} 1, k = j \\ 0, k \neq j \end{cases}$. A measurement is a pair $\left(U_\mu(t), \bar{\delta}_\mu\right)$,

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[5] The set K_A can be even uncountable infinite. In this case (6) is a probability density.

[6] H^* is the adjoint-operator.

[7] $\exists c\epsilon\mathbb{R}, \forall |d\rangle \epsilon H_{\mathbb{C}}, \langle d|d\rangle = 1: \langle d|H|d\rangle \geq c$.

[8] The maps $\{U(t)\}_{t\epsilon\mathbb{R}}$ preserve the linear structure of time $U(t+s) = e^{-iH(t+s)} = e^{-iHs}e^{-iHt} = U(s)U(t), t, s\epsilon\mathbb{R}$.

consisting of a measurement-evolution $U_\mu(t) = e^{-iH_\mu t}$, induced by an appropriate measurement-interaction H_μ, and a collapse to some $\bar{\delta}_\mu$. A necessary condition for an interaction H_μ to induce a measurement-evolution is, that there is a physical system $|A_0\rangle$, called apparatus, such that for $|d\rangle \epsilon H_{\mathbb{C}}$ there holds within some time $T_\mu > 0$

$$|d\rangle \otimes |A_0\rangle \xrightarrow{U_\mu(T_\mu)} \sum_{k=1}^{K_A} d_k |e_k\rangle \otimes |A_k\rangle. \tag{9}$$

The $|A_k\rangle$ are the pointer basis, indicating the result λ_k. The model is silent about what produces collapse to some $|e_\mu\rangle \otimes |A_\mu\rangle$. It is just assumed to be completed within a time interval $\Delta t_\mu \subseteq [0, T_\mu]$ with probability $0 \leq P(\Delta t_\mu) \leq 1$, $P([0, T_\mu]) = 1$ [9], and with probability $pr_\mu = |d_\mu|^2$ for some outcome $\mu \epsilon K_A$. Note that the concept of measurement does nowhere imply an (conscious) observer. Given an initial property $\bar{d}_0 \epsilon S^{K_A}$, let a path $\bar{d}(t) \epsilon S^{K_A}$ be defined by

$$\bar{d}(t) = \begin{cases} \bar{\delta}_j, t = t_j \\ U_j(t)\bar{\delta}_j, t_j < t < t_{j+1} \\ U_0(t)\,\bar{d}_0, t_0 < t < t_1, U_0(t_0) = \mathbb{1} \end{cases}. \tag{10}$$

At the time points $t_j \epsilon \mathbb{R}$ the property $\bar{d}(t)$ is said to collapse to $\bar{\delta}_j$. A pair $(t_j, \bar{\delta}_j) \epsilon \mathbb{R} \times S^{K_A}$ is called a quantum-event. We assume measurements to be built into definition (10) without special notation. The evolution-law (7) is contained in (10), which is actually some kind of stochastic branching process for properties. The nodes are the quantum-events.

Current interpretations of quantum mechanics can be distinguished by their acceptance of the type of evolution-law. Those, which postulate unitary evolution (7) only, have to explain why there are single-valued measurement results. Representatives are e.g. the many-worlds interpretation[9], relational quantum mechanics and Bohmian mechanics. Those, which believe in collapse, try to define how collapse actually works. In this camp are the GRW[10] and transactional interpretations or qbism (for an overview see e.g. [10]). In this paper we just stick to the axiomatic model, which we have introduced, but take a realist attitude to it. In other words, while we don't know, what the "beables" are, we think that the mathematical model represents their behavior in an objective way.

3.4 Some important observations

The model of quantum physics, presented above, clearly distinguishes two different realms. There is the realm of properties, which is an abstract space of amplitudes $\bar{c} \epsilon S^{K_A}$, and there is the empirical realm $\sigma_A \subset \mathbb{R}$, where experimental results or, more generally, empirical facts live. The two are linked by measurement (9). If a system has property $\bar{\delta}_\mu$, then an appropriate measurement finds the corresponding eigenvalue $\lambda_\mu \epsilon \sigma_A$ with certainty. If in a measurement there results an eigenvalue $\lambda_\mu \epsilon \sigma_A$, then the system has property $\bar{\delta}_\mu$. Notice the asymmetry: having property $\bar{\delta}_\mu$ does not mean that there automatically "exists" an eigenvalue λ_μ in the empirical realm. There is an existential hierarchy in the theory

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[9] The many-worlds interpretation actually denies that there is a unique outcome.
[10] Ghirardi, Rimini, Weber

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$$(|c\rangle, A)\epsilon H_{\mathbb{C}} \times L(H_{\mathbb{C}}) \rightarrow \bar{c}\epsilon S^{K_A} \rightarrow \lambda\epsilon\sigma_A. \quad (11)$$

Values $\lambda\epsilon\sigma_A$ do not exist sui generis, but they emerge through measurement. Evidence in this direction is also given by the Kochen-Specker theorem [11]. We want to call the sequence (11) the measurement-eigenvalue link.

If one sticks to the evolution-law of type (7) only, one has to show why experiments have unique results. Simply claiming that they do and not adjusting the law, leads to contradictions of the type of Schrödinger's cat and Wigner's friend. They are discussed in publications to the present day, often without properly recognizing their root-cause, as pointed out in [12]. By the intrinsic randomness of the collapse there is an asymmetry on the time-axis of paths of type (10). After a finite number of collapses at $t_0 < t_1 <\ldots t_J$, there is the interval $P_J = [t_0, t_J]$, where collapses have happened, and the complement $F_J = \mathbb{R} / P_J$, where there still might. In the intervals $]t_j, t_{j+1}[$ time is known to be symmetric, since the evolution is of type (7).

3.5 Space

In paragraph 2.2 we have seen that in classical physics space plays the important role of uniquely identifying property. In quantum mechanics we also need such a property. Indeed, space serves the purpose also in in this case, but only in combination with another property, namely spin $\bar{s}_s\epsilon S^{2|J|+1}$, $s\epsilon\{-J, (-J+1), \ldots, J\}$, for fixed $J\epsilon \left\{\frac{k}{2}\right\}_{k\epsilon\mathbb{N}_0}$. If its spin-number, s, is a fixed half-integer, then a system is uniquely defined by its property $\bar{c}_s = \{c_{\vec{x},s}\}_{\vec{x}\epsilon\mathbb{R}^3}\epsilon S^{\mathbb{R}^3}$, which are the amplitudes for localization at points $\vec{x}\epsilon\mathbb{R}^3$. This is Pauli's exclusion principle. Two fermions, which form matter in its narrow definition, cannot simultaneously have the same property $\bar{c}_s\epsilon S^{\mathbb{R}^3}$. The situation is different for bosons, which have integer spin-number and are the carriers of force. This is among the reasons, why photons are classically represented as vector fields, $\vec{A}$, which can be added at a location.[11] At the empirical level it is virtually impossible to find two different, massive particles at the same point $\vec{x}\epsilon\mathbb{R}^3$. This follows from the Heisenberg uncertainty principle, which implies arbitrarily large momenta to localize a system precisely.

4. Relativity

Let in the sequel the property under consideration be position in space, i.e. $M_A = S^{\mathbb{R}^3}$.[12] By the generalized evolution-law (10) a system randomly assumes values $\vec{x}(t_j)\epsilon\mathbb{R}^3$. The $\vec{x}(t_j)\epsilon\mathbb{R}^3$ spontaneously appear at $t_j\epsilon\mathbb{R}$ by breaking the time-symmetric unitary evolution $U_{j-1}(t)$, and we call the pair $(\vec{x}(t_j), t_j)\epsilon\mathbb{R}^3 \times \mathbb{R}$ an event. These events form (empirical) space-time $(\vec{x}(t_j), t_j)\epsilon\mathbb{R}^3 \times \mathbb{R}$ and are different from the quantum properties $(\bar{c}_s(t), t)\epsilon S^{\mathbb{R}^3} \times \mathbb{R}$, which form quantum space-time with the corresponding quantum-events $(\bar{\delta}_j, t_j)$. At this point space-time just inherits a relational structure from the standard metric of the real numbers. Note, that the collapse does take place in quantum space-time, where the evolution-law (10) lives, and nothing collapses in space-time. There, as a result of collapse, some events $(\vec{x}(t_j), t_j)\epsilon\mathbb{R}^3 \times \mathbb{R}$ come into being. Let us further assume that the probabilities of collapse are such, that a sufficiently large system, consisting of many subsystems, has at any point $t\epsilon\mathbb{R}$ a

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[11] Another reason is the fact that, due to its sharp momentum, a photon cannot be localized according to the uncertainty principle.

[12] There is some evidence for the "supervenience" of space, since in many experiments results are only indirectly measured by pointer-positions.

probability of nearly one to collapse, and there emerges a continuous path $\vec{x}(t)\epsilon\mathbb{R}^3$. This is the transition to the classical realm and can be achieved e.g. by Poisson-type models for collapse [13,14]. With some assumptions on regularity, the path has at every $t\epsilon\mathbb{R}$ a velocity $v_t = \frac{d\vec{x}}{dt}\epsilon\mathbb{R}^3$. It is now possible to take different perspectives on these paths by simply applying (isometric) symmetry transformations, i.e. Galilean transformations, on space-time, $(\vec{x},t) \xrightarrow{G} (\vec{x}',t')$. These transformations enforce different co-ordinate expressions $(\vec{x}'(t'), v'_{t'})$ for the same path, but the laws of physics are reasonably supposed to be independent of perspectives. This has implications.

4.1 Special Relativity

Suppose a physical system in vacuum emits a photon γ and another one absorbs it, a process in the quantum realm. Both, the emitting and absorbing systems, might be macroscopic and hence also lie on paths in space-time. Quantum physics tells us that for all photons in vacuum there is a general relation between their energy and momentum

$$\frac{E_\gamma}{|\vec{p}|} = c. \tag{12}$$

The constant c has the dimension of a velocity and is called the "speed of light". In space-time $(\vec{x},t)\epsilon\mathbb{R}^3 \times \mathbb{R}$ we can write, by simply taking the definition of (coordinate) velocity

$$c = \left|\frac{d\vec{x}}{dt}\right|. \tag{13}$$

This does not mean that there is actually a path for the photon in space-time. Emission is measured as the loss and absorption as the gain of a positive energy amount E_γ, which is a quantum process. Yet, a velocity is definable in space-time and a change of perspective must not change the physics. For this reason and because of (12), relation (13) must hold from every perspective

$$\frac{E'_\gamma}{|\vec{p}'|} = c = \left|\frac{d\vec{x}'}{dt'}\right|. \tag{13}$$

The speed of light remains constant under change of perspective. This fact has direct and well-known consequences for the metric structure of space-time. The space and time component of $\mathbb{R}^3 \times \mathbb{R}$ merge into Minkowski space-time $\mathbb{M}^4$ and the (isometric) symmetry-transformations change to the Lorentz-transformations $\bar{x} = (t,\vec{x}) \xrightarrow{L} \bar{x}' = (t',\vec{x}')$. The metric relations in space-time are constrained by the laws in the quantum realm. The Lorentz-invariant concept of "proper-time" $\tau\epsilon\mathbb{R}$, which is the length of paths in $\mathbb{M}^4$, explicitly generates a dependence of time on the individual systems, which follow the paths. In addition $\mathbb{M}^4$ turns out not to be totally temporally ordered. The "proper-time" of a system can be approximated to an arbitrary degree by sufficiently small light clocks [15]. This fact and the accuracy, by which the ticking of real physical clocks in space-time has experimentally been proven to match the geometric concept of proper-time τ, leads to the conjecture that the exchange of photons is indeed intimately linked to evolution (10) and collapse.

If the measurement-interaction H_ν is the emission of a photon γ of energy $E_\gamma = h\nu$ into some direction $\vec{p}$ by a system at some point $\bar{y}_0 \epsilon \mathbb{M}^4$, and the subsequent absorption by another one in the environment at $\bar{x}_0 \epsilon \mathbb{M}^4$, then it takes the environment in an energy state $|E_0\rangle$ minimally a time of $T_\nu = \frac{1}{\nu}$ to change into an orthogonal state $|E_1\rangle$ [16]

$$|E_1\rangle = U_\nu(T_\nu)|E_0\rangle. \tag{14}$$

We have to translate the evolution $U_\nu(T_\nu)$ into an evolution in quantum space-time. With λ denoting the wave-length of the photon, the bridge to space is the de Broglie-relation $|\vec{p}| = \frac{h}{\lambda}$ [13], which leads together with (12) to

$$c \cdot \frac{1}{\nu} = c \cdot T_\nu = \lambda. \tag{15}$$

We denote by $LC_T(\bar{y})$ the Lorentz-invariant surface of the future-light-cone originating at $\bar{y}\epsilon\mathbb{M}^4$, ranging over a time-span $t_{\bar{y}} \leq t \leq T$. Equation (15) allows the translation of the evolution $U_\nu(T_\nu)$ into a measurement-evolution in quantum-space-time by using the environment, expressed in the "position"-basis $|\phi_{\bar{x}}^0\rangle$ [14], as "apparatus". We get for a system $|c_s\rangle\epsilon H_{\mathbb{C}}$ with property $\bar{c}_s \epsilon S^{\mathbb{M}^4}$

$$|c_s\rangle \otimes |\phi_{\bar{x}}^0\rangle \xrightarrow{T_\nu} \sum_{\bar{x},\bar{y}\epsilon\mathbb{M}^4} c_{\bar{y},s} |\delta_{\bar{y}}\rangle \otimes \left|\delta_{\bar{x}\epsilon LC_{T_\nu}(\bar{y})}\right\rangle. \tag{16}$$

Photon-absorption at some point $\bar{x}_0 = (t_{\bar{x}_0}, \vec{x}_0)\epsilon\mathbb{M}^4$ triggers a collapse to a state $\left|\delta_{\bar{y}_0}\right\rangle \otimes \left|\delta_{\bar{x}_0 \epsilon LC_{T_\nu}(\bar{y}_0)}\right\rangle$. The point of emission $\bar{y}_0\epsilon\mathbb{M}^4$ on the light cone is by (15) also determined, and $|c_s\rangle$ assumes the property $\bar{\delta}_{\bar{y}_0}$. This happens with probability $pr_{\bar{y}_0} = \left|c_{\bar{y}_{0,s}}\right|^2$. The collapse actualizes both, the absorption and emission point, i.e. it generates a space-time interval by pushing $\bar{y}_0$ to the "past".[15] By the laws of relativity and (10), the next actualization of the massive system, $\bar{y}_1\epsilon\mathbb{M}^4$, will lie within the light cone $LC_t(\bar{y}_0)$.

The above process is evidence that the relationship between the quantum realm and $\mathbb{M}^4$ is bidirectional. There is further evidence of this. The canonical commutator relations of the position and momentum operator in (non relativistic) quantum mechanics, for instance, can be shown to arise from a low speed limit $v \ll c$ of the commutators of the Poincaré-algebra. In the same spirit the properties $\bar{c}\epsilon S^{K_A}$ can be won from the space-time symmetries alone, i.e. from irreducible unitary representations $D(G)$ of the symmetry group satisfying $[A\ D(G)] = 0$ [17]. Furthermore, as we have seen, the structure of quantum space-time must be such, that the probabilities (6) are invariant under symmetry transformations of space-time $\bar{x} \xrightarrow{L} \bar{x}'$.[16] The induced transformations $D(L)$ on relativistic quantum space-time are representations of the Lorentz-group. Spin-numbers $J_1, J_2 \epsilon \left\{\frac{k}{2}\right\}_{k\epsilon\mathbb{N}_0}$ determine the dimension $|N| = (2J_1 + 1)(2J_2 +$

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[13] The full de Broglie-relation amounts to a relationship between four-vectors $\bar{p} = \hbar\bar{k}$.

[14] Note that $|\phi_{\bar{x}}^0\rangle$ can be a complicated tensor-product of single systems.

[15] Note the similarity with the Probabilistic Transactional Interpretation [14].

[16] Actually the Poincaré group, if we include translations.

1) of the representation and hence influence the structure (dimension) of quantum space-time. A Lorentz-covariant quantum theory, which further allows the creation and annihilation of particles, is realized by the introduction of quantum fields. In spite of the more complicated structure of relativistic quantum-space-time, the properties of interest in quantum field theory are still amplitudes, namely the momentum-amplitudes for some outgoing particles in a scattering experiment $\bar{c} = \{c_{p,p'}\}_{(p,p')\epsilon\mathbb{M}^4\times\mathbb{M}^4}$ and the conceptual framework including collapse, which we have introduced so far, remains valid.

4.2 Time-symmetry

The probabilities in (6) are empirically accessible in a lab by registering the outcome-frequencies of measurements on identically prepared systems. As already mentioned, given the necessary independence of perspective, we must have

$$pr_k = pr_k{}', \tag{16}$$

if we apply a Lorentz-transformation $\bar{x} \overset{L}{\rightarrow} \bar{x}'$ on space-time. This fact quite unexpectedly has implications for the evolution-law. We have seen that the evolution-law (10) induces a time asymmetry on the path $d(t)$. This fact, together with the measurement-eigenvalue link, leads on the corresponding path through space-time $\bar{x}(t_j)\epsilon\mathbb{M}^4$ to the creation of new local "presents" and a growing "past". The emission and absorption of a positive energy amount, which triggers collapse, additionally implies a direction of this process and hence of time. In quantum space-time between collapses, however, there are the intervals $]t_j, t_{j+1}[$, where evolution (10) is unitary and time, as we have seen, symmetric. This must be so indeed, in order to ensure (16). To see this, we use an argument, originally given in [18,19], which uses some additional notions from relativity and quantum physics, which we just presuppose.

If probabilities are to be independent of perspective in $\mathbb{M}^4$, then the probabilities of two space-like separated measurements have in particular to be independent of their time-order. Let us consider the probabilities of two spin-measurements with values $A, B\epsilon\{\pm 1\}$ on a pair of entangled photons in freely chosen directions $\vec{a}, \vec{b}\epsilon\mathbb{R}^3$. We denote the outcomes by $\mathcal{A} = (A, \vec{a})$ and $\mathcal{B} = (B, \vec{b})$. Assume that in a specific reference frame the measurement $\mathcal{A}$ happens before $\mathcal{B}$ and that there is no influence from the future, what we call no retrocausality. This means that events in the future, in particular $\mathcal{B}$, have no impact on the probabilities of $\mathcal{A}$. Denote by $\lambda_{\mathcal{A}}$ all the variables, which could otherwise influence $\mathcal{A}$ independently of $\vec{a}$. Since $\mathcal{A}$ happens first, and there is no retrocausality, we have by the causal Markov property[17]

$$P_{\mathcal{A}}(A|\vec{a}, B, \vec{b}, \lambda_{\mathcal{A}}) = P_{\mathcal{A}}(A|\vec{a}, \lambda_{\mathcal{A}}). \tag{15}$$

The analogous conclusion holds for a reference frame, in which $\mathcal{B}$ happens before $\mathcal{A}$

$$P_{\mathcal{B}}(B|\vec{b}, A, \vec{a}, \lambda_{\mathcal{B}}) = P_{\mathcal{B}}(B|\vec{b}, \lambda_{\mathcal{B}}). \tag{16}$$

Independence of $P_{\mathcal{A}/\mathcal{B}}$ of time order and spatial translations, together with $\lambda = \lambda_{\mathcal{A}} \cup \lambda_{\mathcal{B}}$ hence define a Bell-model

[17] The causal Markov property says that, if the random variable Z represent the direct causes of a random variable X, then it shields X off from any random variable Y, which is not an effect of X. I.e. $P[X|YZ] = P[X|Z]$.

$$P\big(AB|\vec{a},\vec{b},\lambda\big) = P_{\mathcal{A}}\big(A|B,\vec{a},\vec{b},\lambda\big)\cdot P_{\mathcal{B}}\big(B|\vec{a},\vec{b},\lambda\big) = P(A|\vec{a},\lambda)\cdot P\big(B|\vec{b},\lambda\big). \tag{17}$$

Such models contradict quantum physics theoretically [20] and experimentally [21]. As a consequence, there are either preferential reference frames, contrary to relativity, or we have to drop the assumption of no retrocausality (in at least one reference frame).[18] Consequently there can be no distinction between a future- and a past-direction on the time-axis. Time-symmetry is a necessary condition for compatibility with relativity.

If the retrocausal influence takes a zig-zag path via the variables λ, then every reference frame is equivalent, and retrocausality is a sufficient condition for compatibility with relativity. For the question, whether time-symmetry is sufficient for relativistic compatibility see e.g. [22].

5. General Relativity

We want to outline the step from $\mathbb{M}^4$ as the model of space-time to general Lorentz-manifolds $\mathcal{M}$ as a consequence of quantum-collapse, acceleration and the use of light clocks to define the "rhythm" of reality, described in more detail in [23].

5.1 Einstein Equations

In the quantum realm forces can be understood as momentum-amplitudes generated by exchange of virtual bosons. In empirical space-time they manifest themselves in form of acceleration. Constant acceleration along the x-axis, say, can be built into the metric structure of $\mathbb{M}^4$ by choosing Rindler-coordinates, which are defined in the wedge $|x| = t$ and given by the transformations

$$x = \varrho\cosh(\kappa\vartheta)\,, t = \varrho\sinh(\kappa\vartheta)\,, \quad \kappa\epsilon\mathbb{R}^+, \varrho \geq 0, -\infty < \vartheta < \infty. \tag{18}$$

The corresponding line-element is

$$ds^2 = \left(\frac{\kappa\varrho}{c^2}\right)^2 c^2 d\vartheta^2 - d\rho^2 - dy^2 - dz^2. \tag{19}$$

Assume now that a charged physical system emits a photon with energy $E_\gamma = h\nu$ due to acceleration $a > 0$ along the x-axis, say. From the perspective, in which the system is locally at rest, this event appears like radiation of the induced EM-field at some temperature T_a, $E = k_B T_a$, with k_B denoting the Boltzmann constant. So from this perspective there exists a thermal "clock" of period $\Theta = \frac{1}{k_B T_a}$ [16]. The emission can also be described from the perspective of the photon, leading to a "clock" with period $\Theta = \frac{1}{\omega}$. To maintain consistency, the periods of the clocks should be synchronous, if expressed in their respective rest frames [19], which leads by the help of (19) to the equation [23]

$$\frac{4}{h}k_B T_a \frac{c}{a} = \frac{1}{\pi^2}. \tag{20}$$

[18] Note, that we could also drop the causal Markov property or the free choice of $\vec{a},\vec{b}$. We consider these options more implausible though. Adoption of the first option amounts to giving up a link between correlation and cause, and adoption of the second means accepting a deterministic "conspiracy". In instrumentalist interpretations of the quantum state all the arguments naturally fail, of course.

[19] In case of the photon we take the "oscillatory" rest frame.

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From (20) we directly get the Unruh-Davies temperature

$$T_a = \frac{\hbar a}{2\pi k_B c}. \tag{21}$$

Let us specifically consider the gravitational acceleration $a = g_R$ in the Newtonian limit at distance R of a mass M at relative rest. With the gravitational constant, G, we have

$$g_R = \frac{GM}{R^2}. \tag{22}$$

If we gauge duration with light clocks, we get by (20)

$$\frac{4}{h} k_B T_{g_R} \frac{c}{g_R} = \frac{1}{\pi^2}. \tag{23}$$

With $E = Mc^2$, $l_P = \sqrt{\frac{G\hbar}{c^3}}$, and $A_R = 4\pi R^2$ we derive

$$k_B T_{g_R} A_R = 2l_P^2 Mc^2 = 2l_P^2 E. \tag{24}$$

By using (21) we arrive at

$$g_R \cdot A_R = \frac{4\pi G}{c^2} E. \tag{25}$$

This is a form of the oo-component of the Einstein equations. With V_R denoting the volume of the ball with radius R enclosing M, (25) can also be written

$$\frac{d^2}{dt^2} V_R = \frac{4\pi G}{c^2} E. \tag{26}$$

If we introduce the energy-momentum tensor T_{ab} with energy-density $T_{00} = \frac{E}{V_R}$, add pressure terms in the three space directions $T_{ii}, 1 \leq i \leq 3$, and use the properties of the Ricci-tensor R_{ab}, we can rewrite equation (26) with $T = \sum_{i=0}^{3} T_{ii}$,

$$R_{00} = \frac{8\pi G}{c^4}\left(T_{00} - \frac{1}{2} T \delta_{00}\right). \tag{27}$$ [20]

On a Lorentz-manifold with metric-tensor g_{ab} equations (25,26,27) hold locally and it is shown in [24,25] that under the assumption of a time-like Killing-field $X, \langle X, X \rangle > 0, L_g X = 0$ [21], equation (27) can be covariantly extended to result in the full Einstein equations

$$R_{ab} = \frac{8\pi G}{c^4}\left(T_{ab} - \frac{1}{2} T g_{ab}\right). \tag{28}$$

[20] Note that in order to derive (28) we only made use of the energy transfer, induced by (16). The transfer of the full four-momentum might additionally give rise to a cosmological term [23].

[21] $L_g(\cdot)$ is the Lie-derivative.

Therefore the Einstein equations emerge from locally synchronizing light clocks, which describe the same event in empirical space-time from different perspectives. The symmetry group, which describes change between perspectives, is very large, namely the space $C^\infty(\mathcal{M})$ of local diffeomorphisms on $\mathcal{M}$.

5.2 Newtonian acceleration

The open point is, where the Newtonian acceleration $g_R = \frac{GM}{R^2}$ in (25) stems from. There is a derivation of g_R in [25], which fits very well into our model of an emerging empirical space-time. If a particle with mass m and some "central" mass M come into being at relative rest in locally flat space-time at a distance R of each other, there is a reduction of entropy in quantum space-time ΔS, which must be equalized. We assume a holographic principle, namely that the information contained in a ball of radius R around M is contained on its surface, and that a bit of information is part of the surface-information, once it is at a distance of its Compton length $\lambda = \frac{\hbar}{mc}$ from the surface [25,26].[22] Further assume that entropy, proportionate to m, changes linearly with distance around the Compton length

$$\Delta S = 2\pi k_B \frac{mc}{\hbar} \Delta x. \tag{29}$$

For the energy on the surface, corresponding to the total energy within the ball, we have

$$E = Mc^2 = \frac{1}{2} k_B N T. \tag{30}$$

The number T is the surface-temperature and N denotes the number of bits on the surface, for which we get by the holographic principle and the Planck-length $l_P = \sqrt{\frac{G\hbar}{c^3}}$

$$N = \frac{A_R}{l_P^2} = \frac{4\pi R^2 c^3}{G\hbar}. \tag{31}$$

By (30) and (31) we get for the surface-temperature T

$$T = \frac{MG\hbar}{2\pi R^2 k_B c}. \tag{32}$$

For the total energy-change on the surface we have the entropic-force equation

$$\Delta S \cdot T = F \cdot \Delta x. \tag{33}$$

By plugging (29) and (32) into (33), we arrive at

[22] By symmetry we could also choose m to be the "central" mass.

$$F = G\frac{Mm}{R^2} = mg_R. \tag{34}$$

Therefore, one can think of local gravitational acceleration as a kind of "osmotic pressure" towards the other emerging parts of space-time. Local gravity thus becomes a consequence of the light-induced collapse, the second law and a holographic principle. In fact the arguments in paragraph 5.2 hold, if we take M to be the (Tolman-Komar) mass of a visible homogeneous and isotropic universe, which is supposed to have a conformal time-like Killing-field, right in the spirit of Mach [27,28].

In this line of thinking gravity and the curved structure of empirical space-time emerge as the result of a quantum theory of light-induced events. Given the countable nature of quantum-events (10), the continuity of space-time is probably an idealization. In the context of this work, this does not mean, however, that space-time or the metric are quantized, i.e. are the quanta of some field.

6. Conclusion

We have developed the notions of "time" and "space" from an elementary starting point through different models of nature. On this journey we have learnt that there are different realms of reality, a quantum-realm and an empirical realm, which in turn give additional structure to space and time. If we want to investigate the relationship between the quantum realm and space-time, we need a quantum theory of "events", which we defined by evolution (10).

The analysis shows that space-time inherits its relativistic structure from the quantum realm, whereas in turn the symmetries of relativistic space-time influence the structure of quantum space-time. But, if we mix things up and try to embed quantum properties into space-time for instance, we get into well-known difficulties of non-locality and the like. Non-empirically accessible states in quantum space-time need not be Lorentz-symmetric and collapse does not take place in space-time. If the spin of one of the entangled photons in paragraph 4.2 is empirically measured and corresponds to the value $s = +1$, say, then it is the property $\bar{\delta}_{-1}$ of the other, which is instantly determined. The property $\bar{\delta}_{-1}$ is an element of the quantum realm, and by the hierarchy (11) it still needs a measurement to represent spin $s = -1$ as an empirical value in $\mathbb{M}^4$. This can, of course, be done, but then we cannot just tacitly assume that the probabilities are independent of perspectives, but have to postulate it (16). The consequences have been explained in paragraph 4.2. Evolution law (10) ensures the necessary time-symmetry of quantum space-time between collapses to allow compatibility with relativity.

The evolution-law (10) also induces an asymmetry on the time-axis of space-time and, together with the measurement-eigenvalue link and the photon emission-absorption, the notion of a "present" evolving in a temporal direction to an undetermined "future", while creating an ever larger, determined "past", right in the sense of Heraklitus. This process is however strictly local in space-time and connected to individual systems [29]. None of this can be derived from evolution-law (7). Quantum space-time, relating to (7), is the world of Parmenides. Even some "ultimate" questions might find answers in one realm, but not in the other. The structure of the evolution-law (10) opens the possibility that the universe, if considered as one physical system, had a beginning in space-time, namely its first collapse to a small region of enormous energy density, and that there was an entropy-induced diffusion of enormous amounts of energy inside this region (29). There is nothing we can say about a possible "beginning" of the universe in quantum space-time, though.

Duration can be measured in space-time, by what we have seen, only indirectly through measuring change of properties of appropriate systems with preferably (approximately) constant periods over many measurements. Such systems then serve as "clocks". The fact that observable change in the nearby parts of the universe can be described relative to some specific, scale-setting clocks, like the motion of stars or of the moon, the rotation of earth or the pulse of neutron stars, might have lead to the idea of universal time [3]. The derivation of the relativistic structure of space-time in paragraphs 4 and 5 depends crucially on the choice of light as the mediator of collapse and hence as the scale setting clock. At the beginning of the universe, below the Planck scale, there might have been no such scale-setting clocks and the concept of cosmic time void of any meaning [30]. To conclude from this, that time does not exist, would however be a step too far.

The notions of "time" and "space" have different structure, depending upon the realm we are in, but their original, primitive role is always preserved. Space is a uniquely defining property of physical systems, and time is a necessary logical parameter in order to allow a system to change in the true sense of its meaning. Both notions only exist together with individual physical systems and are distinct from each other, even if they become intertwined in relativistic models of the world.

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