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Is There a Physical Reality Beyond the Axioms of Quantum Mechanics?

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Abstract: Quantum mechanics is an axiomatic theory. The axioms are related to a group of mathematical formulations that attempt to describe the behaviour of the microscopic realm. These axioms have not been derived from simpler, previously accepted statements. Quantum mechanics is based on these axioms, and the theory is in perfect agreement with the experimental results obtained. In our recent articles, an analogy has been proposed for relativistic quantum mechanics. This analogy suggests a deeper level that may be transformed to forms that analogises relativistic quantum mechanics. In this paper, we have tried to demonstrate the possibility of the existence of a deeper level beyond the axioms. In accordance with the adoption of this analogy, quantum axioms may be interpreted as being attributed to a more fundamental level.

Keywords: Foundations of quantum physics; Quantum postulates; Partial observation; Complex velocity; Complex vector; Quantum mechanics axioms

1. Introduction

An axiom may be defined as “a statement that serves as a starting point from which other statements are logically derived. Whether it is meaningful (and, if so, what it means) for an axiom to be ‘true’ is a subject of debate in the philosophy of mathematics.” [1].

This logical definition finds place in the domain of quantum mechanics, where there is no necessity to know what is beyond the physical realm.

Quantum mechanics theory was established in order to describe the microscopic realm. This theory has passed through various stages. A brief summary of its history is given below:

- In 1920, Bohr formulated the Correspondence Principle [2].
- Early attempts to formulate quantum theory were made by Heisenberg, Born, and Jordan [3, 4, 5] in 1925. The approach was based on a non-commutative operator-matrix approach.

- In 1926, Schrödinger derived his wave equation [6, 7, 8, 9, 10, 11]. In other words, his approach was based on the analogy of waves.
- In the same year (1926), Max Born also proposed his statistical approach (Born's postulate) [12].
- In 1927, Werner Heisenberg formulated the uncertainty principle [13].
- Then, in 1928, Dirac proposed his equation of relativistic quantum mechanics [14].

Throughout the given sequence of development, there were two different approaches: the non-commutative operator-matrix approach and the wave approach. These two approaches were unified by the Dirac formulation [15] of the principles of quantum mechanics.

These principles were formulated in an axiomatic form in order to constitute an axiomatic theory, where these achievements would be regarded as axioms forming the mathematical foundation of the quantum mechanics theory. The synthesis of these achievements was realized by John von Neumann in 1932 [16]. Neumann developed the operator theory in Hilbert's space. The resulting principles came to be known as the Dirac-von Neumann axioms of quantum mechanics.

In some literature, these statements of quantum mechanics are known as axioms, postulates, or principles.

1.1. Axioms of quantum mechanics

There is no unanimous agreement with respect to the set of quantum mechanics axioms or postulates. Nottale and Célérier considered the quantum postulates to be categorised into three groups [17]:

- main postulates that cannot be derived from more fundamental ones (axioms);
- secondary postulates that are often presented as "postulates" but can actually be derived from the main ones; and
- statements, often called "principles", which are well known to be mere consequences of the postulates.

In the present article, we are interested in the main postulates, which are as follows:

1. Complex state function (ψ): Each physical system is described by a state function that determines everything that can be known about the system.
   The state function is a complex wave function, which is multiplied by its conjugate in order to carry out probability calculations. For example, the solution of the Schrödinger equation for a free particle may have the following form:
   $$\psi_S = A \exp i \ \ (\boldsymbol{k} \cdot \boldsymbol{x} - \omega t), \tag{1}$$
   where ω, k, and A are angular frequency, wave vector, and amplitude respectively. This is the Schrödinger's wave function. In case of relativistic quantum mechanics, the Dirac wave function (ψ_D) for a free particle (the trial solution) has the following form:
   $$\psi_D(x,t) = u_D(x,t) \exp i(\boldsymbol{k} \cdot \boldsymbol{x} - \omega t), \tag{2}$$
   where ω, k, and $u_D(x,t)$ are angular frequency, wave vector, and a Dirac four-component spinor respectively. The structure of the spinor is due to the nature of the Dirac Hamiltonian for the studied case. It is interesting to see that it has the same complex phase factor for different forms.
2. Schrödinger equation (or Dirac equation): The time evolution of the wave function of a non-relativistic physical system is indicated by the time-dependent Schrödinger equation:

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H}\psi \quad , \tag{3}$$

where ψ_S, $\hat{H}$, and $\hbar$ are the Schrödinger wave function, Hamiltonian, and reduced Planck constant respectively. For axioms of relativistic quantum mechanics, the Hamiltonian is changed to the Dirac Hamiltonian [14]:

$$i\hbar \frac{\partial \psi_D}{\partial t} = (c\alpha \cdot \hat{p} + \beta mc^2)\psi_D, \tag{4}$$

where β and α are the Dirac matrices, $\hat{p}$ is the momentum operator, m is the rest mass of the particle, $\hbar$ is the reduced Planck constant (Dirac constant), ψ_D is the Dirac wave function, and c is the light velocity.

3. Correspondence Principle: For every dynamic variable of classical mechanics, there is a corresponding linear one in quantum mechanics – the Hermitian operator, which, when operating upon the wave function associated with a definite value of the observable (the eigenstate associated with a definite eigenvalue) yields this value times the wave function.
4. Von Neumann's postulate: If a measurement of the observable A yields the value, a_i, then the wave function of the system immediately after the measurement is the corresponding eigenstate, ψ_i.
5. Born's postulate: It is a probabilistic interpretation of the wave function.

1.2. Quantum mechanics as emergent phenomena

The axiomatic structure of quantum mechanics means that the theory is fundamental.

If we consider the nature in physical levels and as emergent levels as seen in Figure 1, the quantum realm may be viewed as an emergent realm of an underlying base. Many attempts have been made to examine this case [18, 19, 20, 21, 22].

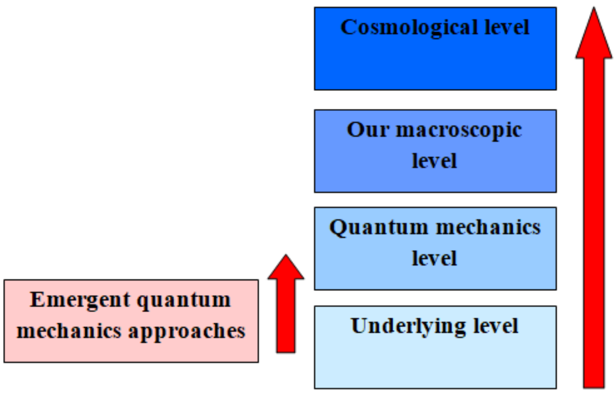

Figure 1. The levels of emergence

Most of these works are influenced by the statistical interpretation of Born's rule, but in many different ways.

However, these attempts are based on the quantum axioms and did not try to explain these axioms. In other words, it looks as though they accepted the axioms (principles) as the mathematical definition of axioms.

In another side, many attempts have been made "to identify a new set of axioms or postulates from which the mathematical structure of quantum theory can be derived, in the hope that with such a reformulation, the features of nature which made quantum theory the way it is might be more easily identified." These attempts are regarded the reconstruction of quantum mechanics [23, 24].

1.3. Attempt at quantum analogy

In an attempt started in 2007, a theory has been constructed to show an emergent concept that can create an analogy for the relativistic quantum mechanics [25, 26, 27, 28, 29]. This project is not in quantum mechanics. The emergence process is not based on statistical techniques. The attempt is based on two postulates [26, 27]:

1. A kinematical system is in an external world (independent of observation). This kinematical system is of two rolling circles in real space and real time.
2. For lab observers, the system must be resolved optically, forming the observable world.

Figure 2 shows the kinematical model mentioned in the first postulate [27, 28]. The development of the system can be found in [27].

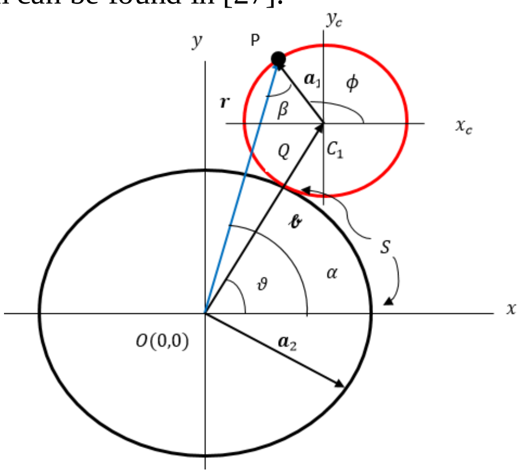

Figure 2. The real model (external world): rolling circles model [27]

This system is characterised by four parameters:

$$\frac{a_2}{a_1} = \frac{\omega_1}{\omega_2} = \mu > 1 \quad . \tag{5}$$

where, a_1 , a_2 ,ω_1, and ω_2 are the radius of the small circle, the radius of the large circle, the angular frequency of the point P, and the anguler frequency of the point Q, the touch point of the two circles respectively (see Figure 2).

This system is a mathematical virtual model. In order to deal with the system as a physical object, it must be observable in a lab. This is an essential approach to prove its physical existence (positivism). It is normal for the lab observer to deal with observables. In classical physics, the observable distinguishability is related to optical resolution (Rayleigh criterion). According to the second postulate, and based on the resolution limit, the system is partially resolved under the conditions [27]:

$$a_1 \ll d_\lambda \ll a_2 \quad , \tag{6a}$$

and:

$$\omega_1 \gg \omega_\lambda \gg \omega_2 \quad , \tag{6b}$$

where d_λ is the spatial resolution, which is the minimum linear distance between two distinguishable points, and the same for ω_λ, which is the frequency resolution. These scaling factors (d_λ and ω_λ) are related to the properties of the used light in the observation process.

Thus, for the lab observer, the partial observation may lead to the conclusion that $a_1 = \omega_2 = 0$, where a_1 and ω_2 can not be resolved. The zero quantity is a practical approximation.

The position vector of point P in Figure 2 is in radial form [27]

$$\boldsymbol{r} = (\boldsymbol{a}_2 + \boldsymbol{b}\sqrt{X})\left\{\cos\ (\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega_1 t + \omega_\beta t) \pm \sqrt{-sin^2(\boldsymbol{k}_2 \cdot \boldsymbol{s} - \omega_1 t + \omega_\beta t) + X}\right\}. \tag{7}$$

It is quite easy to derive the equation of velocity and equation of acceleration as well [27, 28]. These three real forms (position vector, equation of velocity, and equation of acceleration) are obtained from the first postulate. Appling the second postulate, all the three real forms are converted into complex forms [17, 28, 30]. However, under the effect of partial observation (due to the second postulate), one can find the following:

- A complex vector function can be obtained as a transformation of a position vector in a real, two rolling circles model. This complex function may analogise the wave function [27, 29].
- The velocity equation of a point in the two rolling circles model is transformed to a complex velocity equation. This equation analogises the Dirac equation [27, 28].
- The acceleration equation of the point is transformed into a complex acceleration equation. This equation analogises the Klein-Gordon equation [28].

Table 1 shows the results of the transformation (analogical model forms) in comparison with the relativistic quantum forms (without Planck constant).

Table 1. The comparisons [29]

Conventional definition	**Conventional equations of the relativistic quantum mechanics**[1]	Analogical model forms	Analogical definition
Dirac wave function	$\psi_D = u_D \exp i(k \cdot x - \omega t)$	$\boldsymbol{Z} = \boldsymbol{a}_{2m} \exp \pm i(k_{2m} \cdot s - \omega_{1m} t)$	*Z-complex vector*
Dirac equation	$i\frac{\partial\psi}{\partial t} = (-ic\boldsymbol{\alpha} \cdot \boldsymbol{\nabla} + \beta\omega)\psi$	$i\frac{\partial \boldsymbol{Z}}{\partial t} = (-iv\boldsymbol{A} \cdot \boldsymbol{\nabla} + B\omega_{1m})Z$	*Complex velocity equation*
The coefficients	$\boldsymbol{\alpha}$ and β	$\boldsymbol{A}$ and B	*Coefficients*
Property	$\boldsymbol{\alpha}_i\boldsymbol{\alpha}_j + \boldsymbol{\alpha}_j\boldsymbol{\alpha}_i = 0$	$\boldsymbol{A}_\theta \cdot \boldsymbol{A}_\varphi + \boldsymbol{A}_\varphi \cdot \boldsymbol{A}_\theta = 0$	*Property*
Property	$\boldsymbol{\alpha}_i\boldsymbol{\alpha}_i + \boldsymbol{\alpha}_i\boldsymbol{\alpha}_i = 2$	$\boldsymbol{A}_\theta \cdot \boldsymbol{A}_\theta + \boldsymbol{A}_\theta \cdot \boldsymbol{A}_\theta = 2$	*Property*
Property	$\boldsymbol{\alpha}_i^2 = \beta^2 = 1$	$\boldsymbol{A}^2 = B^2 = 1$	*Property*
Property	$\boldsymbol{\alpha}_i\beta + \beta\boldsymbol{\alpha}_i = 0$	$\boldsymbol{A}B + B\boldsymbol{A} = 0$	*Property*
Klein-Gordon equation	$\frac{\partial^2\psi}{\partial t^2} = [c^2\nabla^2 - \omega^2]\psi$	$\frac{\partial^2 \boldsymbol{Z}}{\partial t^2} = [v^2\nabla^2 - {\omega_{1m}}^2]\boldsymbol{Z}$	*Complex acceleratio n equation*

[1]Without Planck constant.

Then, the lab observer appears like a relativistic observer at the quantum level.

In the present paper, we assume that the quantum axioms may be related to a deeper level similar to the case presented above.

2. The axioms

2.1. The first axiom

The position vector (Eq. 7) is transformed to the following:

$$\boldsymbol{\mathcal{Z}}(s,t,0) = \boldsymbol{a}_{2m} \exp \pm i(\boldsymbol{k}_{2m} \cdot \boldsymbol{s} - \omega_{1m} t). \tag{8}$$

This form is similar to the form of the complex state functions or the wave function (ψ) [Eqs. (1) and (2)]. All of them have a similar complex phase factor. Equation (8) appears with a real vector amplitude ($\boldsymbol{a}_{2m}$), whereas the Dirac function is with a Dirac four-component spinor.

2.2. The second axiom

From the position vector Eq. (7), the equation of velocity point P can be derived [28]. Under the partial observation conditions mentioned above, the velocity equation is transformed into the following [28]:

$$i\frac{\partial \boldsymbol{\mathcal{Z}}}{\partial t} = (-iv\boldsymbol{A} \cdot \nabla + B\omega_{1m})\boldsymbol{\mathcal{Z}} \ , \tag{9}$$

where $\boldsymbol{A}$ and B are coefficients related to the rotation of the system.

$$B = \pm 1, \quad \text{and} \quad \boldsymbol{A} = \mp\ i\ \hat{\boldsymbol{e}}_{\vartheta} \quad . \tag{10}$$

The lab observer cannot recognise the rotations of the system, and, mathematically, $\boldsymbol{A}$ and B are not the normal unit vector. Thus, the properties of the rotation unit vectors mentioned above will not be considered by the observer as rotation vectors. In reality, $\boldsymbol{A}$ and B are related to the rotations of the system but are not unit vectors.

Comparing Eq. (9) with the Dirac equation (Table 1), many similarities can be observed. However, it must be mentioned that Eq. (9) is formulated in a two-dimensional space, whereas Dirac's equation is meant for a three-dimensional space.

2.3. The third, fourth, and fifth axioms

The last three axioms (Correspondence Principle, Von Neumann's postulate, and Born's postulate) seem to be techniques for obtaining the physical information from the complex function and might not be related to what lies beyond this function. Thus, these three axioms can be regarded as mathematical tools of the lab observer [27].

3. Conclusions

It may be concluded that if the analogy can offer a full interpretation of the relativistic quantum mechanics, a deeper realm may be considered to be acting behind what we call the observable world. Then, the first two axioms are no more than descriptions of that deeper level, as it deformed due to observation. Can these postulates explain the entanglement and a deeper level of gravity?

For the entanglement, an attempt was made to explain the multi-dimensional complex vector function in [27], but the work still requires more investigation. The flat spacetime has been shown in the analogy of Klein-Gordon equation (Table 1) [28]. The approach of the entanglement explanation may lead to the curved spacetime, and then to the gravity. That part is still under work.

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