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Original Paper

What Might the Matter Wave Be Telling Us of the Nature of Matter?

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Abstract: Various attempts at a thoroughly wave-theoretic explanation of matter have taken as their fundamental ingredient the de Broglie or matter wave. But that wave is superluminal whereas it is implicit in the Lorentz transformation that influences propagate ultimately at the velocity c of light. It is shown that if the de Broglie wave is understood, not as a wave in its own right, but as the relativistically induced modulation of an underlying standing wave comprising counter-propagating influences of velocity c, the energy, momentum, mass and inertia of a massive particle can be explained from the manner in which the modulated wave structure must adapt to a change of inertial frame. With those properties of the particle explained entirely from wave structure, nothing then remains to be apportioned to anything discrete or "solid" within the wave. Consideration may thus be given to the possibility of wave-theoretic explanations of particle trajectories, and to a deeper understanding of the Klein-Gordon, Schrödinger and Dirac equations, all of which were conceived as equations for the de Broglie wave. It is argued that this wave-theoretic interpretation of matter favours a physically realistic, rather than inherently probabilistic, interpretation of quantum mechanics.

Keywords: de Broglie wave; Planck-Einstein relation; wave-particle duality; de Broglie-Bohm theories; Dirac bispinor; configuration space

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1. Introduction

It might be thought that the de Broglie wave can say very little regarding the nature of solid matter. As this "matter wave" is usually understood, it seems to make no sense at all. It has a velocity that is superluminal, increases as the particle slows, and becomes infinite as the particle comes to rest (de Broglie [1]).

There can be no doubt that a massive particle is in some sense wave-like. In accordance with the Planck-Einstein relation,

$$E = \hbar\omega_E, \tag{1}$$

the moving particle exhibits an associated frequency (the Einstein frequency ω_E) and from the de Broglie relation,

$$p = \hbar\kappa_{dB}, \tag{2}$$

a wave number (the de Broglie wave number, κ_{dB}), where E and p are respectively the energy and momentum of the moving particle, and $\hbar$ is the reduced Planck's constant.

Frequency ω_E and wave number κ_{dB} are well-confirmed experimentally. The Planck-Einstein relation defines for both massive and massless particles a consistent scheme relating energies and binding energies to the frequencies of emitted and captured photons. And soon after the suggestion by de Broglie that a beam of electrons might exhibit diffraction when directed through a small enough aperture (de Broglie [2][3]), the scattering of electrons in accordance with the de Broglie wavelength was confirmed by the Davisson-Germer [4] and Thompson [5] experiments.

That massive particles also interfere in the same wave-like manner as photons is demonstrated in a particularly compelling manner in neutron interferometry, a context in which it has been said that the use of the expression "neutron optics" is by no means metaphorical (Rauch and Werner [6], at p. 1). The visibility of this interference may be significant out to the $250th$ interference order and beyond, demonstrating coherence lengths and widths that may be orders of magnitude greater than what might be expected from any measure associated with a solid particle - far greater certainly that the classical particle radius and the Compton wave length (see, for instance, Rauch and Werner [6], Chap 4, Rauch et al [7], and Pushin et al [8]). These lengths are also many orders of magnitude greater of course than the range of the strong force that is primarily responsible for the scattering of the neutron.

Whatever is causing this interference is spatially extended, wave-like, and not at all fictitious. However, the wave,

$$\psi_{dB} = e^{i(\omega_E t - \kappa_{dB} x)}, \tag{3}$$

implied by frequency ω_E and wave number κ_{dB} has the velocity,

$$v_{dB} = \frac{\omega_E}{\kappa_{dB}} = \frac{c^2}{v},$$

which is evidently greater than the limiting velocity c of light.

Faced with this embarrassment, de Broglie was able to show that the classical velocity of the particle could be identified with the group velocity of a suitably constructed superposition of these waves of differing frequency (de Broglie [1], Chap. 1, Sect. II). But such a wave packet spreads with time as Schrödinger found when he sought to contrive from the de Broglie wave a thoroughly wave-theoretic explanation of matter and radiation (see Dorling [9]). Schrödinger was unable to confine either the individual wave or a superposition of these waves to the orbit of an atomic electron. Nor could he reconcile such a superposition with the known precision of the energies of such orbits.

There is a further difficulty. The notion of a superposition of superluminal waves provides no clue whatsoever as to the nature or origin of these waves. Nor beyond the analogy with the photon, is there any apparent reason that a massive particle should have a frequency and wave number directly related to its energy and momentum. Clearly a massive particle is oscillatory, but how is this so? And what then is a particle? Is it wave or particle, the excitation of a quantum field, or something else again?

2. Reinterpreting the matter wave

If those questions have answers, they are unlikely to be found in a superluminal wave having no apparent physical connection with the subluminal particle that it seems to be forever overtaking but never out-runs. I will rely here on an alternative conception of the de Broglie wave, according to which it is not strictly speaking a wave at all, but the relativistically induced modulation of an underlying wave structure having, in the rest frame of the particle, the form of a standing wave.

To an observer for whom the particle is moving, the standing wave becomes a carrier wave subject to a sinusoidal modulation or beating evolving through the carrier wave at the superluminal velocity of the de Broglie wave. It is this modulation that describes the failure of simultaneity in the direction of travel and constitutes the "wave of simultaneity" contemplated in the literature (see, for instance, Rindler [10], p. 121).

That this might be the true interpretation of the de Broglie wave is by no means a new idea. An anticipation of essentially the same effect may be discerned in de Broglie's famous thesis, both in a mechanical model described by de Broglie and in his treatment of the wave in a Minkowski spacetime diagram. This alternative interpretation of the wave has been noticed since on several occasions and in various circumstances (as listed in Shanahan [11] and [12], and see also Mellen [13], Horodecki [14], and particularly Wolff [15] to [17]).

However, it seems to have gone largely unremarked that this interpretation explains immediately the many puzzling features of the de Broglie wave. Considered as a modulation, the wave acquires a physically reasonable origin, the apparent conflict with special relativity is resolved, and it becomes possible to understand why this otherwise anomalous superluminal phenomenon should seem to "pilot" the subluminal structure through the processes of scattering and interference.

It might seem that the underlying wave is empirically invisible. It is the modulation - the de Broglie wave - that defines the energy and momentum of the moving particle. However, it is the standing wave that oscillates at the frequency ω_0 of the particle at rest and, as I will also show, it is the manner in which this underlying carrier wave must adapt to a change of inertial frame that explains the Planck-Einstein and de Broglie relations (Eqns. (1) and (2)), as well as the relativistic equation of motion,

$$E^2 - p^2c^2 = m^2c^4, \tag{4}$$

which relates the energy and momentum of the particle to its rest mass.

In this conception of a massive particle, there is a single modulated wave structure, and it is thus to be distinguished from various other attempts that have been made to make more sense of the de Broglie wave. These have included dual and triple wave proposals, such as those of Horodecki [18] and [19], and Das [20] and [21], which suppose an additional wave moving at the velocity v of the particle.

Nor is this interpretation akin to the “double solution” theory proposed during the 1920s by de Broglie himself (see de Broglie [22] and Vigier [23]), which contemplated coupled solutions of a Schrödinger or Schrödinger-like equation, these being the usual ψ wave function having probabilistic significance, and an additional u wave representing the particle, perhaps as a singularity or “humped” or “extended” particle (Lochak [24] and Martins [25], respectively), or as it might now be termed, a soliton. (For recent discussions, see Fargue [26] and Colin et al [27]).

In the pilot wave theory that de Broglie presented at the fifth Solvay conference in 1927 (de Broglie [28]), the u wave had become a point particle guided by the ψ wave. And so it has remained in Bohm’s rediscovery and revision of de Broglie’s theory as "the causal interpretation" (Bohm [29] and [30]), now more usually referred to as "Bohmian mechanics" (see the review by Goldstein [31]).

The standing wave contemplated here cannot be quite the same thing as the excitation of the quantum field assumed by quantum field theory. In its modal expansions, quantum field theory has carried with it from quantum mechanics the notion of a wave packet. Yet it may be possible to discern in the Lagrangians and quantized fields of quantum field theory a correspondence with the superpositions of counter-propagating waves that will be described in this paper.

In the next section (Sect. 3), I will show how the de Broglie wave emerges from the underlying wave structure, and will consider in Sect. 4 why this interpretation of the wave seems not to have been noticed by de Broglie himself. I will deal with these matters only briefly here as they have been considered more fully elsewhere (see Shanahan [12]).

After establishing a suitably structured wave model in Sect. 5, I will use this model to show in Sect. 6 how the dynamic properties of a particle, including its mass, energy, momentum and inertia, arise from corresponding wave characteristics, and will consider what this might mean for the notion of wave-particle duality. In Sect. 7, I will consider the implications of this wave-theoretic treatment of matter for the existence of realistic particle trajectories. In Sect. 8, I will then canvas briefly some further implications of this interpretation of matter, before concluding with a brief summary in Sect. 9.

3. The modulation

It is a simple matter to show that a modulation with the velocity and wave characteristics of the de Broglie wave emerges from the Lorentz transformation of a standing wave, and that it does so whatever the form of that standing wave. The modulation will emerge in this way whether it is assumed that the standing wave is a solution of the wave equation, or some form of soliton, or simply the superposition of incoming and outgoing electromagnetic or other influences.

Consider the standing wave,

$$R(x, y, z)\, e^{i\omega t}, \tag{5}$$

which is evolving in time at some frequency ω, but for which no assumption has been made as yet as to its manner of spatial variation. Following a boost,

$$\begin{aligned} x' &= \gamma\,(x - vt)\,, \\ t' &= \gamma\left(t - \frac{vx}{c^2}\right), \end{aligned}$$

where γ is the usual Lorentz factor,

$$(1 - \frac{v^2}{c^2})^{-\frac{1}{2}},$$

standing wave (5) becomes the moving wave,

$$R(\gamma\,(x - vt)\,,\, y,\, z)\, e^{i\omega\gamma\left(t - vx/c^2\right)}. \tag{6}$$

in which the spatial factor $R(x, y, z)$ of standing wave (5) has become the carrier wave,

$$R(\gamma\,(x - vt)\,,\, y,\, z), \tag{7}$$

which is evidently moving at the velocity v and, as indicated by the inclusion of the Lorentz factor γ, has suffered the contraction of length predicted by special relativity.

The second factor in wave (6),

$$e^{i\omega\gamma\left(t - vx/c^2\right)}, \tag{8}$$

is a transverse plane wave, which is moving through the carrier wave (7) at the superluminal velocity c^2/v. If the frequency ω is now identified as the natural frequency ω_0 of a massive particle, (or atom or molecule), wave factor (8) can be rewritten in terms of the Einstein frequency,

$$\omega_E = \frac{E}{\hbar}\gamma\omega_0, \tag{9}$$

and de Broglie wave number,

$$\kappa_{dB} = \frac{p}{\hbar} = \gamma\omega_0\,\frac{v}{c^2}, \tag{10}$$

as,

$$e^{i(\omega_E\,t-\kappa_{dB}\,x)}, \tag{11}$$

and is now recognizable as the de Broglie wave, no longer an independent wave, but a modulation. The full composite wave is then,

$$R(\gamma\,(x-vt)\,,\,y,\,z)\,e^{i(\omega_E t-\kappa_{dB} x)}. \tag{12}$$

Once seen as a modulation, rather than a wave in its own right, the superluminal velocity of the wave is no longer that of energy transport and need not be explained away by the awkward device of equating the velocity of the particle with the group velocity of a superposition of such de Broglie waves. It is also only natural that the velocity of this modulation should increase as the particle slows, and become infinite - or more correctly, disappear - as the particle comes to rest.

As will be shown in Sect. 4, it is the full modulated wave, rather than the de Broglie wave considered alone, that provides an understanding of the dynamic properties of a massive particle. Significantly, it is also the full wave rather than the modulation considered alone that displays the full complement of changes in length, time and simultaneity contemplated by special relativity[1].

4. De Broglie's thesis

If this is the true nature of the de Broglie wave, it will be asked why de Broglie did not see that it is so. It is apparent from the famous thesis that he did recognize that

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[1] In the de Broglie wave, $e^{i\omega_E t}$ describes the increased frequency of the moving particle, while $e^{-i\kappa_{dB}x}$ describes the loss of phase due to the failure of simultaneity in the direction of travel. The gain in phase due to the particle's increased frequency is approximately half that due to the loss from the relativity of simultaneity. It is in combination that these two effects describe the net loss in phase and consequent slowing of time observed on a complete particle orbit. It is this slowing of time that explains the "twin effect" - the slower aging of the travelling twin.

the "electron" [2] must be surrounded in its rest frame by what he termed a "periodic phenomenon", which he seems to have assumed to be, and which could only have been, some form of standing wave (see Shanahan [12]).

But because, as de Broglie explained in concluding the thesis, his proposals were "not entirely precise", he left the description of this periodic phenomenon "intentionally vague". It was presumably for that reason that there is nowhere in the thesis a description in mathematical terms of the antecedent standing wave, and nor then could consideration be given to how a standing wave changes under the Lorentz transformation. As was shown in Sect. 3, any such analysis would have revealed a wave of the form (12), in which the de Broglie wave is merely the modulating factor in a composite wave structure.

De Broglie's primary derivation was based on what he called "the theorem of the harmony of phases", essentially the requirement that since the phase of a wave is a scalar, all relatively moving observers must agree on its value at any point of time and space. He began this derivation by referring again to the periodic phenomenon, and on a casual reading it may seem that he then went on to apply the theorem to a spatially extended wave. But what de Broglie eventually transformed was not the spatially extended wave, but a single oscillating point in that wave - the location of the electron which he assumed to be point-like.

And as it so happens, the Lorentz transformation of an oscillating point does generate something that might be mistaken for a wave. Under a boost (in the x-direction), the oscillating point,

$$\delta[x, y, z]\, e^{i\omega_0 t}, \tag{13}$$

(where $\delta[x, y, z]$ is the Dirac delta function) becomes,

$$\delta[\gamma(x - vt),\, y, z]\, e^{i(\omega_E t - \kappa_{dB} x)}, \tag{14}$$

where the second factor has the functional form of the de Broglie wave, but is describing, not an actual wave, but the track through space and time of a moving and oscillating point.

Like a beach ball on a tidal flow, an oscillating point might define the form of a wave as it moves, but is not itself a wave. Under a Lorentz transformation, a point remains a point, and a wave, although changed in form, remains a wave.

De Broglie provided two further demonstrations of his wave, one involving the Lorentz transformation of a mechanical wave contraption, or as it might be called, a toy model, and

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[2] The novelty of what de Broglie was considering is reflected in the descriptions he uses. He refers to the "electron" rather than to particles generally. Until Rutherford's discovery of the proton in 1919 and its naming in 1920, the electron had been the only massive particle known (see Romer [32]). De Broglie was also in newly explored territory with the photon, which he refers to as an "atom of light". De Broglie was one of the very first to take seriously Einstein's proposal (Einstein [33]) that light is not only absorbed and emitted in quanta, but subsists as such.

the other, the transformation of an extended wave in Minkowski spacetime. In each case, what was transformed was not simply an oscillating point, but a spatially extended standing wave (or model thereof), and the wave that resulted from that transformation was not the independent wave supposed by de Broglie, but the modulated wave discussed in Sect. 2.

In effect, de Broglie took a standing wave, Lorentz transformed that wave, and adopted as the result of the transformation, only one of the two wave factors constituting the transformed wave.

It should be said that de Broglie's stated objective in introducing the mechanical contraption was not to derive his wave, but to show how a wave with a velocity greater than that of light might yet be consistent with special relativity provided the velocity of energy transport is less than c. In that objective, the model succeeds very well. Yet this simple mechanical contraption also provides an intuitive illustration of how a modulation with the characteristics of the de Broglie arises from the Lorentz transformation of a standing wave.

All three demonstrations have been analyzed more fully elsewhere (see Shanahan [12]). But the reader might as easily and no doubt more profitably reach an understanding of what de Broglie actually derived by going directly to the primary source, the thesis itself, a document of considerable significance to the evolution of quantum mechanics.

That is not an onerous exercise. These matters were considered by de Broglie in two brief sections, clearly expressed and numbering only some ten pages in all, of the introductory chapter of the thesis, which is readily accessible, not only in the original French, but in German and English translation (see Ref. [1]).

5. The primacy of c

I will show in the next section (Sect. 6) how the underlying standing wave explains the Planck-Einstein and de Broglie relations (Eqns. (1) and (2)) and in turn the relativistic equation of motion (Eqn. 4). With the mass, energy, momentum and inertia of an elementary particle thus explained from the properties of the wave, it will become apparent that there is neither the necessity nor the possibility of apportioning any part of those properties to something "solid" or point-like within the wave.

In considering those properties, it will be helpful to have before us a model that exhibits explicitly the fundamental nature of the velocity c. The primacy of c was demonstrated in the previous section in the process that led from standing wave (5) to modulated wave (12). Yet nothing was stipulated in the formulation of wave (5) regarding the velocity of the counter-propagating influences constituting that standing wave. Those constituent waves might have been, for example, sound or water waves, and it would have followed from the generality of standing wave (5) that the resulting modulation had the velocity c^2/v. It is the Lorentz transformation that imposes the velocity c, and it does so because this transformation assumes (as Einstein saw in 1905 [34]) that all underlying influences evolve ultimately at that velocity.

That this is so is implicit in those thought experiments of Einstein in which light rays pass to and fro within some physical structure, such as a railway carriage or a light clock. If the velocity of light is to be the same for all observers, those structures must contract along the direction of relative motion and experience changes in their oscillatory and thus temporal characteristics replicating precisely the changes defined by those superpositions of counter-propagating light paths.

The argument can be put the other way around. If there were some influence in Nature that evolved at a velocity differing from c, let us say the velocity V, the Lorentz factor would take for that particular effect, the form

$$(1-\frac{v^2}{V^2})^{-\frac{1}{2}},$$

and the laws of physics could not then be the same in all inertial frames (see Shanahan [11]).

While massive particles do not move at velocity c, it is implicit in special relativity that the influences by which these particles interact do develop between and through the particles at velocity c. Refracted light also has a velocity differing from c, but this is the result of interference between waves that do evolve at the velocity c. From interference between the incident wave, and reradiation from moments induced by that wave, the transmitted wave acquires a phase velocity that may be greater or smaller than c. Nonetheless, the front of a pulse of light and any disruption to the waveform develops through the medium at the velocity c (see Gauthier et al [35]).

I adopt as a suitable model,

$$\psi\left(\mathbf{r},t\right)=\frac{1}{2}\left|\mathbf{r}\right|^{-1}[e^{i(\omega_o t-\kappa_o\cdot\mathbf{r})}-e^{i(\omega_o t+\kappa_o\cdot\mathbf{r})}], \tag{15}$$

which is a spherical standing wave centred at $\mathbf{r}=0$, and constructed from incoming and outgoing influences of velocity c, where,

$$\frac{\omega_o}{\kappa_o}=c,$$

(κ_o being not the de Broglie wave number but the wave number that must be associated with a wave of frequency ω_o and velocity c).

This wave has a singularity at the origin and is thus unphysical, but will suffice to show how the dynamic properties of a massive particle might originate in a fully wave-theoretic treatment of matter.

On a boost in the x-direction, model (15) becomes (on taking real parts),

$$\Psi\left(x,y,z,t\right)=\sin\kappa_o\sqrt{\gamma^2(x-vt)^2+y^2+z^2}\,\cos(\omega_E t-\kappa_{dB}x), \tag{16}$$

(where to simplify matters an amplitude factor has also been omitted).

Notice again the composite form of the moving wave. It comprises, as one factor, the carrier wave,

$$\sin \kappa_o \sqrt{\gamma^2(x-vt)^2+y^2+z^2}, \tag{17}$$

of velocity v, which has a relativistically contracted ellipsoidal form, and as modulating factor, the de Broglie wave,

$$\cos(\omega_E t - \kappa_{dB} x),$$

which is of planar form and is moving through the carrier wave at the superluminal velocity c^2/v.

To show how these changes in wave structure might be related to dynamic changes in the particle, it will suffice to concentrate on rays passing through the particle centre and moving forwardly and rearwardly along the direction of travel[3]. In the rest frame of the particle, the superposition of these rays produces the one-dimensional standing wave,

$$\Psi(x,t) = \frac{1}{2}[e^{i(\omega_o t-\kappa_o x)} - e^{i(\omega_o t+\kappa_o x)}] = \sin\kappa_o x \cos\omega_o t, \tag{18}$$

(taking only real parts), but when observed from a frame in which the particle is moving at velocity v, these forwardly and rearwardly moving rays, to be now labelled 1 and 2 respectively, transform as,

$$\begin{aligned} e^{i(\omega_o t-\kappa_o x)} &\rightarrow e^{i(\omega_1 t-\kappa_1 x)}, \\ e^{i(\omega_o t+\kappa_o x)} &\rightarrow e^{i(\omega_2 t+\kappa_2 x)}, \end{aligned}$$

where,

$$\omega_1 = \gamma\omega_0(1+\frac{v}{c}), \quad \omega_2 = \gamma\omega_0(1-\frac{v}{c}), \tag{19}$$

$$\kappa_1 = \gamma\kappa_0(1+\frac{v}{c}), \quad \kappa_2 = \gamma\kappa_0(1-\frac{v}{c}), \tag{20}$$

and standing wave (18) becomes,

$$\Psi\,(x,t) = \frac{1}{2}[e^{i(\omega_1 t-\kappa_1 x)} - e^{i(\omega_2 t+\kappa_2 x)}]/2, \tag{21}$$

which can also be written,

$$\Psi\,(x,t) = \sin(\frac{\omega_1-\omega_2}{2}t - \frac{\kappa_1+\kappa_2}{2}x)\cos(\frac{\omega_1+\omega_2}{2}t - \frac{\kappa_1-\kappa_2}{2}x). \tag{22}$$

Although derived from the one-dimensional wave, the dynamic properties now defined by ω_1, ω_2, κ_1, and κ_2 must also be those of the three-dimensional wave as it moves in the same direction. This is obviously so since the de Broglie wave does not itself vary laterally and, as can be seen from Eqn. (12) or Eqn. (16), the carrier wave moves in its entirety at the common velocity v.

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[3] For a consideration of rays in other directions, see Shanahan [11].

6. Mass, energy, momentum and inertia

The dynamic properties of the particle may now be expressed in terms of the wave characteristics ω_1, ω_2, κ_1, and κ_2 of the moving particle. In wave (22), the first factor,

$$\sin(\frac{\omega_1 - \omega_2}{2}t - \frac{\kappa_1 + \kappa_2}{2}x),$$

is the carrier wave, the velocity of which is,

$$v = \frac{\omega_1 - \omega_2}{\kappa_1 + \kappa_2},$$

while the Lorentz factor becomes,

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} = \frac{\omega_1 + \omega_2}{2\omega_0}. \tag{23}$$

The second factor,

$$\cos(\frac{\omega_1 + \omega_2}{2}t - \frac{\kappa_1 - \kappa_2}{2}x),$$

in wave (22) is the de Broglie wave, from which the Einstein frequency and de Broglie wave number are therefore, respectively,

$$\omega_E = \frac{\omega_1 + \omega_2}{2}, \tag{24}$$

and,

$$\kappa_{dB} = \frac{\kappa_1 - \kappa_2}{2}. \tag{25}$$

With the use of Eqns. (1) and (2), and in natural units in which $\hbar = c = 1$, Eqns.(24) and (25) become,

$$E = \frac{\omega_1 + \omega_2}{2}, \tag{26}$$

and,

$$p = \frac{\omega_1 - \omega_2}{2}, \tag{27}$$

so that the energy and momentum of the particle are explained in an intuitive way as, respectively, the sum of and the difference between the energies of forwardly and rearwardly moving waves[4].

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[4] For a particle at rest, Eqn. (4) reduces of course to Einstein's famous,

$$E = mc^2,$$

and it is interesting that this equation was derived in Einstein's second relativistic paper of 1905 (Einstein [36]) with the aid of a thought experiment involving pulses of light emitted in opposite directions. Einstein's treatment was in terms of changes in the energies of these waves, rather than, as here, changes in the wave characteristics of the counterpropagating waves.

In the same natural units, it follows from Eqns. (19) and (20) that,

$$m = \omega_0 = \sqrt{\omega_1 \omega_2}, \tag{28}$$

while the relativistic equation of motion (4) becomes,

$$E^2 - p^2 = m^2, \tag{29}$$

which is just the equality,

$$(\frac{\omega_1 + \omega_2}{2})^2 - (\frac{\omega_1 - \omega_2}{2})^2 = \omega_1 \omega_2 = \omega_0^2. \tag{30}$$

It can thus be seen that it is because a massive particle is in some sense a standing wave that the equation of motion (29) is of non-linear form and is not, for instance,

$$E - p = m.$$

If inertia is now interpreted, not simply as the resistance of a massive particle to changes in its state of motion, but at a more fundamental level, as the resistance of a wave to changes in its oscillatory state, we have in Eqns. (26) to (30), a consistent scheme for the treatment in terms of wave characteristics of the energy, momentum, inertia and mass of an elementary particle[5].

And just as the mass m is the Lorentz invariant for the four-vector (E, p), and the frequency ω_0 is the corresponding invariant for the four-vector (ω, κ), the antecedent standing wave becomes the invariant form to which the composite travelling wave reverts in the inertial frame of the particle.

If the superluminal de Broglie wave were the only wave associated with a massive particle, it would be necessary to suppose something more, perhaps something small and solid, within the wave. But the equivalence of dynamic properties and wave characteristics described by Eqns. (26) to (30) leaves no part of those dynamic properties to be apportioned to anything other than the modulated wave structure. Moreover, the wave structure moves with the velocity of the particle and as illustrated by its modelling above may have a

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[5] Other relationships may also be expressed very simply in terms of ω_1 and ω_2, for instance the rapidity, which serves as a measure of relativistic velocity, becomes,

$$\phi = \ln \omega_1 - \ln \omega_2.$$

while in solutions to the Dirac equation,

$$\begin{aligned} E + p &= \frac{\omega_1 + \omega_2}{2} + \frac{\omega_1 - \omega_2}{2} = \omega_1, \\ E - p &= \frac{\omega_1 + \omega_2}{2} - \frac{\omega_1 - \omega_2}{2} = \omega_2. \end{aligned}$$

well-defined centre following a well-defined trajectory. The presence of something solid within the wave is thus redundant - a discontinuity in the wave and an embarrassment to the theory of the wave.

If a massive particle is wave-like, one might ask what is doing the waving, what it is waving in and, if the particle is a standing wave, what is constraining the wave at its extremities. Some things at least are reasonably clear. The wave must presumably be non-dispersive and linear, and the medium, if there is one, elastic and of linear response. As to its boundary conditions, it is sufficient to suppose that every particle is constrained by its interactions with every other particle (as contemplated by Wheeler and Feynman [37] and [38]).

7. Realistic trajectories?

If the de Broglie wave is the modulation contemplated in this paper, it is not so much piloting the particle, but like the bowsprit of a sailing boat, turning with the underlying wave structure. Yet if the existence of the carrier wave were to go unnoticed, it might well seem that the de Broglie wave is somehow guiding the particle. It is after all the wave vector of the de Broglie wave that identifies the momentum of the particle.

It is also this role of the de Broglie wave that explains the relevance to de Broglie-Bohm theories of the wave functions that emerge as solutions of the Schrödinger equation. That equation was conceived as an equation for the de Broglie wave (see Bloch [39] and Bacciagaluppi and Valentini [40], esp. Chaps. 2 and 11), as also were those other wave equations of quantum mechanics for massive particles - the Klein-Gordon, Pauli and Dirac equations (see for the last, Dirac [41]).

In constructing a wave equation that would have solutions consistent with the Planck-Einstein and de Broglie relations (Eqns. (1) and (2)), Schrödinger made the substitutions[6],

$$p \rightarrow i\hbar\frac{\partial}{\partial x},$$
$$E \rightarrow i\hbar\frac{\partial}{\partial t},$$

in the non-relativistic equation of motion,

$$E = \frac{p^2}{2} + V(\mathbf{r}, t),$$

[6] This is the most direct way of deriving the equations, and seems to have been the way they were initially obtained by Schrödinger (Moore [42], pp. 142-143), but in his published papers, Schrödinger gave other derivations (see Schrödinger [43])

to obtain the non-relativistic Schrödinger equation,

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V(\mathbf{r},t)\psi, \tag{31}$$

and likewise in the relativistic equation of motion,

$$E^2 - p^2c^2 = m^2c^4,$$

(which is Eqn. (4) above) to obtain (in free space) the relativistic equation,

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\psi - \nabla^2\psi + \frac{m^2c^2}{\hbar^2}\psi = 0,$$

now called the Klein-Gordon equation.

Because these equations were based on the Planck-Einstein and de Broglie relations, the frequency and wave vector of any solution to the equation must be consistent with the energy and momentum of the particle. But in the absence of the carrier wave, the best that these wave functions can generally do is identify permissible energies and possible trajectories.

For a particle of well–defined momentum moving freely in the absence of a constraining potential, the solution of the Klein-Gordon equation is simply the de Broglie wave of Eqn. (3), that is to say, a plane wave of superluminal velocity,

$$\psi = e^{i(\omega_E t - \kappa_{dB} x)},$$

that identifies the energy and momentum of the particle, but can say nothing at all regarding its location. Even in this elementary case, the wave function can be seen to be, as Schrödinger himself suggested, a "smeared out" superposition of all possible trajectories (Schrödinger [44]). In his report to the Solvay conference of 1927, Schrödinger described the wave function as "something that continuously fills the entire space and of which one would obtain a 'snapshot' if one dragged the classical system, with the camera shutter open, through all its configurations" (see Bacciagaluppi and Valentini [40], p. 411).

In standard quantum mechanics, where the wave function is a probability wave, the de Broglie wave of Eqn. (3) seems to be saying that the particle could be anywhere at all in the Universe, except at the nodes of the sinusoid where the probability falls to zero. This difficulty is addressed in the standard interpretation by assuming that the particle is localized within a wave packet, but as discussed in Sect. 1, such a wave packet spreads with time and very soon the particle may be almost anywhere at all. If the de Broglie wave is recognized instead as a modulation, there is no such difficulty. The location and trajectory of the particle are fixed by the carrier wave.

In this single particle situation, it is only necessary to suppose that a particle has some definite momentum to see an instance of the incompleteness of quantum mechanics asserted by Einstein, Podolsky and Rosen in their famous EPR paper of 1935 [45]. Without the underlying carrier wave to locate the particle, the de Broglie wave itself is incomplete, as also must be the quantum mechanics that supposes that this anomalous superluminal wave is the only wave associated with a massive particle.

As a further comment on the meaning of these equations, it is significant I suggest that the Dirac bispinor achieves a partial recovery of the modulated structure contended for in this paper. As discussed above, the Dirac equation was constructed as a equation for the de Broglie wave (Dirac [41]). But the equation was contrived in such a way that its solutions are able to suggest the spin and helicity of the electron.

Its author accomplished that impressive feat by factorizing the Klein-Gordon equation, or as he recounted on several occasions, by "playing around with equations". But what is of some interest in the context of the present discussion is that the Dirac equation can instead be reverse engineered from a superposition of null spinors propagating, as has been said, on diametrically opposite sides of the light cone (see Ryder [46] and Steane [47]).

Considered in this way, the Dirac electron becomes in effect a subluminal wave structure moving at the velocity of the particle, but assembled from counter-propagating waves of velocity c, and subject to the superluminal modulation contemplated above.

8. Discussion

Schrödinger had initially argued that his wave functions should be interpreted in a physically realistic sense as charge density waves. But notwithstanding the dissent of Einstein and the misgivings of de Broglie and Schrödinger, it was the probabilistic interpretation of Born that became what is still today the orthodox or standard interpretation of quantum mechanics (for the interesting debates that led to this, see Jammer [48] and Bacciagaluppi and Valentini [40]).

Yet the protagonists in that contest had supposed the independent de Broglie wave described by de Broglie in his thesis of 1923 [1]. To a significant extent, the arguments given for abandoning physical reality were thus based on a phenomenon that, according to the understanding of the de Broglie wave espoused in this paper, does not in fact exist.

One such argument concerned the many body problem. For a system of n mutually interacting particles, the Schrödinger wave function describes the evolution, not of separable waves, but of a single point in a configuration space of $3n$ dimensions. In classical mechanics, it is merely a matter of convenience whether such a problem is considered in a $3n$ configuration space, or in phase space, or in ordinary space of three dimensions (see Goldstein [49], esp. Chap 10-8). But in quantum mechanics the necessity for an analysis in configuration space seems to be an unavoidable consequence of the wave-like nature of the interacting particles. This has meant in effect, not only that the wave function must be treated as an indivisible whole, but that the measurement probabilities for the individual particles cannot be factored out from the probabilities for the composite system.

This reliance on configuration space drew criticism, not only from the proponents of the rival matrix mechanics, but also from Einstein who favoured the continuity of evolution implicit in a wave-theoretic approach, but was concerned by the inability of Schrödinger's wave functions to describe individual trajectories in three dimensions (see Howard [50]). It was also objected that although Schrödinger's suggestion of a charge density wave might explain the wave-like properties of the particle, it left its particulate properties largely unexplained and said nothing at all regarding the relationships between these various properties (see generally Tomonaga [51], Vol. 2, chap. 6).

Those particular concerns have been addressed in the discussion above. According to the argument of Sect. 6, it is the modulated wave structure that explains the relationship between the wave and particulate properties of the particle, while as shown in Sect. 7, it is the underlying carrier wave that localizes the particle and is thus capable of describing, at least in principle, a physically realistic trajectory. Nor with the superluminality of de Broglie wave explained in a manner consistent with special relativity is there any need to suppose that the particle itself could be anywhere at all in a rapidly spreading wave packet. It is suggested that once definite particle trajectories can be contemplated, a reliance on configuration space should not be an obstacle to a physically realistic theory. Any system of mutually interfering waveforms, real or probabilistic, that combine to form a composite whole of reduced energy might well be as non-factorizable as the wave functions of Schrödinger.

There is also the mystery of self (single particle) interference, which still remains a significant challenge for physical reality. But this phenomenon presents difficulties for all interpretations of quantum mechanics, including standard quantum mechanics, where the notion of a divisible probability wave seems no less mysterious than the self interference itself. On the other hand, the possibility that a particle is an extended wave structure, rather than a small solid or point-like object, may be one step toward an explanation of this curious effect (see Shanahan [52]).

The apparent nonlocality of entanglement stands as a further obstacle to a coherent treatment of quantum mechanics. This nonlocality is usually explained as arising from the non-factorizability of measurement probabilities referred to above, but it is not at all apparent why nonlocality should be the inevitable consequence of non-factorizabilty. While a nonlocal prediction might well arise from a non-relativistic equation, it might reasonably have been expected that the curious correlations between entangled particles would become increasingly attenuated - in the same manner as the electromagnetic force - with the increasing separation of the particles. Or as had been suggested by Furry [53] ("the Furry hypothesis") that the wave function would then factorize spontaneously. But it would seem from a series of Bell's experiments, culminating with the recent spate of “loophole free” experiments (that commenced with that of Hensen et al [54]), that entanglement is indeed nonlocal.

Whether Nature is ultimately revealed to be local or nonlocal, there is in this a considerable mystery. If, on the one hand, Nature is locally causal, there must exist some experimental loophole or correlating or holistic effect that has as yet eluded recognition, perhaps some aspect of statistical mechanics or electromagnetic theory, or even the superdeterminism of ’t Hooft’s cellular automaton interpretation of quantum mechanics (’t Hooft [55]).

On the other hand, if Nature is nonlocal, it remains to be explained how these superluminal quantum influences of unknown ontology are to be reconciled with those other influences, electromagnetic and gravitational, that do respect the limiting velocity c and yet in some situations must coexist, and presumably compete, with those superluminal effects.

9. Conclusion

From a reconsideration of the de Broglie wave, three simplifying reconciliations have been proposed. In the *first*, the de Broglie wave merges with the particle in a single modulated wave structure. In the *second*, the wave-particle duality of matter is resolved in favour of a wave-theoretic treatment of matter and radiation. And in the *third*, the superluminality of de Broglie wave is reconciled with the limiting velocity c of special relativity.

If a massive particle is indeed the modulated structure proposed, there is of course much else to consider. It might be asked, for instance, how one such wave structure might interact with another, or whether whether spin, parity and charge might correspond to asymmetries in that wave structure, and how trajectories are to be identified in situations more complicated than the simple example considered in Sect. 7.

That such questions are worth pursuing is suggested by what has already been demonstrated in the discussion above. Despite the many species of massive particles comprehended by the standard model, and irrespective of the many free parameters and other puzzling features of that model, it is only necessary to suppose that a massive particle is some form of standing wave to see why every such species of particle has a mass, energy and momentum determined exactly by its wave characteristics.

What the de Broglie wave seems to be telling us then is that at a fundamental level a massive particle has the form of a standing wave comprising counter-propagating influences evolving at the velocity c of light.

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