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Original Paper

Hidden-time in Quantum Mechanics

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Abstract: This paper aims to give an alternative interpretation of quantum mechanics based on conjecturing the existence of a hidden infinitesimal complex time. It is shown that many features of quantum mechanics emerge from the conjectured hidden-time. The goal of this paper is to better understand quantum phenomena or at least to render them more logical to our understanding.

Keywords: hidden-time; ensemble of positions; Brownian motion; measurement; arrow of time

I. Introduction

Conventionally, the startling features of quantum mechanics such as the "particle-wave duality" are illustrated by the double-slit experiment. When particles such as electrons are sent one at a time through a double-slit plate, single random impacts are observed on a detector screen behind the plate as expected out of individual particles. However, when the electrons are allowed to build up one by one, the cumulative effect of a great number of impacts on the detector screen reveals an interference pattern of light and dark bands characteristic of waves arriving at the screen from the two slits. This phenomenon seems to entail that the electrons embody a wave-like feature in addition to their particle nature hence illustrating a particle-wave duality structure.

When the electrons are made to build up one by one while detectors are placed at the slits to find out through which slit each electron went, the interference pattern disappears, and the electrons behave solely as particles. It seems thus impossible to observe interference and to simultaneously know through which slit the particle has passed. The best explanation given by conventional quantum mechanics is that the same electron seems to pass *simultaneously* through both slits when no detectors are present and through only one slit when detectors are present [1]. This seemingly paradoxical statement seems to be in conformity with the experimental data.

Conventionally, when no detectors are present, the state-vector $|\psi\rangle$ (or wave function ψ) of the electron is said to evolve per a deterministic continuous unitary evolution U whereas, when detectors measure from which slit the electron passes, the deterministic evolution of the state-vector $|\psi\rangle$ is transformed into a probabilistic discontinuous and non-linear state reduction R as explained by Penrose [2]. The two processes U and R create a conflict in the formalism of quantum mechanics. Different ontologies have been proposed to interpret the strange

combination of the deterministic continuous U process with the probabilistic discontinuous R process.

According to the Copenhagen interpretation [3, 4], the state-vector $|\psi\rangle$ and the U and R processes should be regarded as a description of the experimenter's knowledge. There exist several other interpretations amongst which the Everett interpretation or what is more commonly known as the many-world interpretation [5], according to which there is no wave function collapse and all measurement results exist but in different worlds. In line with this interpretation, it is claimed [6] that when a measurement is conducted on an electron in a superposition state, a deterministic branching takes place where on one branch a first detector detects the electron while a second detector doesn't and at the same time but on the other branch (i.e. another world), the first detector doesn't detect the electron while the second detector does detect it. However, this interpretation pauses some probabilistic as well as ontological problems.

Another model is the De Broglie-Bohm deterministic theory according to which particles always have definite positions and velocities. In the double-slit experiment, the particle goes through only one slit, but its behaviour is affected by a quantum potential (or sometimes referred to as a guiding wave) that encapsulates instantaneous information about the whole environment. The guiding wave influences the motion of the particle in a way that produces the same experimental predictions. This theory seems to make more sense of quantum mechanics than the other interpretations as discussed in detail by Jean Bricmont in his book "Making Sense of Quantum Mechanics [7].

A review of these different models is described by Shan Gao in his book "The Meaning of the Wave Function" [8]. Gao considers that there are two fundamental problems in the conceptual foundations of quantum mechanics. The first concerns the physical meaning of the wave function and the second is the measurement problem. Gao proposes a new ontological interpretation of the wave function in terms of random discontinuous motion RDM of particles and shows the implications of this new interpretation on the measurement problem.

We believe that the main mystery of the physical meaning of the wave function and the measurement problem resides in the superposition principle that makes a quantum system take different and sometimes conflicting states at the same instant of time t.

In this paper, we conjecture the existence of a hidden infinitesimal complex time that discriminates the different states taken by the quantum system making thus sense of some remarkable features of quantum mechanics.

II. Hidden time

1. Basic hypothesis: Extended-time

We propose to show that many features of quantum mechanics emerge out logically from a hypothetical *hidden-time*.

According to this hypothesis, time referred hereafter as "*extended-time*" is considered to be three-dimensional and thus, space-time becomes a six-dimensional differentiable manifold composed of a collection of points, each of which corresponds to a unique position in ordinary space defined in $\mathbf{R^3}$ and a unique date in time defined in **R-C** where **R** is the set of real numbers and **C** the set of complex numbers.

The main reason for introducing complex time is due to the fact that in Quantum Mechanics, complex variables do not seem to be just an *artefact* for making calculations but rather an intrinsic property as evidenced by the complex valued probability amplitudes. Another reason is that complex-time helps to avoid causality violation while still being consistent with relativity and in particular, with the constancy of the speed of light in all inertial frames. Indeed, the proper time for light $(\Delta\tau)^2 = (\Delta t)^2 - (\Delta X)^2 = 0$ and thus, if one or more

real time dimension(s) is(are) added, the sum of their squares can never be non-trivially equal to zero. Meanwhile, the square of an extra real time dimension is positive and the square of an extra imaginary time dimension is negative and their sum can therefore be made equal to zero. *Extended-time* can be represented in a three-dimensional orthogonal coordinate system **R-C** composed of a real line **R** 'orthogonal' to a complex plane **C**, wherein, the real line **R** corresponds to the "*ordinary-time*". The 3d representation of *extended-time* can be 'geometrically' perceived as a 'cylinder' whose longitudinal axis is the ordinary-time-line. Moreover, the cylinder must be imagined as partitioned into slices perpendicular to the ordinary-time-line such that each slice, hereafter called "*t-slice*" is associated to a corresponding ordinary-time index t. In other words, each ordinary-time-index t is paired with exactly one *t-slice* and vice-versa. There is thus a one-to-one correspondence between the set of ordinary-time indices t and the set of *t-slices*.

A physical-system (for example, a particle) can thus be considered as evolving with respect to an *extended-temporal-set* $S(t,s)$ of "*extended-time-indices*" $w=(t,s)$ where $t \in \boldsymbol{R}$ and $s \in \boldsymbol{C}$ and where each ordinary-time-index t is associated with a *sub-temporal-set* $C(t)$ of complex numbers made up of "*sub-time-indices" s* belonging to the corresponding "*t-slice*".

For the sake of simplicity of notation and in order to highlight the fact that each ordinary-time-index t is associated with a plurality of sub-time-indices, discrete subscripts s_k are used to differentiate between the different sub-time-indices even if the sub-temporal set $C(t)$ may be continuous. A *sub-temporal-set* $C(t)$ can thus, be defined as follows:

$$\forall\, t \in \boldsymbol{R},\ \exists\, C(t) \subset \boldsymbol{C}\ ; C(t) = \{s_k\}_k\ \ s_k \in \boldsymbol{C} \quad (1)$$

For any quantum entity, there is a one-to-one correspondence between the set of ordinary-time indices $\{t\}$ and the set of *sub-temporal-sets* $\{C(t)\}$. The transition from one ordinary-time-instant t to another takes place bloc-by-bloc (i.e. a pure quantum state at a sub-time index evolves through ordinary time as part of a bloc of states).

On the other hand, the *extended-temporal-set* $S(t,s_k)$ can be defined as follows :

$$S(t,s_k) = \{w=(t,s_k),\ t \in \boldsymbol{R}\ ,\ s_k \in C(t)\ \} \quad (2)$$

2. Ensemble of positions and momenta

Sub-time enables different states of the physical system to occur at the same ordinary-time-index t simply because they take place at different sub-time-indices s_k. However, being made up of complex numbers, there is no natural order between the sub-time-indices. Thus, our knowledge of the dynamics of the physical system with respect to sub-time would at most be restricted to knowing an ensemble $\phi(x,p)$ of its different position x and momentum p states (where $x \in \boldsymbol{R}^3, p \in \boldsymbol{R}^3$), as well as the sub-time-indices $s_k \in C(t)$ associated to these states without knowing their order.

The ensemble $\phi(x,p)$ of points associated to the *sub-temporal-set* $C(t)$ can be expressed as a set of position states $x(\mathrm{t},s_k)$ and momentum states $p(\mathrm{t},s_k)$ in a *phase space* as follows:

$$\phi(x,p) = \{\, x(\mathrm{t},s_k), p(\mathrm{t},s_k)\ ;\ \forall\, t \in \boldsymbol{R}, s_k \in C(t)\} \quad (3)$$

The ensemble $\phi(x,p)$ represents the set of position-momentum couples (x,p) associated to the sub-temporal-set $C(t)$ and thus, the position $x(\mathrm{t},s_k)$ as well as the momentum $p(\mathrm{t},s_k)$ of the physical system are well defined.

3. Ensemble of positions

If we are interested to know the different positions visited by the physical system and/or the different momenta taken by the system then, we are obliged to define two separate ensembles: a position ensemble $\phi(x)$ defining the set of position states and a momentum ensemble $\phi(p)$ defining a set of momentum states. Indeed, positions and momenta may be

recurrent and the recurrence of a position does not necessarily match that of the momentum. However, by reorganising the original ensemble $\phi(x,p)$ into two separate secondary ensembles $\phi(x)$ and $\phi(p)$, we pay the price of losing the "coupling-link" between each position x and its corresponding momentum p.

In this section, we are interested in the ensemble of positions. For simplicity, as well as for the uncertainty principle, we shall consider that space is subdivided into a great number of infinitesimal cells $\{x_k\}_k$, where each cell is represented by a discrete position-cell $x_k \subset \boldsymbol{R}^3$.

At any given ordinary-time index t, a position-cell x_k may be visited more than once, and thus, let $D_k(t) = \{s_{kj}\}_j$ be the set of sub-time-indices $s_{kj} \in C(t)$ at which the physical-system visits the position-cell x_k, and let δs_{kj} be the "*sub-time-presence*" of the physical-system at the j[th] visit to the position-cell x_k.

The ensemble of positions visited by the physical-system at any given ordinary-time index t, can then, be defined as follows:

$$\phi(x_k) = \{\, x_k(\mathrm{t}, s_{kj})\,;\ \forall\, t \in \boldsymbol{R}, s_{kj} \in D_k(t) \subset C(t);\ x_k \in \{x_k\}_k\} \qquad (4)$$

A base of position vectors $X = \{|x_k\rangle, \mathrm{k} = 1, \dots., \mathrm{n}, \dots\}$ can be defined out of the ensemble of positions $\phi(x_k)$, such that at any given extended-time-index $w = (t, s_{kj})$, the position-state $|x_k\rangle$ is defined as the state $|\psi(t, s_{kj})\rangle$ of the physical-system at that extended-time-index:

$$|\psi(t, s_{kj})\rangle = |x_k\rangle \text{ where } t \in \boldsymbol{R}, s_{kj} \in D_k(t) \qquad (5)$$

However, to express the state $|\psi(t)\rangle$ of the physical-system at a given ordinary-time-index t, we need to sum-up appropriately the sub-time-indices belonging to each set $D_k(t)$ associated to the corresponding position-state $|x_k\rangle$.

Summing over j of all infinitesimal sub-time-presences δs_{kj} during which the position-cell x_k is visited by the physical system leads to a unique *"resultant-sub-time-presence"* $a_k(t)$ of the physical system at the position-cell x_k, defined as follows:

$$a_k(t) = \textstyle\sum_j \delta s_{kj} \qquad (6)$$

where $a_k(t)$ represents a *"resultant-sub-time-presence"* of the physical system at the position-cell x_k and is associated with a corresponding set $D_k(t)$ of sub-time-indices. As the sub-time-presences δs_{kj} are complex numbers, the *resultant-sub-time-presence* $a_k(t)$ encompasses an auto-interference of the physical-system with itself at each visited position.

On the other hand, let $L(t)$ be a measure (hence a real number) defining the *total-sub-timing-length*, as:

$$L(t)^2 = \textstyle\sum_k \|a_k(t)\|^2 \qquad (7)$$

Then, by considering equations (5), (6) and (7), the state vector $|\psi(t)\rangle$ of the physical-system at a given ordinary-time-index t can be expressed as follows:

$$|\psi(t)\rangle = \textstyle\sum_k \frac{a_k}{L} |x_k\rangle \qquad (8)$$

where each coefficient a_k/L represents the *resultant-sub-time-fraction* spent by the physical system at the position-cell x_k (i.e. base vector $|x_k\rangle$) and where each one of a_k and L depends on the ordinary-time-index t.

Thus, the notion of a state vector $|\psi(t)\rangle$ emerges out of the ensemble of positions $\phi(x_k)$. However, some information concerning the details of the positions of the physical-system is lost and the state vector $|\psi(t)\rangle$ represents a global view of the state of the physical-system as if sub-time is frozen. The state vector $|\psi(t)\rangle$ gives a sort of an "*ordinary-time's-eye-view*", i.e. a static "*sub-timeless-picture*" of the physical system's state.

The state-vector $|\psi(t)\rangle$ defined according to equation (8) is identical to the normalized conventional state-vector where the *resultant-sub-time-fraction* coefficients a_k/L are referred

to as probability amplitudes. As will be shown in section 4.1.1, the probability of the physical system at a given position-state $|x_k\rangle$ is the mod-square of the resultant-sub-time-fraction (see equation (11). In other words, using the language of the wave function $\psi(x,t)$, the mod-square of the resultant-sub-time-fraction represents the mod-square of the probability amplitude $|\psi(x,t)|^2$.

Thus, as in conventional quantum mechanics, the state-vector $|\psi(t)\rangle$ (or wave function $\psi(x,t)$) can be defined as a ray in a Hilbert space over the complex numbers. However, according to the present hidden-time model, it is interpreted as the distribution of the different positions occupied by a physical system at a given ordinary-time-index t, each position $|x_k\rangle$ being affected with the *resultant-sub-time-fraction* a_k/L spent by the physical system at that position $|x_k\rangle$.

Therefore, according to this model, for any given ordinary-time index, a particle is localised at a determined position at every sub-time index such that the total mass M and the total charge Q of the particle is localised at that position. The motion of the particle with respect to its sub-temporal-set $C(t)$ forms the mass and charge distributions of the particle.

According to Gao [8], "*the charge distribution of a quantum system in space can be generated by either (1) a continuous charge distribution with density* $|\psi(x,t)|^2 Q$ *or (2) the motion of a discrete point charge* Q *with time spent* $|\psi(x,t)|^2 dvdt$ *in the infinitesimal spatial volume* dv *around x in the infinitesimal time interval* $[t,\ t+dt]$. *For the first possibility, the charge distribution exists throughout space at the same time. For the second possibility, at every instant there is only a localised, point-like particle with the total charge of the system, and its motion during an infinitesimal time interval forms the effective charge distribution. Concretely speaking, at a particular instant the charge density of the particle in each position is either zero (if the particle is not there) or singular (if the particle is there).*"

According to the present hidden-time model, the charge (or masse) distribution of a particle is a synthesis of the above two possibilities: it is a continuous charge distribution with respect to ordinary time t generated by the motion of the discrete point charge Q with respect to sub-time. In other words, at any given ordinary time index t, the effective charge Q is distributed throughout a spatial extension visited by the particle during its motion with respect to its sub-temporal-set $C(t)$ knowing that at each sub-time index $s_k \in C(t)$ the particle with its total charge Q is localised at a determined position x_k.

4. Evolution of a particle with respect to extended-time

The evolution of a particle with respect to extended-time is twofold, or a two-step-process composed of a sub-temporal-kinematics and an ordinary-time-dynamics.

The sub-temporal-kinematics can be viewed as the evolution of the particle with respect to its corresponding *sub-temporal-set* $C(t)$ while ordinary-time t is kept "frozen". The positions and momenta experienced by the particle at a given ordinary-time-index t represents its "*sub-history*" with respect to sub-time.

On the other hand, the ordinary-time-dynamics can be viewed as the evolution of the "*sub-history*" as an indivisible entity with respect to ordinary-time t while the corresponding *sub-temporal-set* $C(t)$ is considered to be "frozen". Indeed, at each ordinary-time-index t, the sub-history of the particle forms an indivisible whole made up of a sub-timeless-bloc.

4.1 Sub-temporal-kinematics

The sub-temporal-kinematics of a particle can be described through a probabilistic approach or a statistical approach. The probabilistic approach consists in determining the probabilities of the different positions occupied by the particle during its evolution with respect

to the *sub-temporal-set* $C(t)$. The statistical approach consists in applying a statistical method to study the fluctuations of the particle with respect to sub-time.

4.1.1 Probabilistic approach to sub-temporal-kinematics

We are interested in determining the probability of the particle being at any position-state $|x_k\rangle$. We have no knowledge of the 'transitions' between the different position-states, but we do know that when the particle is in the position-state $|x_k\rangle$ at a sub-time index s_k, it is not in any other position-state $|x_j\rangle$ $(j \neq k)$. When we are interested in one particular position-state $|x_k\rangle$, the kinematics can be reduced into two position states consisting of the position-state of interest $|x_k\rangle$ and another global position-state $|X\rangle$ that encapsulates all other position-cells $(x_j)_{j\neq k}$.

We are not interested in what is going on inside the global position-state $|X\rangle$ and therefore, the kinematics can be simply reduced to a 'single' transition from the position-state of interest $|x_k\rangle$ to the global position-state $|X\rangle$ knowing that all passages at the position-cell x_k have already been accounted for by the *resultant-sub-time-presence* $a_k(t)$ associated to the position-state $|x_k\rangle$.

Suppose that the particle 'stays' at the position-state of interest $|x_k\rangle$ during a real sub-time period $\|a_k(t)\|$. As there are only two position-states, the sub-time-period of transition from the position-state of interest $|x_k\rangle$ into the global position-state $|X\rangle$ is equivalent to the sub-time-period of presence at the position-state $|x_k\rangle$. Therefore, the transition from the position-state of interest $|x_k\rangle$ into the global position-state $|X\rangle$ is also equal to the real sub-time period $\|a_k(t)\|$.

Let u denote a dummy real variable of the segment $[0, L(t)]$, where $L(t)$ is the *total-sub-timing-length* and let $Y(u)$ be a random variable describing the position-state of the particle at the variable u. Suppose that at u the particle starts at the position-state $|x_k\rangle$ and at u' it enters the global position-state $|X\rangle$. The real sub-time-period spent by the particle at the position-state $|x_k\rangle$ is $\Delta u = u' - u = \|a_k(t)\|$. The probability corresponding to the particle being at the position-state $|x_k\rangle$ is thus equal to the probability of transition from the position-state $|x_k\rangle$ to the global position-state $|X\rangle$. The probability of the particle being at the position-state $|x_k\rangle$ can thus be defined according to the following Markovian probability:

$$P(Y = |x_k\rangle) = P(Y(u') = |X\rangle \quad | \quad Y(u) = |x_k\rangle) = \delta_{u,u'} + q_{u,u'}\,\Delta u/L\,(t) \qquad (9)$$

where $\delta_{u,u'}$ is the Kronecker symbol which is equal to zero when there is a transition (i.e. $u' \neq u$) between $|x_k\rangle$ and $|X\rangle$ and is equal to 1 otherwise, $\Delta u/L$ is the relative transition period and $q_{u,u'}$ represents the rate of transition from the position-state $|x_k\rangle$ into the global position-state $|X\rangle$. As there are only two states and as the single relative transition period $\Delta u/L$ between the two states is already normalised, then the rate of the transition $q_{u,u'}$ is simply equal to $(1 - \delta_{u,u'})\Delta u/L$.

Thus, equation (9) becomes:

$$P(Y = |x_k\rangle) = \delta_{u,u'} + (1 - \delta_{u,u'})(\Delta u/L(t))^2 = \delta_{u,u'} + (1 - \delta_{u,u'})(\|a_k(t)\|/L(t))^2 \qquad (10)$$

The probability $P(Y = |x_k\rangle)$ indicated in equation (10) has two exclusive terms depending on whether there is a transition or not from the position-state $|x_k\rangle$ into the global position-state $|X\rangle$.

If there is a transition from the position-state $|x_k\rangle$ into the global position-state $|X\rangle$, then $\delta_{u,u'} = 0$ and thus, the probability of the particle to be at the position-state $|x_k\rangle$ is:

$$P(Y = |x_k\rangle) = (\|a_k(t)\|/L(t))^2 \qquad (11)$$

This probability can be considered as a "*density of presence*" of the particle at the position-state $|x_k\rangle$. Indeed, it is highly reasonable to consider that for a given physical-time-

index t, the greater is the value of the *normalised-mod-sub-time-period* $\Delta u = \|a_k\|/L$ spent by the particle at the position-state $|x_k\rangle$, the higher is the probability to find the particle in that position-state $|x_k\rangle$.

However, if there is no transition, then the probability of the particle to be at the state $|x_k\rangle$ is:

$$P(Y = |x_k\rangle) = \delta_{u,u} = 1 \qquad (12)$$

This latter case can be considered as an act of measurement between a "measuring device" and the particle during which one state of the particle is "selected". Indeed, when a state $|x_k\rangle$ is "selected", then there is no more transition from this selected state $|x_k\rangle$ into the global state $|X\rangle$, and thus, the probability of the particle to be at the selected state $|x_k\rangle$ is 1.

It should be noted that the probability for the particle to be at a given position as defined in equation (11) is simply Born's rule which emerges from the notion of *resultant-sub-time-presence* $a_k(t)$ of the physical system at that position. However, unlike conventional quantum mechanics, here probability is the same as in classical probability and is simply a convenient way to fill the gap of our ignorance of the dynamics taking place between the different positions of the particle within sub-time. Nevertheless, it emerges out of the real physical state of the particle.

On the other hand, the "*centroid*" or "*mean value*" of the particle's position at a given ordinary-time index t can be defined as the arithmetic mean of all positions weighted by the corresponding probabilities (i.e. *densities of presence*), as follows:

$$\langle x(t)\rangle = \bar{X} = \sum_k (\|a_k(t)\|/L)^2 \, x_k(t) \qquad (13)$$

The centroid according to equation (13) is simply the expectation value as defined in conventional quantum mechanics.

4.1.2 Statistical approach to sub-temporal-kinematics

In this section, we propose to use Brownian motion in order to have a stochastic but approximate picture of the sub-temporal-kinematics governing the motion of the particle.

A particle can be imagined to be in a *fluid of virtual particles.* The sub-time-interactions at any given ordinary-time-index t, with the virtual-particles push the particle in all directions according to a Brownian motion with respect to the *sub-temporal-set* $C(t)$.

However, for simplicity reasons we shall consider the Brownian motion with respect to the segment $[0, L(t)]$ where $L(t)$ is the *total-sub-timing-length* which is derived from the complex *sub-temporal-set* $C(t)$ at a given ordinary-time-index t.

Indeed, a mapping can be constructed between the set of *"mod-sub-time-presences"* $\{\|a_k(t)\|\}_k$ and the segment $[0, L(t)]$. Each *mod-sub-time-presence* $\|a_k(t)\|$ can be considered as a segment of a certain length and thus, it can be mapped onto a corresponding sub-segment $[u_{k-1}, u_k]$ belonging to $[0, L(t)]$ and whose length is proportional to that of the *mod-sub-time-presence* $\|a_k(t)\|$ (i.e.$\|a_k(t)\| = \mathrm{C}\,[u_{k-1}, u_k]$, where C is a constant).

Let $\rho = \rho(x, u)$ be the probability density that the particle is at the position x ($x \in \boldsymbol{R}^3$) at the real-sub-time-point u where $u \in [0, L(t)]$. According to the Brownian model the probability density $\rho(x, u)$ satisfies the following diffusion equation:

$$\frac{\partial \rho}{\partial u} = \nu \Delta \rho \qquad (14)$$

where ν is a diffusion coefficient defined by Nelson [9] as $\nu = \hbar/2m$.

Let $\bar{X}(t, 0)$ be the medium position of the particle at a given ordinary-time-index t and at $u = 0$. Here, ordinary-time-index t is just a parameter and the probability density $\rho(x(u), u)$ solution of the above equation (14) has the following form:

$$\rho(x, u) = \frac{1}{(4\pi\nu u)^{3/2}} e^{-(x-\bar{X})^2/4\nu u} \qquad (15)$$

Thus, at a given ordinary-time-index t, the ensemble of positions "spreads" from a Dirac function at $x(t,u) = \bar{X}(t,0)$ to a dispersed bell-like Gaussian at $u = L(t)$. In particular, at $u = L$, the probability density $\rho(x,L)$ has the following form:

$$\rho(x,L) = \frac{1}{(4\pi\nu L)^{3/2}} e^{-(x-\bar{X})^2/4\nu L} = \frac{m}{(2\pi\hbar L)^{3/2}} e^{-\frac{1}{2}\left(\frac{x-\bar{X}}{\sqrt{(2\hbar L/m)}}\right)^2} \quad (16)$$

The variance σ^2 is hence defined as follows:

$$\sigma^2 = 2\nu L = \hbar L/m \quad (17)$$

The total-sub-timing-length L at a given ordinary-time-index t, can therefore be expressed as:

$$L = \sigma^2 m/\hbar \quad (18)$$

The total-sub-timing-length $L(t)$ of a particle can be experimentally determined in function of the standard deviation σ. For example, in the double slit experiment, the standard deviation σ can be estimated by gradually increasing the distance between the two slits until almost no more interference would take place and this last distance would give a rough estimate of the standard deviation σ. Indeed, interference would stop when the maximal spread of the particle is smaller than the distance between the two slits.

At any given ordinary-time-index t, equation (16) can be geometrically interpreted as a "*spatial bloc*" composed of all the positions visited by the particle. Being Gaussian, the density of the spatial bloc is high at its centre i.e. medium position $\bar{X}(t,0)$, where the density of probability is maximal and decreases as we go away from the centre. The "*spatial bloc*" can be imagined to be a density-inhomogeneous "*ball of sub-positions*" presenting roughly a radius equals to the standard deviation σ as defined in equation (17).

However, the "*ball of sub-positions*" should only be considered as a partial picture because the analysis is made with respect to the segment $[0, L(t)]$ and not with respect to the sub-time indices. Being made up of complex numbers the sub-time indices would somehow lead to variations in phase within the *ball of sub-positions*.

4.2 Ordinary-time-dynamics

The ordinary-time-dynamics of the particle can be described by applying the laws of classical point-mechanics on the *ball of sub-positions* representing the particle or by applying Schrodinger's equation.

4.2.1 Ordinary-time-dynamics according to point-mechanics

The ordinary-time dynamics can be viewed as the evolution of the *ball of sub-positions* as an indivisible entity (or a *sub-timeless-bloc*), with respect to ordinary-time t. The movement of the *ball of sub-positions* with respect to ordinary time t can be described by the movement of its centre $\bar{X}$ (i.e. centroid) according to Newton's laws of point mechanics, as follows:

$$\frac{dP}{dt} = F = -grad\, V \quad (19)$$

and

$$P = m\frac{dX}{dt} \quad (20)$$

As an example, consider a free particle moving along a given direction with a constant momentum $\overrightarrow{P(t)} = \overrightarrow{P_0}$. For any given ordinary-time-index t, the medium position $\overrightarrow{X(t)}$ of the free particle is expressed as follows:

$$\overrightarrow{X(t)} = \overrightarrow{P_0}t/m \quad (21)$$

Thus, the probability density at a given ordinary time t of equation (16) becomes:

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$$\rho(\vec{x},L)=\frac{m}{(2\pi\hbar L)^{3/2}}e^{-\frac{1}{2}\left(\frac{\vec{x}-\overrightarrow{P_0}t/m}{\sqrt{(2\hbar L/m)}}\right)^2} \qquad (22)$$

The resultant probability density is therefore a Gaussian distribution whose centre moves with respect to ordinary-time t, at the velocity $\overrightarrow{P_0}/m$.

The free particle can thus be imagined to be a cloud of 'positions' whose radius is roughly equal to the standard deviation σ and whose density is dispersed according to a Gaussian distribution around its 'centroid' (i.e. medium position $\vec{X}(t)$) which is moving at the velocity $\overrightarrow{P_0}/m$ with respect to ordinary time t, along a fixed direction.

The movement of the particle can be imagined to be a process of diffusion of its *ball of sub-positions* throughout space. This movement explains the wave-like structure of the particle. Indeed, let the free particle be moving at the velocity $\overrightarrow{P_0}/m$ along a given direction $\vec{k}$. This implies that the centre $\vec{X}(t)$ of the *ball of sub-positions* is moving at the velocity $\overrightarrow{P_0}/m$ along the direction $\vec{k}$.

The surface of the ball can be assimilated to a wave-front with respect to the centre $\vec{X}(t)$ of the ball. In particular, the outer surface of the hemisphere in the direction $\vec{k}$ (i.e. front-hemisphere) can be assimilated to a wave-front perpendicular to the direction $\vec{k}$ and separated from the centre $\vec{X}(t)$ by the distance $\left|\vec{x}-\vec{X}\right|$ which corresponds to the radius of the ball.

The equation of the wave-front can be expressed as $\vec{k}.\left(\vec{x}-\vec{X}\right)=K$, where K is a constant. The wave function in the position representation $\langle x|p\rangle$ can therefore be assimilated to a plane wave given by the following expression:

$$\langle x|p\rangle=\psi_0 e^{i\vec{k}.(\vec{x}-\vec{X})} \qquad (23)$$

Let X=0 and by introducing de Broglie's relation of momentum $\vec{p}=\hbar\vec{k}$, the normalised wave function in the position representation $\langle x|\psi\rangle$ becomes:

$$\langle x|p\rangle=\frac{1}{h^{3/2}}e^{i\vec{p}.\vec{x}/\hbar} \qquad (24)$$

4.2.2 Ordinary-time-dynamics according to Schrodinger's equation

In section 3, the state-vector $|\psi(t)\rangle$ (or wave function $\psi(x,t)$) has been interpreted as the distribution of the different positions occupied by a particle at a given ordinary-time-index t and where each position $|x_k\rangle$ is affected with the *resultant-sub-time-fraction* a_k/L spent by the particle at that position $|x_k\rangle$. The *resultant-sub-time-fraction* a_k/L is a complex number measuring the relative abundance of a position with respect to all other positions visited by the particle. Some positions are more or less *dense* or *concentrated* than others and the movement of the particle with respect to ordinary time t diffuses this density (or concentration) of positions as shown in the precedent section.

It has also been shown in section 4.1.1 that the mod-square of the *resultant-sub-time-fraction* a_k/L represents the probability of the particle to be at the position-state $|x_k\rangle$. It can thus be straightforwardly deduced that the flow of probabilities with respect to ordinary time is governed by Schrodinger's equation.

4.3 Application of the two-step-process of a particle's evolution

In view of the above, the evolution of a particle with respect to extended-time is composed of a sub-temporal-kinematics and an ordinary-time-dynamics. The sub-temporal-kinematics can be described in a probabilistic manner or a statistical manner while the ordinary-time-dynamics can be described by Schrodinger's equation.

For example the dynamics of a free particle can be described by a diffusion equation with respect to "*real-sub-time*" and by Schrodinger's equation with respect to ordinary time as follows:

$$\begin{cases} \frac{\partial \rho}{\partial u} = \nu \Delta \rho & \rightarrow \quad \rho(x,t) = \|\psi(x,t)\|^2 \\ \psi(x,t) \sim \sqrt{\rho(x,t)} & \rightarrow \quad i\hbar \frac{\partial \psi(t)}{\partial t} = \frac{-\hbar^2}{2m} \psi(t) \end{cases} \qquad (25)$$

The first step consists in determining the sub-temporal-kinematics of the particle by solving the diffusion equation. As seen in section 4.1.2, the solution of the diffusion equation at a given ordinary time index t can be expressed by the following Gaussian:

$$\rho(x,t) = \frac{1}{(4\pi\nu L)^{3/2}} e^{-(x-\bar{X})^2/4\nu L} = \frac{1}{(2\pi\sigma^2)^{3/2}} e^{-(x-\bar{X})^2/2\sigma^2} \qquad (26)$$

where $\sigma^2 = 2\nu L$ and $\bar{X}$ is the medium position at a given ordinary time index t. As seen in section 4.2.1 the medium position moves with a determined momentum P with respect to ordinary time.

It should be reminded that the Gaussian distribution $\rho(x,t)$ represents the distribution of actual and real positions of the particle at a given ordinary time index t.

The wave function in the position representation $\psi(x,t)$ at a given ordinary time index t can thus be expressed as follows:

$$\psi(x,t) = \langle x|\psi\rangle = \sqrt{\rho(x,t)} = \frac{1}{(2\pi\sigma^2)^{3/4}} e^{-(x-\bar{X})^2/4\sigma^2} \qquad (27)$$

The wave function in the momentum representation $\psi(p,t)$ can be derived from the above equation and can therefore be expressed as follows:

$$\psi(p,t) = \frac{1}{(2\pi\hbar^2/4\sigma^2)^{3/4}} e^{-(p-P)^2\sigma^2/\hbar^2} \qquad (28)$$

The probability density $\|\psi(p,t)\|^2$ of the momentum wave function is also a Gaussian centred on P with dispersion $\sigma_p = \hbar/2\sigma$.

The second step consists in using Schrodinger's equation to determine the evolution of the particle with respect to ordinary time t.

The evolution of the free particle can be easily determined by using the basic time-evolution equation of energy because the Hamiltonian of a free particle is only a function of momentum.

Let the above momentum wave function $\psi(p,t)$ of equation (28) be the initial state $\psi_0 = \psi(p,0)$, then its evolution t units of ordinary-time later is:

$$\psi(x,t) = \int dp \langle x|p\rangle \langle p|\psi_0\rangle e^{-p^2 t/2m\hbar^2} \qquad (29)$$

The result of the above integral can be easily calculated [10] and out of which the probability density of the momenta of the particle $\|\psi(p,t)\|^2$ at time t can be expressed as:

$$\|\psi(p,t)\|^2 = Ae^{-(x-Pt/m)^2\sigma^2/(2\hbar^4/|b|^4} \qquad (30)$$

where $b^2 = \left(\frac{\sigma^2}{\hbar^2} + \frac{it}{2m\hbar}\right)$. This is a Gaussian distribution whose centre moves with the velocity P/m.

The above example shows the two-step process for determining the evolution of a free particle. At the first step the sub-temporal-kinematics gives the particle's wave function at a given ordinary time t and then at the second step Schrodinger's equation is used to determine the evolution of this wave function with respect to ordinary time.

On the other hand, the first step can be described probabilistically as in equations (10)-(12) and thus, the dynamics of a particle can also be described as follows:

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$$\begin{cases} P(Y=|x_k\rangle) = \delta_{u,u'} + (1-\delta_{u,u'})(\|a_k(t)\|/L(t))^2 \rightarrow & |\psi(t)\rangle = (1-\delta_{u,u'})\sum_k \frac{a_k}{L}|x_k\rangle + \delta_{u,u'}|x_k\rangle \\ |\psi(t)\rangle \rightarrow & i\hbar\frac{\partial|\psi(t)\rangle}{\partial t} = H|\psi(t)\rangle \end{cases} \quad (31)$$

where $\delta_{u,u'}$ is equal to zero when there is a transition ($u' \neq u$) between any position-state $|x_k\rangle$ and the global position-state $|X\rangle$ (that englobes all other possible states) and is equal to 1 when there exists a position-state $|x_k\rangle$ from which no other transition takes place.

Once the notion of a state vector $|\psi(t)\rangle$ has been extracted from the underlying sub-temporal-kinematics, the evolution of the state vector $|\psi(t)\rangle$ with respect to ordinary time t can be studied on its own (for example, according to Schrodinger's equation) without referring back to the real distribution or configuration of positions with respect to sub-time.

The above two-step or two-scale dynamics can be compared to the situation of temperature T or entropy S of a system. These entities T and S emerge out of a statistical analysis of the system's constituent particles. Once these statistical entities T and S are determined, their evolution can be studied without referring back to the underlying configurations of the constituent particles. However, the inverse is not true, i.e. if we know the temperature of a system we can't deduce its exact configuration (i.e. we can't know the position and momentum of each constituent particle) at any given time.

In a similar manner, the wave function of a particle can be used to study the evolution of the particle. However, once a measurement is undertaken, it is necessary to refer back to the underlying sub-temporal structure as will be described in the next section.

5. Measurement

Strictly speaking, according to the hidden-time model there is no 'measurement problem' simply because there is no undetermined state of the system with respect to extended-time. Indeed, at any extended-time index, the state of the system (for example, the position or momentum of a particle) is well determined.

The problem arises only when we make abstraction of sub-time. In that case, it is reasonable to suppose that an act of measurement at a given ordinary-time t, conducted by a device on a particle is possible only when the position and sub-time of the particle are 'correlated' with those of the device, i.e. when the spatial location of the interaction is common to both and when at least some sub-time indices relative to the particle coincide with some of those relative to the device. Indeed, by analogy to ordinary time, it would not be possible for example to measure today at a given location of space an event that happened yesterday at that same location.

Suppose that the interaction between the device and the particle took place at a common extended time $(t, \Delta s_n(t))$ such that during the sub-time-period $\Delta s_n(t)$, the particle is located at the position-cell x_n corresponding to the position-state $|x_n\rangle$. Measurement means the selection and retention of a position-state $|x_n\rangle$ during the common sub-time-period $\Delta s_n(t)$. This implies that the position of the particle is reduced to the position-cell x_n as if it is narrowly confined in a very deep potential well whose width is equal to that of the position-cell x_n. As we shall see in the next section this confinement of the particle or coupling between the particle and the device corresponds to an entanglement between the two entities.

Equations (11) and (12) show that the selection of a position-state $|x_n\rangle$ means that the transitions with respect to sub-time-indices would stop at the selected position-state $|x_n\rangle$ and probability becomes equal to 1. This entails that $(\|\Delta s_n(t)\|/L(t))^2 = 1$ implying that the *total-sub-timing-length* $L(t)$ is equal to $\|\Delta s_n(t)\|$. It can thus be deduced that either the *sub-temporal-set* $C(t)$ associated to the particle is reduced to the limited sub-time period $\Delta s_n(t)$ or the limited sub-time period $\Delta s_n(t)$ becomes cyclic for a period equivalent to that of the whole sub-temporal-set $C(t)$.

It should be remarked that the act of measurement is a two-way action-reaction process: the action consists in the selection of a position-cell and the reaction consists in either the limiting of the whole sub-temporal-set $C(t)$ to the sub-time period $\Delta s_n(t)$ or to the fact that the latter becomes cyclic for a period equivalent to that of the whole sub-temporal-set $C(t)$.

The act of measurement in which the position-state $|x_n\rangle$ has been selected during the sub-time-period $\Delta s_n(t)$ can be mathematically expressed as the multiplication of the state vector $|\psi(t)\rangle$ by a Dirac measure (or indicator function) $\delta_{s_{nj}}(\Delta s_n(t))$, as follows:

$$\delta_{s_{nj}}(\Delta s_n(t))|\psi(t)\rangle = \delta_{s_{nj}}(\Delta s_n(t))\left(\sum_k \frac{a_k}{L}|x_k\rangle\right) \quad (32)$$

where the Dirac measure is defined as:

$$\delta_{s_{nj}}(\Delta s_n) = \begin{cases} 1 & if\ s_{nj} \in \Delta s_n \\ 0 & if\ not \end{cases} \quad (33)$$

However, as defined in equation (5) the state $|\psi(s_{kj}, t)\rangle$ of the particle at a given sub-time index s_{kj} is simply equal to the eigenvector $|x_k\rangle$ and thus, equation (32) becomes:

$$\delta_{s_{nj}}(\Delta s_n(t))|\psi(t)\rangle = \delta_{s_{nj}}(\Delta s_n(t))\left(\sum_k \frac{a_k}{L}|\psi(s_{kj}, t)\rangle\right) \quad (34)$$

The action of Dirac's measure transforms equation (34) into:

$$\delta_{s_{nj}}(\Delta s_n(t))|\psi(t)\rangle = \frac{a_n}{L}\left(\psi(s_{nj}, t)\rangle\right)_{s_{nj}\in\Delta s_n} = \left(\psi(s_{nj}, t)\rangle\right)_{s_{nj}\in\Delta s_n} = |x_n\rangle \quad (35)$$

where $a_n/L = 1$ because sub-time indices are restricted to a specific sub-time-period $\Delta s_n(t)$ associated to the *resultant-sub-time-presence* $a_n(t)$.

To sum up, at each ordinary time index t, a particle has its proper sub-temporal-set $C(t)$ during which its state keeps changing until an act of measurement is performed on the particle that stops the transitions at a selected position-state and the state of the particle is restricted to the selected state during its 'future' proper sub-temporal-sets $C(t)$ at least for a certain period of ordinary time.

On the other hand, as seen in the precedent sections a particle can be imagined as a *ball of sub-positions* behaving as a wave. However, when measurement is conducted, one position (or at least a limited sub-set of positions) is selected, hence, reducing the *ball-of-sub-positions* into almost a single position and thus the particle behaves as a classical point particle. This explains the particle-wave duality. Moreover, restricting the position of the particle to almost one position restricts in its turn (by reaction) its whole sub-temporal-set $C(t)$ into a limited sub-time period Δs_n or extends the period of the latter to that of the whole sub-temporal-set $C(t)$.

In view of the above, the act of measurement is a dynamic process that modifies the wave function as in conventional quantum mechanics and thus, after measurement the evolution according to Schrodinger's equation should be updated by the new wave function. However, this modification has a physical origin stemming out from the underlying kinematics of the particle with respect to sub-time.

However, the term 'measurement' is misleading because it is not what we would call a measurement in the ordinary sense. It is rather a dynamic interaction between the device and the particle where the set of states of the particle at a given ordinary time index t, are restricted or confined to a particular sub-set of states corresponding to a limited sub-time period.

6. Entanglement

Consider a quantum system composed of two particles. Let $X^{(1)} = (x^{(1)}, y^{(1)}, z^{(1)})$ and $X^{(2)} = (x^{(2)}, y^{(2)}, z^{(2)})$ represent the positions of the first and second particles respectively. The ensemble of positions of both particles at a given ordinary time t can then, be defined as follows:

$$\phi(X^{(1)},X^{(2)}) = \left\{ X_k^{(1)}(\mathrm{t}, s_{kj}),\ X_p^{(2)}(\mathrm{t}, s_{pi}) \right\} \quad (36)$$

Each position $X_k^{(1)}$ of the first particle may be visited at different sub-time indices s_{kj} and similarly for the second particle.

By using the same analysis as in the above section 3, the state vector $|\psi(t)\rangle$ of the physical-system (i.e. two particles) at a given ordinary-time-index t can be expressed in a six-configuration space, as follows:

$$|\psi(t)\rangle = \sum_{k,p} \frac{a_{kp}}{L} \left| X_k^{(1)}\ X_p^{(2)} \right\rangle \quad (37)$$

where $\left| X_k^{(1)}\ X_p^{(2)} \right\rangle$ is a state eigenbasis of the first and second particles.

However, by regrouping the couples of positions associated to the same sub-time indices, the ensemble of positions of equation (4) can be expressed as follows:

$$\phi(X^{(1)},X^{(2)}) = \left\{ \left(X_k^{(1)}(\mathrm{t}, s_{ki}),\ X_k^{(2)}(\mathrm{t}, s_{ki}) \right), \left(X_l^{(1)}(\mathrm{t}, s_{lj}),\ 0 \right), \left(0,\ X_m^{(2)}(\mathrm{t}, s_{mj}) \right) \right\} \quad (38)$$

The above expression indicates that some positions $X_k^{(1)}$ and $X_k^{(2)}$ of both particles may be associated to the same sub-time indices s_{ki} and are thus coupled together whereas, some positions $X_l^{(1)}$ of the first particle and some other positions $X_m^{(2)}$ of the second particle may be associated to different sub-time indices s_{lj} and s_{mj} and are thus independent and cannot be coupled together.

Suppose $\left\{ \left| X_n^{(1)} \right\rangle \right\}$ and $\left\{ \left| X_m^{(2)} \right\rangle \right\}$ are two state eigenbasis of the first and second particles respectively.

Some of the sub-time indices may be common to both particles and among these common sub-time-indices some of them may be associated with the same corresponding couples of positions $(X_k^{(1)}, X_k^{(2)})$. All other positions cannot be coupled together because they don't have common sub-time indices.

Therefore, using a similar analysis as in the above section 3, the state vector $|\psi(t)\rangle$ of the physical-system at a given ordinary-time-index t can be expressed as a sum of two terms where the first term corresponds to the couples of positions $(X_k^{(1)}, X_k^{(2)})$ having the same sub-time indices while the second term corresponds to independent positions of both particles that are not visited by the same sub-time indices, as follows:

$$|\psi(t)\rangle = \sum_k \frac{a_k}{L} \left| X_k^{(1)}\ X_k^{(2)} \right\rangle + \sum_{ij} \frac{b_i c_j}{L} \left| X_i^{(1)} \right\rangle \otimes \left| X_j^{(2)} \right\rangle \quad (39)$$

The first term cannot be expressed as a tensor product of two vectors:

$$\sum_k \frac{a_k}{L} \left| X_k^{(1)}\ X_k^{(2)} \right\rangle \neq \sum_k \frac{b_k c_k}{L} \left| X_k^{(1)} \right\rangle \otimes \left| X_k^{(2)} \right\rangle \quad (40)$$

Indeed, it is in general impossible to find N coefficients b_k and N coefficients c_k such that NxN equations $a_k = b_k c_k$ be satisfied because there are 2N unknowns and N^2 equations.

Thus, the states of the particles associated to common sub-time indices are entangled (first term of equation (39)) while those associated to different sub-time indices are not entangled (second term of equation (39)). It can thus be concluded that if the second term is negligible, then both particles are entangled whereas, if the first term is negligible, then the particles are not entangled.

Consider the case where the composite second tem of equation (39) is equal to zero such that the state-vector $|\psi(t)\rangle$ is defined by completely entangled particles as follows:

$$|\psi(t)\rangle = \sum_k \frac{a_k}{L} \left| X_k^{(1)}\ X_k^{(2)} \right\rangle \quad (41)$$

At each outcome (referenced by the index k), both particles are associated with a unique *resultant-sub-time-fraction* a_k/L because they have a common sub-time or more precisely a common *sub-temporal set* $C(t)$ regardless of the distance separating them. This common *sub-temporal set* $C(t)$ acts as a common '*proper sub-time-clock'* for both entangled particles. On the other hand, sub-time is itself a proper dimension and does not need to be spatially located. Moreover, sub-time is not an *information* or *energy* that needs to travel or propagates from one location to another. Thus, each sub-time-index s_k (a tick of the common sub-time-clock) links the states of both particles whatever is the distance between them.

Applying an act of measurement according to the definition of equation (32) on the above state-vector $|\psi(t)\rangle$ of equation (41) gives:

$$\delta_{s_{nj}}(\Delta s_n(t))|\psi(t)\rangle = \delta_{s_{nj}}(\Delta s_n(t))\sum_k\left(\frac{a_k}{L}|\psi(s_k,t)\rangle\right) = \left(\psi(s_{nj},t)\rangle\right)_{s_{nj}\in\Delta s_n} \quad (42)$$

and thus according to equation (35):

$$\delta_{s_{nj}}(\Delta s_n(t))|\psi(t)\rangle = \left(\psi(s_{nj},t)\rangle\right)_{s_{nj}\in\Delta s_n} = \left|X_n^{(1)}\right\rangle\left|X_n^{(1)}\right\rangle \quad (43)$$

Measurement acts on the state-vector $|\psi(t)\rangle$ as a whole affecting the common *sub-temporal set* $C(t)$ of both particles, and thus, the outcome is defined for both particles whether the act of measurement is locally conducted on one particle or the other.

Indeed, suppose that a measuring operation has been conducted on the first particle and a given state $\left|X_n^{(1)}\right\rangle$ has been selected. In reaction to this selection, the *sub-temporal set* $C(t)$ is either restricted to a sub-time domain common to the state $\left|X_n^{(1)}\right\rangle\left|X_n^{(1)}\right\rangle$ of both particles which is frozen because there are no more transitions into other states or the common sub-time domain becomes cyclic for a period equivalent to the whole *sub-temporal set* $C(t)$ such that the visited states become periodic for a period equivalent to that of the whole *sub-temporal set* $C(t)$. It can be imagined as if the common 'proper sub-time clock' has been modified by the selection of a state on either side and thus, the transition to other states on both sides is also modified accordingly.

By using Bell's terminology, the *sub-temporal set* $C(t)$) can be considered as a *non-local beable* for entangled particles that enables them to be separated in space but united in sub-time. Entangled particles are thus, inherently coexistent in ordinary time as well as sub-time independent of their distance from each other because they are governed by the same and unique extended-time. In other terms, they are non-locally connected through their common extended-time and no information is needed to be transferred from one particle to the other in order to synchronise their extended time or their state. Entangled states can be said to be 'co-being' states.

On the other hand, particles that do not have a common extended-time are considered to be non-locally separated.

It is to be noted that the notion of entanglement according to the above model is not incompatible with relativity. Indeed, relativity is concerned with the distance between events in space-time as defined by Minkowski metric. However, according to this model, two events are said to be entangled if they have a common extended-time. In other words, the metric of entanglement is binary and the 'entangled-distance' between two events can be said to be either 'zero' if they have common extended-time or 'infinite' otherwise. It can alternatively be defined by a notion of 'binary connectivity' of '1' or '0' where entangled events have a connectivity of '1' and un-entangled events have a connectivity of '0', though un-entangled events (as well as entangled events) can still be connected through space-time according to the laws of relativity.

It should be noted that entanglement between two events does not take place through space-time but only via extended-time and thus, the extended-time common to two events is inherently unobservable but it has observable effects exemplified by the properties of entanglement. Thus, even if entangled events are said to have inherently common extended-time (i.e. same ordinary time-index and same sub-time index (or indices)), yet for an observer and because of relativity the ordinary-time indices of both events could of course be different depending on his state of motion.

In a relativistic space-time representation, each set of entangled events can be represented by a connected graph where the vertices of the graph would change under a Lorentz transformation whereas, the graph itself remains invariant.

7. Uncertainty

As seen in the previous sections, a free particle can be imagined to be a *ball of sub-positions* that occupies a certain volume in space at each ordinary-time index t and out of which a state-vector is derived that evolves according to the usual Schrodinger equation with respect to ordinary time t.

At each ordinary-time index t, the particle has a certain distribution of positions x and a certain distribution of momenta p and it has been shown in section 4.3 that the probability densities of position $\|\psi(x,t)\|^2$ and momentum $\|\psi(p,t)\|^2$ of the wave functions of a free particle are Gaussians with dispersions σ_x and $\sigma_p = \hbar/2\sigma_x$ respectively and thus:

$$\sigma_x \sigma_p = \hbar/2 \qquad (44)$$

Equation (44) expresses the uncertainty principle between position and momentum of a single free particle and at a given ordinary-time-index t.

More generally, Fourier analysis shows that the variance $Var(x)$ for the distribution of position x and the variance $Var(p)$ for the distribution of momenta p satisfy the following well-known classical mathematical relation [1], [10]:

$$Var(x)Var(p) \geq h/4 \qquad (45)$$

However, it should be emphasised that according to the present model the above uncertainty relation (45) comes from the statistical distribution at *one* given ordinary-time-index t of the different positions and momenta of a *single particle* visited by the different sub-time-indices. It does not come from a statistical distribution of the different positions and momenta of different particles neither from a statistical distribution of the different positions and momenta of a particle at different ordinary-time-instants t. It should be clear that here we are considering only one particle at only one ordinary-time index t and the statistical distribution is simply generated by the hidden sub-time.

8. Quantum Mechanics Recipe

To sum up, the hidden-time model has a six-dimensional space-time structure: three ordinary spatial dimensions, one ordinary real-time dimension and a complex sub-time dimension. Each particle has a definite position and a definite momentum at each extended-time-instant $w = (t, s)$. However, at each ordinary-time instant t, a particle has a plurality of positions and a plurality of momenta associated to corresponding sub-time indices, hence forming a *ball of sub-positions*. Each *sub-position* is associated with a *resultant-sub-time-fraction* a_k/L generated by the sub-time indices during which the sub-position is visited by the particle. The *ball of sub-positions* along with the corresponding *resultant-sub-time-fractions* a_k/L can be represented by a state vector defining the quantum state of the particle. The centre of the *ball of sub-positions* is defined by a *centroid* whose dynamics can be described by

classical point mechanics. The evolution of the *ball of sub-positions* with respect to ordinary time is governed by Schrodinger's equation.

By borrowing the term '*quantum mechanics recipe*' from Maudlin's book [11], it can be said that the model of hidden-time conforms entirely to the *quantum mechanics recipe* and the same tools and equations are used to solve the same quantum problems. What is changed is the meaning of quantum mechanics showing how superposition, probabilities and Born rules emerge out of a hidden-time.

For example, in the double slit experiment, when no detectors are present, the electron is assimilated to a *ball of sub-positions* having plurality of positions diffused throughout a certain region of space. Thus, the electron having different positions at the same ordinary time index t but at different sub-time indices, behaves as a wave that passes through both slits, hence generating interference. However, when detectors are present, and an electron's position is 'measured', the *ball of sub-positions* is reduced into almost a single position and thus the electron behaves as a classical point particle passing through only one slit before impacting on the screen.

The present model of hidden-time is intended to give a realistic interpretation of quantum mechanics without adding any new laws to the original version of quantum mechanics. However, the interpretation of these laws is changed.

9. Nature of sub-time and Arrow of Time

Sub-time has the main minimal properties of ordinary time in that it permits an object to have different states (for e.g. occupying different positions at different sub-time indices) and it permits two different objects to occupy the same position at different sub-time indices.

However, being made up of complex numbers, sub-time is distinguished from ordinary time by two features. The first concerns the fact that sub-time has no relation of order. The second feature concerns the fact that it has phase factors produced by its real and imaginary parts.

The first feature entails an '*arrow of time*'. Indeed, unlike ordinary time, even if we know the sub-time-indices of several 'sub-events', it would still not be possible to give an ordered sequence of these sub-events. Thus, once sub-events have taken place with respect to sub-time, it is not possible to reverse the dynamics because there is no inherent order. This of course, leads to irreversibility with respect to the global extended time because each ordinary-time-index is associated with an irreversible set of complex sub-time-indices. The hidden irreversible sub-time-indices at each ordinary-time-index induces the 'arrow of time'.

The second feature generates interference between the sub-time-indices when summed up which in its turn leads to interference between the states visited by these sub-time-indices.

It should be noted that the only temporal notion that sub-time possess is that a physical-system may have one state at one sub-time index and another state at another sub-time index. It has no order relation and thus, no notions of earlier-than, later-than and simultaneous with. It does not even have any undirected asymmetric order such as 'in-between'.

Thus, its temporal notion is very minimalistic. It is even more minimalistic than what Mc Taggart calls a C-series. By extension to Mc Taggart's notions, the minimal property of sub-time may be called a *D-series* which has the sole temporal feature of permitting an entity to have different properties at different sub-time indices.

By applying a similar terminology to that used in the philosophy of time [12], the states of a physical system that takes place at different sub-time indices but at a unique ordinary-time index may be considered as '*sub-timeless'* and may be said to form a '*minimal-block-sub-universe'*. It combines a physical system's 'sub-everywhenism' with a *D-series* relation.

It remains to be asked whether sub-time really exist (One may ask "does sub-time really exist?"). This question is as answerable or rather unanswerable as whether ordinary time is real. Ordinary time is imposed on us in order to structure and better understand our environment and we believe that complex sub-time would also be imposed on us for understanding quantum phenomena whether it has a genuine reality or not this is not the question. Indeed, we don't even know whether ordinary time has a genuine reality or is simply an object that we impose on the outer world to have a better comprehension. In this work, the goal of introducing sub-time is simply to better understand quantum phenomena or at least to render them more logical to our understanding.

10. Discussion

In his book [8], Gao proposes an ontology of quantum mechanics where the wave function represents the physical state of a single system. In particular, he considers that *the superposition principle of quantum mechanics requires that the charge distribution of a quantum system such as an electron is effective; at every instant there is only a localised particle with the total charge of the system, while during an infinitesimal time interval around the instant the ergodic notion of the particle forms the effective charge distribution at that instant.*

He also considers that an *observable assumes an eigenvalue at each instant, and its value spreads all eigenvalues during an infinitesimal time interval around the given instant. The time spent in each eigenvalue is proportional to the modulus squared of the wave function of the system.*

A 'time instant' in Gao's model may be interpreted as an infinitesimal time interval (i.e. not a geometrical point) such that all eigenvalues of an observable are distributed within the infinitesimal time interval and such that the time spent in each eigenvalue is proportional to the modulus squared of the wave function of the system.

The philosophical approach of the superposition principle in the present hidden-time model may be considered to be similar to that of Gao's whereas, the structure is different. In the hidden-time model, a 'time instant' is also considered not to be a geometrical point. However the difference with respect to Gao's model is that the 'time instant' has a certain infinitesimal thickness 'perpendicular' to the direction of time and the eigenvalues of an observable are distributed within that thickness. On the other hand, this time-thickness is complex such that the 'complex-time' 'spent' in each eigenvalue is proportional to the corresponding probability amplitude of the state vector. This entails that the square of the complex-time spent in each eigenvalue is proportional to the modulus squared of the wave function.

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