Synchronization of thermal Clocks and entropic Corrections of Gravity

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Abstract

There are so called MOND corrections to the general relativistic laws of gravity, able to explain phenomena like the rotation of large spiral galaxies or gravitational lensing by certain galaxy clusters. We show that these corrections can be derived in the framework of synchronizing thermal clocks. We develop a general formula, which reproduces the deep MOND correction at large scales and defines the boundary-acceleration a_0 beyond which corrections are necessary.

Key Words: General Relativity, Cosmological Horizon, Entropy, Vacuum Energy, Dark Matter, MOND.

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In [1] it is shown that the synchronization of a thermal clock in the local rest frame of an observer with acceleration κ, and a single-photon clock[1] in vacuum leads to the relation

$$\frac{4}{h}k_B T_\kappa \frac{c}{\kappa} = \frac{1}{\pi^2}. \tag{1}$$

In (1) k_B and h denote, as usual, the Boltzmann and Planck-constants. The temperature T_κ is hence

$$T_\kappa = \frac{\hbar\kappa}{2\pi c k_B},$$

which is the Unruh-Davies temperature [2,3]. If we choose for κ the Newtonian acceleration, we derive from (1) relations between energy, temperature and entropy, which turn space-time into a curved medium. The notion of entropy formally enters the equations as the surface area in a space-like section of space-time [1,4,5].

Concretely, we work in a weak gravitational field induced by a stationary mass M_b,[2] where the subscript b stands for the baryonic origin of matter. The corresponding Schwarzschild - solution has the line-element

$$ds^2 = h(r)dt^2 - \frac{1}{h(r)}dr^2 - r^2 d\Omega^2, \tag{2}$$

where

[1] We mean in this context rather the core of a clock, since we do not need devices to indicate time.
[2] So the mass M_b is not too big and there is no formation of black holes.

$$h(r) = \big(1 + 2\Phi(r)\big), \Phi(r) = -\frac{M_b G}{c^2 r}. \tag{3}$$

$\Phi(r)$ is the Newtonian potential. The key-relation, which is derived in $[1]$ and which is valid on a space-like section at $t = 0$, say, is

$$T_{g(r)} \cdot \frac{k_B A(r)}{4 l_P^2} = \frac{1}{2} E_b. \tag{4}$$

$T_{g(r)}$ denotes the Unruh-Davies temperature corresponding to the surface acceleration $g(r) = \frac{M_b G}{r^2}$, $l_P = \sqrt{\frac{G\hbar}{c^3}}$ is the Planck-length and $A(r) = 4\pi r^2$ the surface area of a sphere with radius r. The energy E_b is induced by the baryonic mass $E_b = M_b c^2$. Equation (4) allows us to formally introduce an entropy term

$$S_b(r) = \frac{k_B A(r)}{4 l_P^2}. \tag{5}$$

This entropy can be shown to arise from the space-like entanglement of microstates in vacuum $[6 - 8]$. For our purpose we introduce it on the heuristic level in analogy to the thermodynamic relation $Q = TS$. Equation (4) can also be written as

$$g(r)A(r) = \frac{4\pi G}{c^2} E_b. \tag{6}$$

It is explained in $[1]$ that it is possible to generalize relation (6), valid in a local rest frame, to a covariant version on arbitrary static Lorentz manifolds and to derive the Einstein equations from there. A key-ingredient is the strict area law (5). It is well known that the gravitational laws need a correction to explain phenomena like the flattening of the rotation-velocity of large galaxies in the outer limits of the spiral arms and anomalies in gravitational lensing by certain galaxy clusters. As a natural solution astronomers assumed the presence of additional, invisible, non-baryonic matter M_d, so called dark matter, in order to fit the data.[3] So far the hunt to detect dark matter has been elusive. In addition, the idea of dark matter unpleasantly adds free ad hoc parameters to the theory. Corrections to the laws of gravity, so called MOND[4] corrections, have been introduced $[9,10]$[5] in order to fit the data. There have been suggestions to explain MOND by thermodynamic aspects of the vacuum $[11 - 15]$. In $[11]$ E. Verlinde recently proposes an elaborate entropic origin for the corrections and we show in this paper that the framework of synchronizing thermal clocks is ideally suited to derive a general correction formula, which produces the deep MOND correction at large scales and the correct boundary between the regimes of MOND and general relativity.

[3] The idea was pioneered by Fritz Zwicky in the 1930-ties.
[4] Modified Newtonian Dynamics
[5] MOND has been pioneered by M.Milgrom in the 1970-ties.

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De-Sitter space is a spherically symmetric vacuum solution of Einstein's equations with positive cosmological constant $\Lambda > 0$. It is suited to serve as a model for an expanding universe, like the one we seem to live in. The line-element in a static coordinate patch takes the already familiar form

$$ds^2 = h(r)dt^2 - \frac{1}{h(r)}dr^2 - r^2 d\Omega^2, \tag{7}$$

this time with

$$h(r) = \left(1 - \frac{r^2}{R_\infty^2}\right). \tag{8}$$

R_∞ denotes the Hubble scale, i.e. the radius of a cosmic horizon, which accelerates with the Hubble parameter H_0 [16]

$$a_{R_\infty} = cH_0 = \frac{c^2}{R_\infty} = c^2\sqrt{\frac{\Lambda}{3}}. \tag{9}$$

We will need the following relationship: with $a_0 = \frac{a_{R_\infty}}{2}$ we have

$$T_{a(r)} = \frac{r}{R_\infty}T_{a_0}. \tag{10}$$

Relationship (10) is motivated by the cosmological constant. The energy density of the vacuum is supposed to be constant, in spite of expansion, and there holds

$$\frac{8\pi G}{c^4}\bar{e}_{vac} = \Lambda. \tag{11}$$

Therefore we get for the energy within a region of radius r right in the spirit of (4)

$$\frac{4}{3}\pi r^3 \frac{\bar{e}_{vac}}{2} = T_{a(r)}S(r). \tag{12}$$

Starting from (12) we get the following chain of expressions

$$\frac{4}{3}\pi r^3 \frac{\bar{e}_{vac}}{2} = T_{a(r)}k_B \frac{4\pi r^2}{4l_P^2}, \tag{13}$$

and by (11)

$$\frac{c^4 r}{4\pi}\cdot\frac{\Lambda}{3} = T_{a(r)}\frac{k_B c^3}{\hbar}, \tag{14}$$

which turns by (9) into

$$\frac{\hbar r}{4\pi k_B} \cdot \frac{c^2}{R_\infty^2} = T_{a(r)} c, \tag{15}$$

and finally by the definition of a_0

$$\frac{\hbar}{4\pi k_B c} \cdot \frac{r}{R_\infty} a_{R_\infty} = T_{a_0} \frac{r}{R_\infty} = T_{a(r).} \tag{16}$$

The existence of the horizon seems to cause a challenge to the description of entanglement entropy by a mere area law and there is no satisfactory microscopic description of a space-time with positive cosmological constant yet [11]. Some recent work [17,18] suggests that information and hence entropy is not localized at the horizon but is stored in the bulk space-time. We introduce on the spatial section at $t = 0$, say, like in [11] the following entropy term

$$S_d(r) = \frac{r}{R_\infty} \frac{k_B A(r)}{4 l_P^2}. \tag{17}$$[6]

We index this entropy by the small letter d to indicate its dark vacuum-nature. At the horizon $r = R_\infty$, $S_d(R_\infty)$ coincides with the Bekenstein-Hawking entropy [4,5], as it should. With the help of (10) and (17) we are going to introduce a dark vacuum-energy term E_d by defining

$$E_d = T_{a(r)} S_d(r). \tag{18}$$

This dark energy E_d ultimately stems from the cosmological constant $\Lambda > 0$ (9) and is explicitly of entropic form. We will argue that it is this energy, which is responsible for the corrections to the laws of gravity and also gives rise to apparent dark matter M_d.

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The presence of a baryonic mass M_b in a causal patch of the vacuum with positive cosmological constant $\Lambda > 0$ is described by a combination of two geometries, the Schwarzschild-de Sitter geometry, differing from (3) and (8) in the factor $h(r)$

$$h(r) = \left(1 - \frac{r^2}{R_\infty^2} - \frac{2M_b G}{c^2 r}\right). \tag{19}$$

If the dark energy contribution is supposed to originate from the surface-gravity $g(r) = -c^2 \frac{d}{dr} \Phi(r)$, induced by M_b, then together with (4) there must hold

[6] Of course, there ist the possibility to explain $S_d(r)$ heuristically by observing that by (16) there holds $T_{a(r)} S(r) = T_{a_0} S_d(r)$ and that from a vacuum-perspective there is only the uniform a_0-accleration of space. The definition of E_d in (18) is then based on a mixed perspective, like the metric (19).

$$T_{g(r)}S_b(r) = E_d = T_{a(r)}S_d(r).$$

Hence

$$T_{g(r)}\frac{k_B A(r)}{4l_P^2} = T_{a(r)}\frac{r}{R_\infty}\frac{k_B A(r)}{4l_P^2}. \tag{20}$$

(20) can be interpreted as the synchronization of two thermal clocks [1]. By (10) we get from (20)

$$\frac{a^2(r)}{a_0} = g(r) = \frac{M_b G}{r^2}. \tag{21}$$

Hence

$$a(r) = \frac{\sqrt{M_b G a_0}}{r}. \tag{22}$$

(22) is exactly the MOND correction in [9]. Note that for large scales, $|\Phi(r)| \ll 1$, $a(r)$ clearly dominates the gravitational acceleration. Assuming now that the acceleration $a(r)$ is due to the gravitational potential, induced by some apparent dark matter M_d

$$a(r) = \frac{M_d G}{r^2}, \tag{23}$$

we get by suitable manipulation of (21)

$$\frac{M_d^2 G}{r^2} = M_b a_0, \tag{24}$$

or

$$M_d = \sqrt{\frac{M_b a_0}{G}} \cdot r. \tag{25}$$

Dark matter is therefore spread out over space, as it is needed to flatten the velocity curves. To gain further insight, we can also write (24) in integrated form

$$\int_0^R \frac{M_d^2(r) G}{r^2} dr = M_b a_0 R. \tag{26}$$

We have assumed that M_b is a point-mass at the center of the coordinate system, comprising all the mass within $r \leq R$. Nothing in the steps from equation (20) to (26), prevents us to assume that $M_b = M_b(r)$ is distributed over the range $r \leq R$. It makes, however, a difference in the last step. Assuming that (26) is correct with $M_b = M_b(r)$ distributed across space, we get by differentiation with respect to the radius

$$\frac{M_d^2(r)G}{r^2} = M_b(r)a_0 + \frac{dM_b(r)}{dr}a_0 r. \tag{27}$$

Since $\frac{dM_b(r)}{dr} \geq 0$, the last term in (27) is positive and hence (25) underestimates the apparent dark matter. In this paper we do not pursue this line of thought.[7] Equations (25-27) give us relations between the baryonic matter M_b and an apparent dark matter M_d. All that is real though, is the dark, entropic energy contribution E_d with a corresponding acceleration, which affects baryonic mass. M_d is an auxiliary parameter to save the laws of gravity (23).

3.1 The boundary be.1 een Mb o u and General Rela.ivi.y

Observation tells us that the boundary, below which general relativity is accurate and beyond which MOND plays a role, lies at a range, where acceleration is $\sim a_0$. So far our theory does not allow us to derive this fact. Observation also tells us that at shorter ranges, general relativity is a very accurate description of gravity, which is at least plausible, since there we find $S_b(r) \geq S_d(r)$. The key to the answer will be the fact that adding baryonic mass to the vacuum has an influence on dark energy E_d. We follow the exposition in [11], where first the displacement of the de-Sitter horizon by addition of a mass M_b and the resulting change $S_{M_b}(R_\infty)$ in horizon entropy is being calculated. There holds

$$S_{M_b}(R_\infty) = -\frac{2\pi c k_B M_b R_\infty}{\hbar}. \tag{28}$$

So adding mass reduces the horizon entropy. To calculate the entropy change in regions considerably smaller than the horizon, where the term $\frac{r^2}{R_\infty^2}$ in the metric (19) is negligible, we take the derivative of (5) with respect to the geodesic distance $dr = \big(1 + \Phi(r)\big)ds$ in the Schwarzschild metric to obtain

$$\frac{d}{ds}\left(\frac{A(r)}{4l_P^2}\right)\Bigg|_0^{M_b} = \Phi(r)\frac{d}{dr}\frac{A(r)}{4l_P^2}\Bigg|_0^{M_b} = -\frac{2\pi c k_B M_b}{\hbar}. \tag{29}$$

Since the entropy change (29) lives strictly speaking in two geometries, with and without mass, we define the entropy change simply to be the derivative with respect to r only. This leads after integration to the definition

$$S_{M_b}(r) = -\frac{2\pi c k_B M_b r}{\hbar}.\,[8] \tag{30}$$

Note that (30) coincides with the change at the horizon for $r = R_\infty$. After multiplying (30) both in the nominator and the denominator by R_∞, we finally get

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[7] At very large scale the additional term is of importance for agreement with observation [11].
[8] This is the Bekenstein-bound with respect to the mass-induced energy $E_b = M_b c^2$.

$$S_{M_b}(r) = S_{M_b}(R_\infty)\frac{r}{R_\infty}. \tag{31}$$

We are now in a position to calculate where the presence of dark energy influences gravity. As long as effective dark entropy $S_d^{ef}(r)$, which is the net dark entropy after addition of mass M_b, is bigger than zero, dark energy plays a role. So we find the condition

$$S_d^{ef}(r) = S_d(r) - |S_{M_b}(r)| = k_B\left(\frac{A(r)}{4l_P^2} - \frac{2\pi c M_b R_\infty}{\hbar}\right)\frac{r}{R_\infty} \geq 0, \tag{32}$$

hence

$$\frac{2\pi c M_b R_\infty}{\hbar} \leq \frac{A(r)}{4l_P^2},$$

or

$$r \geq r_0 = \sqrt{\frac{M_b G}{a_0}}. \tag{33}$$

Note that we have $a(r_0) = g(r_0) = a_0$. This confirms empirical evidence that the boundary between the regimes of general relativity and MOND lies at acceleration $\sim a_0$.

3.2 Effective dark energy

In (20) we used for synchronization total dark entropy $S_d(r)$, of which we now know that it is reduced, but positive at ranges $r > r_0$. We want to account for this and define the effective dark energy E_d^{ef} by

$$E_d^{ef} = T_{\bar{a}(r)} S_d^{ef}(r).[9] \tag{34}$$

Like in (20) we now synchronize two thermal clocks ($r > r_0$)

$$T_{g(r)} S_b(r) = T_{\bar{a}(r)} S_d^{ef}(r) = T_{\bar{a}(r)} S_d(r) - T_{\bar{a}(r)}|S_{M_b}(r)|. \tag{35}$$

By applying (10) on the dark, non-baryonic entropy, we get

$$\frac{\bar{a}^2(r)}{a_0} S_b(r) - \bar{a}(r)|S_{M_b}(r)| - g(r) S_b(r) = 0. \tag{36}$$

(36) is a quadratic equation with one positive solution

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[9] We use the notation $\bar{a}(r)$ for effective acceleration.

$$\bar{a}(r) = \frac{\left(\frac{|S_{M_b}(r)|}{S_b(r)}\right) + \sqrt{\left(\frac{|S_{M_b}(r)|}{S_b(r)}\right)^2 + \frac{4g(r)}{a_0}}}{\frac{2}{a_0}}. \tag{37}$$

It turns out that the Newtonian potential can be written as $|\Phi(r)| = \frac{1}{2}\frac{|S_{M_b}(r)|}{S_b(r)}$ and hence (37) turns into

$$\bar{a}(r) = \frac{2|\Phi(r)| + \sqrt{4|\Phi^2(r)| + \frac{4|\Phi^2(r)|c^4}{M_b G a_0}}}{\frac{2}{a_0}}.$$

After factoring out, we finally arrive at our main result

$$\bar{a}(r) = |\Phi(r)|a_0\left(1 + \sqrt{1 + \frac{c^4}{M_b G a_0}}\right). \tag{38}$$

Note that formula (38) links directly to observation. In particular, if $|\Phi(r)|a_0 \ll 1$, we find [10]

$$\bar{a}(r) \approx |\Phi(r)|a_0\sqrt{\frac{c^4}{M_b G a_0}} = \frac{\sqrt{M_b G a_0}}{r}. \tag{39}$$

So we reproduce (22). For $r = r_0$, we make use of that fact that by (32) there holds $S_b(r_0) = S_{M_b}(R_\infty)$ and hence $|\Phi(r)| = \frac{r_0}{2R_\infty}$. Since $\frac{r_0}{R_\infty}a_0 \ll 1$,[11] (38) turns indeed into

$$\bar{a}(r_0) \approx \frac{r_0}{2R_\infty}a_0\sqrt{\frac{c^4}{M_b G a_0}} = \sqrt{\frac{c^4}{4R_\infty^2}} = a_0. \tag{40}$$

In the last step we made use of the definition of a_0. On the other hand we get for $g(r_0)$, as calculated before

$$g(r_0) = \frac{c^2|\Phi(r_0)|}{r_0} = \frac{c^2}{2R_\infty} = a_0. \tag{41}$$

These results coincide with the ones in 3.1, which confirm the empirical fact that the boundary between regimes lies at acceleration $\sim a_0$. For distances $r < r_0$, $g(r)$ describes gravity accurately, since there is no effective dark energy left.

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[10] $\frac{c^4}{M_b G a_0} \sim 10^{12}$

[11] $a_0 \sim 10^{-10}\frac{m}{s^2}$

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In [1] the idea of synchronizing thermal clocks in order to observe the universe by a unique rhythm is introduced. This general principle, based on considerations in the quantum-world [19], turns out to be very powerful and allows us to derive that space-time needs to be a curved medium, governed by Einstein's equations, if $\Lambda = 0$. Entropy is introduced in a formal way and by making assumption (17) in the presence of $\Lambda > 0$, we are able to apply the synchronization principle again. We thus derive corrections to general relativity, resulting in the main formula (38), which corresponds to the MOND corrections (22) at large scales. The approach also explains why the boundary between the two regimes, general relativity and MOND, lies at accelerations $\sim a_0$.
The source of the corrections lies in the assumption that in a de-Sitter vacuum the strict area law (5) for entropy does not hold and that dark energy cannot enter the scene just by adding it to the Einstein equations, a fact, which by assumption (17) follows from the derivation of Einstein equations in [1].

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