Emergence of Spacetime Through Interaction

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June 7, 2016

Abstract

We study kinematics of atoms and molecules in quantum shape dynamics. We analyzed a model universe where there is only electrical force between protons and electrons. A similar model where there is only gravitational attraction between masses was investigated by Barbour, Koslowski and Mercati before. Our results is an expansion of the ideas there. We found that absolute spacetime emerges by coarse graining the internal atomic and molecular degrees of freedom. It may be that spacetime only exists because of the presence of quantum degrees of freedoms. In the orthodox way of thinking in gravitation, emergence of spacetime is related to the gravitational degrees of freedom. Here we show that interaction alone is enough for the emergence of spacetime. In terms of our results, any force that can be attractive would give the same conclusions. This is especially important from the quantum gravity perspective.

1 Introduction

Shape dynamics is a fully relational theory of gravitation. In the case of N body problem, it states that only the relative distances and angles between them are dynamical [1]. In this scenario universe cannot have a nonvanishing angular momentum otherwise it would define an absolute space in which the universe is rotating [2]. Similarly nonvanishing total energy implies an external absolute time according to which the universe evolves, therefore we

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require total energy to vanish [2]. Another constraint comes from working in the center of mass frame, the total momentum should be zero. Some works [3] also put another constraint on the system, the vanishing of the dilational momentum: $\sum_a \mathbf{r}^a \cdot \mathbf{p}^a = 0$. This is required for scale invariance [3] however we will not impose it. Overall, we have three constraints:

- $\mathrm{H} = \sum_a E^a = 0$
- $\mathbf{P} = \sum_a \mathbf{p}^a = 0$
- $\mathbf{L} = \sum_a \mathbf{r}^a \times \mathbf{p}^a = 0$

We consider a universe populated by N_p protons and N_e electrons with masses m_p and m_e and with charges $-e$ and e. The total energy is:

$$H = \sum_a \frac{\mathbf{p}_a^2}{2m_a} + \frac{1}{2}\sum_{a\neq b} \frac{kq_a q_b}{|\mathbf{r}^a - \mathbf{r}^b|}, \tag{1}$$

where $k = 1/4\pi\epsilon_0$. We need to calculate Poisson brackets of commutators with each other in order to classify them. This distinction will be important when we quantize the theory. We do calculations in Appendix A and we list the results here:

$$\{P_i, P_j\} = 0, \tag{2}$$
$$\{L_i, L_j\} = \varepsilon^{ijk} L_k, \tag{3}$$
$$\{L_i, P_j\} = \varepsilon^{ijk} P_k, \tag{4}$$
$$\{H, P_i\} = 0, \tag{5}$$
$$\{H, L_i\} = 0. \tag{6}$$

The results of these commmutators are either zero or another constraint. Hence they vanish weakly. Therefore all the constraints are first class in the terminology of Dirac [4]. We did the Dirac analytis in order to show that our system is consistent and have seen that is it well defined.

2 Quantization of the Model

We quantize the model by promoting positions and momenta to quantum operators. The Poission bracket $\{\cdot,\cdot\}$ is mapped to $i\hbar[\cdot,\cdot]$. Between position and momenta we now have:

$$[\hat{r}_i^a, \hat{p}_j^b] = i\hbar\delta_b^a\delta_j^i. \tag{7}$$

Momenta are represented by operators, $\hat{p}_i^a = -i\hbar\partial/\partial r_i^a$. It is time to consider what happens to constraints in this case. As the readers can verify easily, the constraint algebra survives the quantization. In particular there is no anomaly. In the presence of dilational momentum constraint ref. [3] reports the existence of scale anomaly. It is then argued in [3] that this anomaly may give rise to a gravitational arrow of time. However, the arrow of time is not discussed in this Letter.

At the quantum level the constraints become operators acting on the quantum state of the system. For example the Hamiltonian constraint becomes:

$$H\psi = \sum_a -\frac{\hbar^2}{2m}\nabla_a^2\psi + V\psi = 0, \tag{8}$$

where ∇_a stands for gradient operator with respect to particle a: $(\partial/\partial r_x^a, \partial/\partial r_y^a, \partial/\partial r_z^a)$. We see that we have obtained a time independent Schrödinger equation. Wavefunctions do not evolve in time and are static. This is similar to what happens with the Wheeler-DeWitt equation.

The momentum and angular momentum constraints become:

$$\mathbf{P}\psi = -i\hbar\sum_a \nabla_a\psi = 0, \tag{9}$$

$$\mathbf{L}\psi = -i\hbar\sum_a \mathbf{r}^a \times \nabla_a\psi = 0. \tag{10}$$

We interpret equations (8) (9) and (10) as operator equations that determines the allowed kinematic states of the system.

3 Non-Formation of a Single Hydrogen Atom

In this section we suppose there is only one proton and electron in the universe. Classically, with the constraints in mind, shape dynamics tells us that proton and electron collide head-on with no angular momentum. Let us see if this description changes once the quantum mechanics is taken into account.

Rather than using $\mathbf{r}^1$ and $\mathbf{r}^2$ let us use the center of mass vector $\mathbf{R}$ and $\mathbf{r} = \mathbf{r}^2 - \mathbf{r}^1$ as coordinates. Here the first and second indices refer to proton and electron respectively. In this variables the Hamiltonian becomes:

$$H = \frac{\mathbf{p}_R^2}{2M} + \frac{\mathbf{p}_r^2}{2\mu} - \frac{ke^2}{r}, \tag{11}$$

where $M = m_p + m_e$ is the total mass and $1/\mu = 1/m_p + 1/m_e$ is the reduced mass. Center of mass degree of freedom is decoupled from the relative degrees of freedom. Because we work in the center of mass frame $\mathbf{R} = 0$. The rest of the Hamiltonian give rise to a hydrogen atom. The solutions of the hydrogen atom is well known in the literature. Readers may see [5].

However the eigenfunctions of the Hamiltonian produces an energy -13.6 eV$/n^2$ where $n = 1, 2, \ldots$ In order to satisfy the Hamiltonian constraint $H\psi = 0$ it is seen that n must approach to infinity: $n \to \infty$. On the other hand the angular momentum constraint requires $\mathbf{R} \times \mathbf{p}_R\psi + \mathbf{r} \times \mathbf{p}_r\psi = 0$. The first term vanishes because $\mathbf{R} = 0$. The other term vanishes only if the electron occupies an $l = 0$ state.

Moreover we have the momentum constraint. The wavefunction of the elctron depends only on the magnitude of $\mathbf{r}^2$, and wavefunction of the proton is a delta function. For the product of these two, the sum $-i\hbar \sum_a \nabla_a \psi$ does not vanish. The momentum constraint cannot be satisfied by an electron and proton pair. Hence we conclude that hydrogen atom does not form if there is only one electron and one proton in the universe.

4 Formation of Two Hydrogen Atoms

Let us suppose now there are two protons and two electrons in the universe. Because there are more degrees of freedom, we suspect there are many ways to satisfy the shape dynamics constraints. First we suppose that particles are far from each other. Then we will focus on the possibility of hydrogen atoms.

Suppose two atoms do form. This case is more interesting because the part of the Hamiltonian that describes each atom can take negative energy eigenvalues due to the potential term. These eigenvalues can then be compensated by nonzero atomic momentum, hence the Hamiltonian constraint is satisfied. At maximum the total kinetic energy of atoms can be 27.2 eV.

By arranging the motion of atoms there is a way to satisfy the Hamiltonian and momentum constraints. However the angular momentum constraint is more involved. If the total classical angular momentum is not zero, the atoms may pass each other with a perpendicular offset cancelling the expectation value of quantum angular momentum. We conclude that two hydrogen atoms can form. This observation has important results revealed in the next section.

5 Formation of a Hydrogen Molecule

It is time to consider the formation of a hydrogen molecule. First, we think that there are two electrons and two protons. Second we will look at the case where there are many particles in the universe.

If the atoms are close to one other, we expect they form a hydrogen molecule. This situation is more complex. Because electrons would no longer occupy definite angular momentum states if a molecule forms. Only the z component of angular momentum is quantized: molecular orbitals are common eigenfunctions of L_z and H.

Suppose that molecules form. When they form the expectation value of $\mathbf{L}$ will be nonzero. In order to satisfy the angular momentum constraint the molecule must rotate in a way to cancel the expectation value of $\mathbf{L}$. However this cannot occur. Because the $\langle \mathbf{L} \rangle$ rotates with the molecule as well. It is true that *time average* of angular momentum may vanish but this is still contrary to shape dynamic constraints. Entanglement does not solve the problem as well. Because two hydrogen atoms cannot be entangled in a singlet state. This is because they are bosons. We therefore conclude that if there are two hydrogen atoms in the universe, a hydrogen molecule cannot form.

6 Formation of Atomic and Molecular Hydrogen gas and the Emergence of Spacetime

In a universe with N_p protons and N_e electrons hydrogen atoms and molecules will form possibly with the exception of a few unbound protons and electrons. Hydrogen molecules will form if there is a subsystem of the rest of the system whose angular momentum cancels that of the hydrogen molecules. For example bypassing two hydrogen atoms with zero momentum in total may yield a total angular momentum in the reverse direction of the angular momentum of the molecule. The rest of the system must compensate for nonzero quantum angular momentum. See Figure 1.

The momentum constraint can be satisfied by requiring the subsystem that compensates the nonzero angular momentum by classical motion to have zero momentum. Again the solution comes from bypassing atoms and molecules. Therefore we conclude that in a system more than four particles bound states may occur.

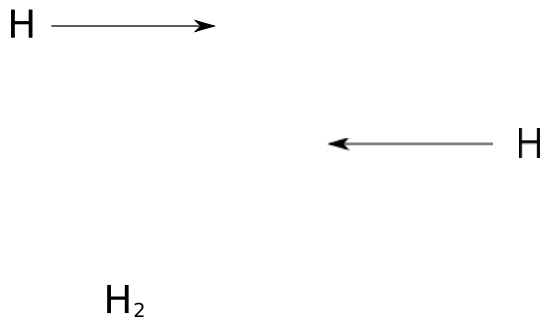

Figure 1: Bypassing two hydrogen atoms with zero momentum in total yielding a total angular momentum in the reverse direction of the angular momentum of the molecule.

7 Conclusion

In this paper we analyzed a model universe where there is only electrical force between protons and electrons. In ref. [2] a similar model where there is only gravitational attraction between masses is investigated. Our results are expansions of the ideas there. We found that single hydrogen atom or a single hydrogen molecule does not form. If there are more than four particles, bound states may occur.

This observation is especially striking from the point of view of classical shape dynamics. When considered as point particles, two atoms and one hydrogen molecule cannot rotate in a way to acquire angular momentum because it violates the angular momentum constraint and defines an absolute space in which things can rotate [2]. However when atoms are regarded as having internal structure, we have seen that a two hydrogen atoms can bypass each other in a way to cancel the expectation value of quantum angular momentum of the hydrogen molecule. From this observation we find that absolute spacetime emerges by coarse graining of quantum degrees of freedom. More precisely by coarse graining the internal structure of bound states. This conclusion is especially important from the quantum gravity perspective. It may be that spacetime only exist because of the presence of quantum degrees of freedom.

What is more, we did not make use of gravitational force in our model but electromagnetism. However an interaction with attractive forces (as in our case, it is not required that the force should attractive between all of the particles) would have the same effect in terms of our conclusions. Contrary to mainstream thinking in quantum gravity, we conclude that it is not gravitation that causes the emergence of spacetime but interactions with attractive force.

8 Acknowledgements

F.S.D. would like to thank Julian Barbour for useful discussions.

A Constraint Analysis

Commutation of momentum components:

$$\{P_i, P_j\} = \sum_{ab} \left\{p_i^a, p_j^b\right\} = 0. \tag{12}$$

Commutation of angular momentum components (We adopt Einstein summation convention: repeated indeces are summed over):

$$\{L_i, L_j\} = \left\{\sum_a \varepsilon^{ikl} r_k^a p_l^a, \sum_b \varepsilon^{jmn} r_m^b p_n^b\right\} \tag{13}$$

$$= \sum_{ab} \varepsilon^{ikl}\varepsilon^{jmn}[p_l^a r_m^b \left\{r_k^a, p_n^b\right\} + r_k^a p_n^b \left\{p_l^a, r_m^b\right\}] \tag{14}$$

The Poisson brackets will yield a product of Kronecker deltas: $\delta_b^a\delta_n^k$ and $-\delta_b^a\delta_m^l$

$$= \sum_a \varepsilon^{ikl}\varepsilon^{jmk} r_m^a p_l^a - \sum_a \varepsilon^{ikl}\varepsilon^{jln} r_k^a p_n^a \tag{15}$$

The sum of two Levi-Civita symbols over one index yields two Kronecker deltas: $\varepsilon^{ijk}\varepsilon^{ilm} = \delta_l^j\delta_m^k - \delta_m^j\delta_l^k$.

$$= \sum_a (r_i^a p_j^a - r_j^a p_i^a) \tag{16}$$

$$= \varepsilon^{ijk} L_k \tag{17}$$

Commutation of momentum and angular momentum components:

$$\{L_i, P_j\} = \left\{ \sum_a \varepsilon^{ilm} r_l^a p_m^a, \sum_b p_j^b \right\} \tag{18}$$

$$= \sum_{ab} \varepsilon^{ilm} p_m^a \left\{ r_l^a, p_j^b \right\} \tag{19}$$

$$= \varepsilon^{ijm} \sum_a p_m^a \tag{20}$$

$$= \varepsilon^{ijk} P_k \tag{21}$$

Commutation of Hamiltonian and momentum:

$$\{H, P_i\} = \left\{ \sum_a \frac{(\mathbf{p^a})^2}{2m_a} + V, \sum_b p_i^b \right\} \tag{22}$$

$$= \sum_b \left\{ V, p_i^b \right\} \tag{23}$$

$$= \sum_b \frac{\partial V}{\partial r_i^b} \tag{24}$$

$$= 0 \tag{25}$$

This is because V is only a function of interparticle separations. This is expected because the commutator $\{P_i, H\}$ equals the total external force on the system which is zero. Commutation of Hamiltonian and angular momentum:

$$\{H, L_i\} = \left\{ \sum_a \frac{(\mathbf{p^a})^2}{2m_a} + V, \sum_b \varepsilon^{ijk} r_k^b p_k^b \right\} \tag{26}$$

$$= \sum_{ab} \frac{p_l^a \left\{ p_l^a, r_j^b \right\} p_k^b \varepsilon^{ijk}}{m_a} + \sum_b \varepsilon^{ijk} r_j^b \left\{ V, p_k^b \right\} \tag{27}$$

The first term will yield a sum over $\mathbf{p}^a \times \mathbf{p}^a$ which is zero.

$$= \sum_b \varepsilon^{ijk} r_j^b \frac{\partial V}{\partial r_k^b} \tag{28}$$

$$= \sum_b \mathbf{r^b} \times \nabla_b V \big|_i \tag{29}$$

where ∇_b stands for gradient with respect to particle b: $(\partial/\partial r_x^b, \partial/\partial r_y^b, \partial/\partial r_z^b)$.

$$= 0 \tag{30}$$

It vanishes because V only depends on the interparticle separations. This is a check that the angular momentum constraint is preserved as time goes on.

All in all, we have the following relations:

$$\{P_i, P_j\} = 0, \tag{31}$$
$$\{L_i, L_j\} = \varepsilon^{ijk} L_k, \tag{32}$$
$$\{L_i, P_j\} = \varepsilon^{ijk} P_k, \tag{33}$$
$$\{H, P_i\} = 0, \tag{34}$$
$$\{H, L_i\} = 0. \tag{35}$$

References

[1] J. Barbour. Shape Dynamics. An Introduction. May 2011. arXiv:1105.0183.

[2] J. Barbour, T. Koslowski, and F. Mercati. Identification of a Gravitational Arrow of Time. *Phys. Rev. Lett.*, 113(18):181101, October 2014. arXiv:1409.0917.

[3] J. Barbour, M. Lostaglio, and F. Mercati. Scale anomaly as the origin of time. *Gen. Relat. Gravit.*, 45:911–938, May 2013.

[4] Paul AM Dirac. *Lectures on quantum mechanics*. Courier Corporation, 2013.

[5] David Jeffery Griffiths. *Introduction to quantum mechanics*. Pearson, 2005.