Original paper

Resolving the Problem of Definite Outcomes of Measurements

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Abstract: The entangled "Schrodinger's cat state" of a quantum and its measurement apparatus is not a paradoxical superposition of states but is instead a non-paradoxical superposition of nonlocal coherent correlations between states: An un-decayed nucleus is correlated with a live cat, *and* a decayed nucleus is correlated with a dead cat. This elucidation of entanglement is demonstrated by quantum-theoretical analysis and by experiments performed in 1990 using entangled photon pairs. Thus the cat state does not predict a dead-and-alive cat. Instead of indefinite superpositions, it predicts mixtures of definite eigenvalues even though the subsystems are not actually in the corresponding eigenstates, a situation that implies a (trivial) revision of the standard eigenvalue-eigenstate rule. Because the subsystem *states* are not mixed even though the subsystem *eigenvalues* are mixed, this analysis avoids two common objections to such a resolution, namely improper density operators and basis ambiguity. Thus, entanglement transfers coherence from the superposed quantum to correlations between the quantum and its measuring apparatus, permitting instantaneous collapse without interrupting the global unitary evolution. This resolves a key part of the measurement problem.

Keywords: problem of definite outcomes • measurement problem • Schrodinger's cat • nonlocality • two-photon interferometry

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1. Introduction and background

This paper presents a suggested resolution of the problem of definite outcomes, a key part of the quantum measurement problem. This introductory section discusses the pertinence and precise nature of the measurement problem, von Neumann's well-known ideal measurement scheme, the extent to which decoherence resolves the problem, the "Schrodinger's cat state" that results from an ideal measurement, and previous analyses of the problem of definite outcomes. As we'll see in later sections, clarification of the cat state's properties, especially its nonlocal aspects, is key to resolving the problem of outcomes.

Although von Neumann first posed the measurement problem in 1932 {1}, the issue persists. Thirty-three participants in a 2013 quantum foundations conference responded to a multiple-choice questionnaire as follows: The measurement problem is (a) a pseudo-problem (27 percent agreed), (b) solved by decoherence (16 percent), (c) solved or will be solved in another way (39 percent), (d) a severe difficulty threatening quantum mechanics (24 percent), (e) none of the above (27 percent) {2}. In 2011, 17 recognized quantum foundations experts provided one-page written responses to several questions. One question was "The quantum measurement problem: serious roadblock or dissolvable pseudo-issue?" Nine of the 17 said it's an unsolved roadblock. Six said it's not a roadblock but only a pseudo-issue because quantum physics does not describe reality, it describes only our own knowledge of a veiled microscopic world. Of the remaining two experts, one said decoherence solves the measurement problem and the other said the many-worlds interpretation solves it {3}. Summarizing both polls, most experts think measurement is a real problem but there's little agreement as to how to solve it or whether it's already been solved, and a strong minority think it's not a problem because quantum physics is merely "epistemic" (about knowledge) rather than "ontological" (about reality).

The problem can be compactly and adequately formalized in terms of a quantum S whose Hilbert space is spanned by just two orthonormal eigenstates $|s_i>$ $(i=1,2)$ of some observable, in a superposition state

$$|\psi>_S = c_1|s_1> + c_2|s_2> \qquad (1)$$

where $|c_1|^2+|c_2|^2=1$. "Measurement" is the process of correlating the eigenstates $|s_1>$ and $|s_2>$ with states $|a_1>$ and $|a_2>$, respectively, of a macroscopic detection apparatus A that is assumed to be describable by quantum physics, so that direct macroscopic observation of A can tell us the microscopic state of S.

As pointed out in the measurement analyses of Bassi and Ghirardi {4}{5}, the simplicity of von Neumann's scheme {1} for an ideal measurement process allows one to grasp immediately the difficulties associated with quantum measurements. This scheme supposes that A has a "ready state" $|a_0>$ in which A is ready to measure S, and that $|a_0>$, $|a_1>$, $|a_2>$ are mutually orthogonal. If the measurement interaction is linear (as the Schrodinger evolution certainly is) and does not disturb eigenstates of the measured observable, the initial correlated composite state $|s_i\rangle|a_0\rangle$ evolves into the correlated measured state $|s_i\rangle|a_i\rangle$ $(i=1,2)$. Thus the linearity of the evolution guarantees that the superposition (1), when measured, evolves into the final correlated state

$$|\psi\rangle_{SA} = c_1|s_1\rangle|a_1\rangle + c_2|s_2\rangle|a_2\rangle. \quad (2)$$

(2) is often called the "post-measurement state," but this term would presume, inappropriately, that (2) represents the end of the measurement process. So I'll call (2) the "measurement state" (MS). Since Schrodinger spoke of entanglement as "not one but rather *the* characteristic feature of quantum mechanics," it's well worth noting that the MS is just such an entangled state.

The measurement problem is that the state we actually observe at the end of such a measurement is not the MS. Instead, experiment shows that the MS "collapses" randomly to one of its two terms. In other words, the actually observed final outcome is the composite-system mixture

$$\text{either } |s_1\rangle|a_1\rangle \text{ or } |s_2\rangle|a_2\rangle \quad (3)$$

whereas the superposition (2) amounts to "both $|s_1\rangle|a_1\rangle$ and $|s_2\rangle|a_2\rangle$." von Neumann and others have dealt with this fundamental inconsistency by simply postulating that, somehow, the final state following a realistic, i.e. non-idealized, measurement is the collapsed state (3), but then of course the problem becomes one of demonstrating consistency between this postulate and the other principles of quantum physics. One would obviously prefer to derive (3) from the other quantum principles, especially since (3) appears to contradict the superposition (2).

In hopes of maintaining an unambiguous and realistic discussion, I'll often phrase my arguments in terms of one or the other of two familiar examples. The first is the double-slit (or beam-splitter) experiment with a single electron (or photon) *S* coming through two slits or a beam-splitter, a "which-path" detector *A* in place at the slits or beam-splitter, and a detection screen or "particle" detectors at the far end. The second example is Schrodinger's much-overworked cat, where *S* is a radioactive nucleus, the $|s_i\rangle$ are undecayed and decayed states of the nucleus, *A* is a cat attached to vial of poison attached to a hammer attached to a Geiger counter at the nucleus, and the $|a_i\rangle$ are the alive and dead states of the cat {6}.

The "delayed choice" version of the double-slit experiment is especially instructive {7}. When the quantum is in a simple superposition (1) of following both paths, repeated trials reveal an interference pattern. But when the quantum correlates with a which-path detector as in (2), the interference pattern instantly collapses into an incoherent mixture. It's a revealing experiment because careful timing and fast switching, one photon at a time, between detection and non-detection shows that the correlation between *S* and *A* causes the collapse, implicating the entangled MS directly in the collapse process.

As another way of viewing the measurement problem, the MS can be thought of as a superposition of the two composite states $|b_i\rangle = |s_i\rangle|a_i\rangle$ ($i=1,2$). The MS then becomes the macroscopic superposition

$$c_1|b_1\rangle + c_2|b_2\rangle \tag{4}$$

representing an alive-cat-plus-undecayed-nucleus superposed with a dead-cat-plus-decayed-nucleus, which is ridiculous because this is not what we would see and it cannot be this easy to create such a macroscopic superposition. Both (2) and (4) seem to describe a superposition of both composite outcomes b_1 and b_2, so there appears to be no "definite" (i.e. single) outcome of the measurement.

This paper presents a proposed resolution of only this part of the measurement problem, namely the problem of definite outcomes.

The MS cannot represent the final state of the composite system *SA*. It's a pure state, which therefor has zero entropy, but the final situation (3) cannot have zero entropy because it is a mixture and is irreversibly macroscopically recorded.

Decoherence theory {8} sheds considerable light by showing how the natural environment can measure a quantum system, transforming superpositions into the MS, where the $|a_i\rangle$ are now states of the environment. Decoherence occurs when the $|a_i\rangle$ are not initially orthogonal, implying that they do not fully distinguish between the eigenstates $|s_i\rangle$, and a series of system-environment interactions then ensues that eventually orthogonalizes the $|a_i\rangle$. This process "decoheres"--removes the interferences and coherence from--the superposition (1). Because information about *S* is now widely and irreversibly dispersed in the environment, this process shows how measurements can become irreversible {9}. A similar dispersal occurs in the case of measurements by an apparatus rather than by the environment, with the many-body apparatus playing the role of the environment {9}. Decoherence also responds to John Bell's complaint:

> To restrict quantum mechanics to be exclusively about piddling laboratory operations is to betray the great enterprise. A serious formulation will not exclude the big world outside the laboratory {10}.

The environment, by means of the decoherence process, does indeed perform the vast majority of quantum measurements throughout the natural universe.

Although decoherence shows how measurements become irreversible and universal, it doesn't solve the problem of definite outcomes. As Stephen Adler puts it:

> The quantum measurement problem consists in the observation that [the MS] is *not* what is observed as the outcome of a measurement! What is seen is not the superposition of [the MS], but rather *either* the unit normalized state [$|s_1>|a_1>$] *or* the unit normalized state [$|s_2>|a_2>$]" (the emphasis is Adler's, the square brackets are mine) {11}.

In other words, decoherence simply gets us back to the MS, which still exhibits the problem of definite outcomes.

The entangled MS is subtle. It's clearly a superposition but precisely what is superposed? Are dead and alive states of Schrodinger's cat superposed? Is a decayed an undecayed nucleus superposed? The answer to these questions is "no." A perhaps surprising feature of the MS that is often, and unaccountably, ignored is that *neither S nor A is in a superposition.* This is directly provable by assuming *S* or *A* is in a superposition and deriving a contradiction {12}. Perhaps, then, the entire composite system is superposed. But this just leads us back to the macroscopic superposition (4) which, as noted, is ridiculous. We will find, in Section 2, that there is another way of viewing the superposed MS, and that both quantum theory and experiment support this view.

The key feature of the MS is entanglement. Entangled subsystems are correlated nonlocally {13}. In fact, nonlocality is written all over the measurement process in general and the double-slit experiment in particular {14}. With both paths available and no which-path detector, repeated trials show an interference pattern on downstream detectors, showing each quantum to be in a superposition of following both paths--a nonlocalized state. Furthermore, with both paths available and a which-slit detector present at only *one* path, a non-interfering mixture forms. Every measurement trial conforms to this pattern, including those in which the quantum follows the undetected path--a nonlocal effect. It's well known that entangled states such as the MS are nonlocal {15}, implying that subsystem correlations can be altered instantly across arbitrary distances and in violation of John Bell's locality conditions.

A photon, electron, or other fundamental quantum is a single thing. Any change in its state must happen simultaneously throughout the quantum's entire spatial extent {14}. Furthermore, two fully entangled quanta, such as an entangled photon pair, or an electron and a which-slit detector, act in a similar unified fashion {15}. The same is surely true of any maximally-entangled N-body system: It is a single quantum, just as an isolated electron or photon is a single quantum, only it carries N excitations instead of one. This unity of the quantum is the source of nonlocality. But despite widespread experimental and theoretical support for a nonlocal relationship between a quantum and its measuring apparatus, many analyses of quantum measurement pay scant attention to nonlocality.

There's a big difference between a simple superposition (1) and a superposition (2) of a composite system made of subsystems that can move relative to each other. Serge Haroche advises us that a system should be considered non-composite whenever the binding between its parts is much stronger than the interactions involved in its dynamics {16}. By this criterion, a system and its detector must be considered separate subsystems. It would be a big mistake to consider the MS to be a simple superposition (4), because such an identification would miss important physics, namely the nonlocal aspects of the entanglement: *A* and *S* can in fact be moved arbitrarily far apart without diminishing the nonlocal unity of their correlations. Thus Schrodinger's cat is not a simple superposition of "live-cat-and-undecayed-nucleus" and "dead-cat-and-decayed-nucleus." The MS must be regarded as a state of entanglement between subsystems rather than a simple superposition of the composite system *SA*.

The MS is a pure state, not a mixture. Yet the desired end-point of the measurement analysis is the mixture (3). The strategy of previous measurement analyses {17}{18}{19} has been to regard the MS (2) as unacceptable on the grounds that it is a pure state that entails indefinite (i.e. superposed) outcomes of the apparatus. These analyses instead argue that the initial state, prior to establishing the measurement correlations, should be a mixture of apparatus states because of the apparatus' macroscopic nature, and an appropriate choice of this initial mixture would then lead to the final mixture (3) of pure states in each of which the apparatus shows a definite outcome. But it then turns out that no such initial mixture exists.

This paper takes a different tack via a closer look at the MS itself. It's not what it at first appears to be.

Section 2 shows that, once its nonlocal features are taken into account, the MS is not a paradoxical superposition of subsystem states. Theory and experiment show the MS to instead be a non-paradoxical superposition of nonlocal *correlations between* subsystem states, and the outcomes to indeed be mixtures of just the desired sort but mixtures of eigenvalues rather than eigenstates. Little was known about the nonlocal properties of entangled states until 1964 when John Bell showed such states permit the instant establishment, across arbitrarily large distances, of nonlocal correlations that cannot be explained locally by hidden variables or prior common causes {20}. Aspect and others, whose nonlocality experiments can be regarded as experimental investigations of the MS, verified experimentally that entangled states indeed violate local realism by establishing distant correlations superluminally {13}. Section 2 reviews such an experiment and the associated theory to show that the MS is not a superposition of states at all, but rather a superposition of correlations between states.

Section 3 discusses implications for the measurement problem. In terms of Schrodinger's cat, the MS (in which the phase angle between the superposed branches is fixed at zero) says simply that an undecayed nucleus is 100% positively correlated with an alive cat, *and* a decayed nucleus is 100% positively correlated with a dead cat. The italicized word "*and*" indicates the superposition, which is not paradoxical.

Section 4 summarizes the conclusions.

2. Nonlocality and measurement

This section analyzes the MS (2) based on experiment and on quantum theory. It is a superposition, but precisely what is superposed? The MS is prone to many misconceptions, for example the misconception, discussed in Section 1, that it describes a superposition of either subsystem A (e.g. a cat) or subsystem S (e.g. a nucleus). As an entangled state, it violates Bell's inequality and has nonlocal aspects {15}. Does this nonlocality have anything to do with the measurement problem?

We'll see, in fact, that the nonlocal aspects of the MS are key to understanding the measurement problem. To understand these aspects, there is no need for either subsystem to be macroscopic. Microscopic experiments involving pairs of photons entangled in the MS (2) have been performed for decades, with Aspect's experiment {13} a characteristic and leading example. Such experiments turn out to be quite relevant to the measurement problem.

There is in fact at least one beam splitter experiment that strikingly highlights the nonlocal relationship between a quantum and its detector. In 1991, Zou, Wang, and Mandel {21} performed an experiment employing entangled photon pairs in which one photon goes through a beam splitter while its entangled partner acts as a distant detector for the first photon. The first photon passes through the beam splitter and impacts a detector, while the second photon flies to a distant location. The ingenious geometry of these paths entails that, by either (1) inserting or (2) not inserting a barrier along one of the second photon's two superposed paths, this second photon either (1) detects or (2) doesn't detect, respectively, the path of the first photon. So the second photon acts as an optional which-path detector for the distant first photon. As expected, on those trials in which the barrier is inserted in the second photon's path, the first photon impacts the screen in a *mixture* of definitely coming through one or the other slit, and on those trials in which the barrier is not inserted, the first photon impacts the screen in an interference pattern implying an indefinite *superposition* of coming through both slits. This is astonishingly nonlocal. The first photon could be sent into space, and the decision to insert or not insert the second photon's barrier made on Earth when the first photon was halfway to the next star, yet the first photon would presumably jump between mixture and superposition depending on whether the second photon could or could not perform a which-slit detection, i.e. depending on whether the two photons were *nonlocally entangled* or *not entangled*.

Experiments conducted in 1990 by Rarity and Tapster {22} and concurrently by Ou, Zou, Wang, and Mandel {23} reveal the true nature of the MS. These "RTO" (for Rarity, Tapster, and Ou) experiments involve paired photons whose phases and momenta are entangled in the MS. For pedagogical discussions of these experiments, see {24} and {25}. Microscopic subsystems S and A are essential for our analysis of the MS. With a macroscopic which-path detector, the phase relations and correlations between S and A are frozen: $|s_1>$ is 100 percent positively correlated with $|a_1>$, and $|s_2>$ is 100 percent positively correlated with $|a_2>$, with no possibility of variation and thus no opportunity to observe interference. Phase variations over repeated trials are needed to create interference effects and thus to understand which entities are superposed.

RTO's photon pairs are entangled in the MS with $c_1 = c_2 = 1/\sqrt{2}$. In each trial, a central source creates two entangled photons S and A by parametric down-conversion. Each pair is emitted into two superposed branches, shown in Figure 1 as a solid line and a dashed line. Note carefully that the solid correlation *S1-A1* is superposed with the dashed correlation *S2-A2*. This superposition of two branches, each branch representing a correlation between a state of S and a state of A, is the essence of the MS. Thus S is emitted into two beams (dashed and solid) and so is A, and the beams are entangled in the MS (2). To observe interference, one beam of S is phase-shifted through φ_S (see Figure 1) and one beam of A is phase-shifted through φ_A. S's and A's two beams are overlapped at beam splitters and monitored by photon detectors S1, S2, A1, A2 as shown.

Without entanglement, the set-up would be simply two beam splitter interference experiments, with each photon emerging in two superposed beams that interfere at separate detectors at phase shifts φ_S and φ_A respectively. The detectors would observe each photon interfering with itself.

Entanglement changes everything: Even though the detectors might be separated widely, the entangled MS entails that *each photon acts as an ideal which-path detector for the other photon*. Entanglement collapses, or "measures," both single-photon superpositions, so that *S and A impact their detectors randomly with no phase-shift dependence*. In the RTO experiment, we see quite clearly the causal connections between entanglement, measurement, and collapse from superposition to mixture.

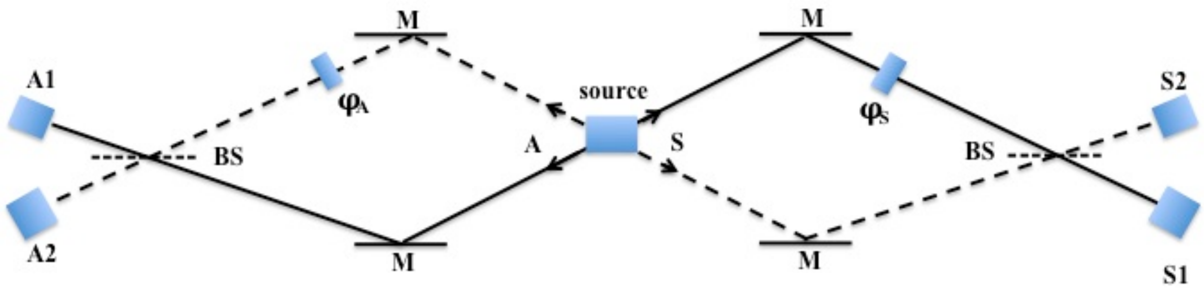

Figure 1. The RTO experiments. In each trial, the source emits entangled photons *A* and *S* into a superposition of solid and dashed paths to create the entangled MS. The set-up can be viewed as a double double-slit experiment with each photon going into two superposed paths (solid and dashed) with one path phase-shifted. Mirrors M and beam splitters BS recombine the beams so they can interfere. Without entanglement, each photon would coherently interfere as a function of its own phase shift. Entanglement destroys this interference, so each photon exhibits an incoherent mixture of eigenvalues while remaining in the coherent MS.

This collapsed, or mixed, outcome of both subsystems, caused by the entanglement, is precisely what the reduced density operators for both subsystems predict. It's a standard theorem {8} of quantum physics that, when a composite system is in an entangled pure state, all expectation values for either subsystem alone can be extracted from the reduced density operators. For the case of the MS (2), these reduced density operators are

$$\rho_S = Tr_A(\rho_{SA}) = |s_1\rangle\, |c_1|^2 \langle s_1| + |s_2\rangle\, |c_2|^2 \langle s_2|, \tag{5a}$$

$$\rho_A = Tr_S(\rho_{SA}) = |a_1\rangle\, |c_1|^2 \langle a_1| + |a_2\rangle\, |c_2|^2 \langle a_2|, \tag{5b}$$

where ρ_{SA} is the density operator $|\psi\rangle_{SA\,SA}\langle\psi|$ formed from the MS. Thus, when the photons are entangled in the MS, each photon exhibits a mixture of definite outcomes with no sign of superposition or interference. Note the agreement between the theoretical reduced density operators and experiment.

Thus the outcomes observed separately or "locally" at each subsystem are mixtures, not superpositions. But we must be careful: *S and A are not in the states (5), because S and A are actually in the global MS.*

Nevertheless, (5) predicts all measurement statistics for *S* and *A* separately. That is, (5) predicts what a local observer observes, and what RTO observe in their experiment, even though (5) doesn't predict the actual pure *state* (2) that a global observer would detect, because the global observer would detect *S-A* correlations of which local observers are unaware. To put it another way, the locally-observed *outcomes* or *values* are mixed, but the globally-observed *state* is not a mixture, it is a pure state.

This implies that Schrodinger's cat is either alive or dead, not both, even though the global state is the pure entangled state (2). The outcome of a measurement is not some abstract state in Hilbert space but rather an eigenvalue--a number or property such as "alive" or "dead." The "local states" (5) correctly predict the eigenvalues--the observed local outcomes--exhibited by *S* and *A* when the state (2) obtains. Because neither subsystem is actually in the mixture (5), these reduced density operators are called "improper mixtures."

Thus, standard quantum theory predicts that *S* and *A* exhibit mixtures rather than superpositions, but mixtures of eigenvalues not mixtures of states. John Bell famously disagreed with such a replacement of *both/and* by *either/or*:

> The idea that elimination of coherence, in one way or another, implies the replacement of "and" by "or," is a very common one among solvers of the "measurement problem." It has always puzzled me {10}.

Contrary to Bell, the RTO experiment demonstrates that the "and" of a superposition does indeed get replaced by the "or" of an improper mixture when a superposed quantum loses coherence by becoming entangled with another quantum.

But quantum dynamics is unitary, implying that the global MS remains coherent despite the incoherence of its local subsystems. Where has the coherence gone? The answer: It resides in the coherent relationship between Figure 1's solid and dashed MS branches! This global coherence is observed experimentally in coincidence measurements comparing the impact points of entangled pairs. Quantum theory predicts, and the RTO measurements confirm, that *the degree of correlation between paired photons S and A* varies coherently as the cosine of the *difference* φ_S-φ_A between the two local phase shifts, as graphed in Figure 2. In Figure 2, perfect correlation (+1) means the photon detectors always agree: Either both register state 1 or both register state 2. Perfect anti-correlation (-1) means the detectors always disagree: If one registers state 1, the other registers state 2. In either case, the outcome at *S* is predictable from the outcome at *A*. Zero correlation means the detectors agree on a random 50 percent of trials, and disagree on 50 percent, so the outcome at *S* is not at all predictable from the outcome at *A*. Other degrees of correlation represent intermediate situations; for example, a correlation of +0.5 implies a 75 percent probability of agreement, while a correlation of -0.5 implies a 75 percent probability of disagreement.

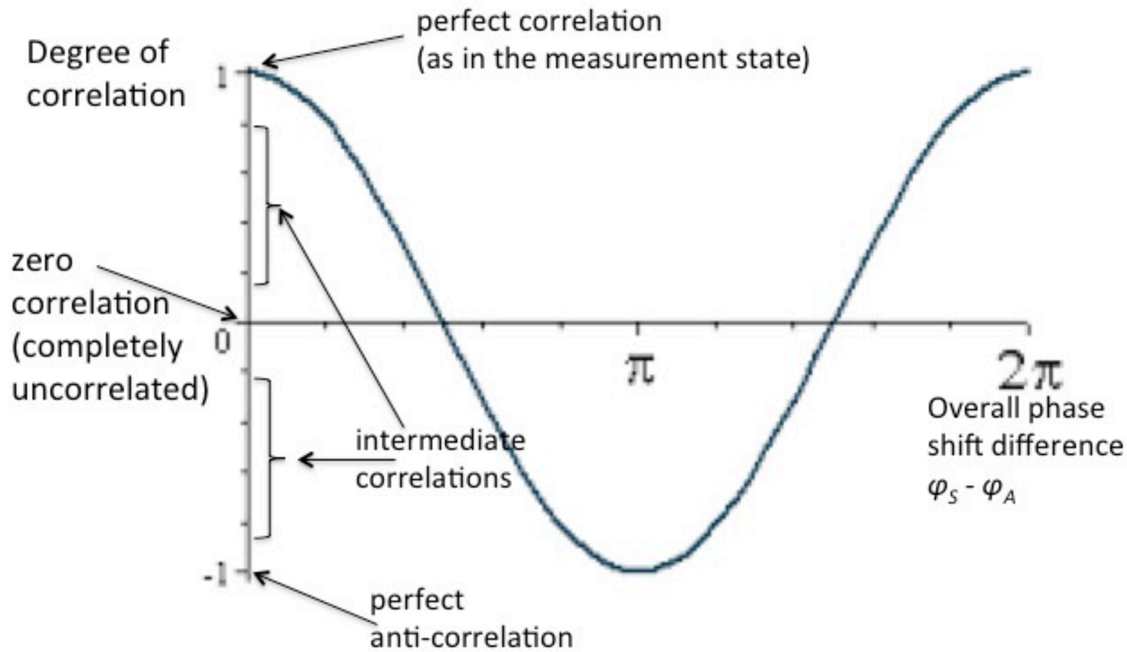

Figure 2. Nonlocal interference of the coherent correlations between the entangled photons of the RTO experiments. As the nonlocal phase difference $(\varphi_S - \varphi_A)$ varies, the degree of correlation shows coherent interference.

In the equivalent entangled double-double-slit experiment with two screens, each photon "knows" the impact point (equivalently, the phase shift) of the *other* photon and instantaneously adjusts its own impact point in order to form an interference pattern as a function of the *difference* between the two photons' phase shifts! This seems strikingly nonlocal, and indeed the outcomes violate Bell's inequality.

What is the source of the interference demonstrated by Figure 2, i.e. what entities are interfering? Quantum physics predicts the following probabilities for the four possible correlated pairs of outcomes of the RTO experiment {24}:

$$P(A1, S1) = P(A2, S2) = 1/4\ [1 + \cos(\varphi_S - \varphi_A + w)] \tag{6a}$$

$$P(A1, S2) = P(A2, S1) = 1/4\ [1 - \cos(\varphi_S - \varphi_A + w)] \tag{6b}$$

where the phase angle w is determined by fixed parameters of the experimental setup in Figure 1, namely the phase shifts upon reflection and transmission at beam splitters, and the fixed (with both phase shifters set at zero) path lengths for the two photons; in Figure 2, w has been set to zero by an appropriate choice of origin for the nonlocal phase difference $\varphi_S - \varphi_A$.

With $w=0$, the probability that measurements of A and S will yield the *same* eigenvalues (either both 1 or both 2) is

$$\text{P(same)} = P(A1, S1) + P(A2, S2) = 1/2\ [1 + \cos(\varphi_S - \varphi_A)] \qquad (7a)$$

while the probability that the two values will be *different* is

$$\text{P(diff)} = P(A1, S2) + P(A2, S1) = 1/2\ [1 - \cos(\varphi_S - \varphi_A)]. \qquad (7b)$$

Thus quantum physics predicts the degree of correlation, defined as C = P(same) - P(diff), is simply $\cos(\varphi_S - \varphi_A)$, as graphed in Figure 2. When the two photons are in phase with each other ($\varphi_S - \varphi_A = 0$), their correlations (S1-A1 and S2-A2) reinforce each other positively ($C = +1$); when the photons are exactly out of phase with each other ($\varphi_S - \varphi_A = \pi$), their correlations reinforce each other negatively ($C = -1$); and when they are 90-degrees out of phase with each other ($\varphi_S - \varphi_A = \pi/2$), their correlations cancel out ($C = 0$). Clearly, the interference is between the A1-S1 correlation (the solid line in Figure 1) and the A2-S2 correlation (the dashed line).

Thus the MS is a *nonlocal coherent superposition of correlations.* The superposition is nonlocal because, as in all interference phenomena, alteration of the phase of either branch instantly changes the interference pattern, and such alterations can be made at either of the arbitrarily-distant local sites *A* or *S*. But since it's an interference not of *states* but only of *correlations between states*, neither a local observer of *A* nor a local observer of *S* can detect it; global data, gathered from both *A* and *S,* is required to detect changes of correlations and thus detect the interference. So nonlocality cannot be used to send superluminal signals. Nature's tactic here is ingenious: She must be nonlocal in order to preserve the unity or "coherence" of the spatially extended "bi-quantum" (pair of entangled photons), yet she must not violate relativistic causality. Thus she accomplishes nonlocality entirely by means of correlations.

As verification that the interference cannot be observed locally, we calculate from (5) that P(A1) = P(A1, S1) + P(A1, S2) = 1/2, and similarly P(A2) = P(S1) = P(S2) = 1/2. Thus the local states at *A* and at *S* are phase-independent mixtures, as predicted by (5); if this were not the case, instant messages could be sent. The 2-photon correlations, however, depend on both local phases, as can be seen directly from Figure 1: For example, if we use the phase shifter φ_S to add a quarter wavelength to the length of the solid branch of the global superposition in Figure 1, we shift the phase relation between detectors A1 and S1 by 90 degrees. This could for example shift the degree of correlation C (Figure 2) between the two photons from +1 to 0--from perfectly correlated to entirely uncorrelated.

Such instantaneous nonlocal phase shifts demonstrate the unity of the quantum, in this case the unity of the entangled spatially extended photon pair: An action at one end of the entangled state shown in Figure 1 instantly affects the physical situation at the other end (says Bell's inequality).

Thus the "+" sign in the MS (2) represents neither a superposition of states of S nor a superposition of states of A nor a superposition of states of the composite system SA. Nature must prohibit nonlocal alteration of states because this would permit instantaneous signaling. The MS entails only that the outcome s_1 is coherently correlated with the outcome a_1, *and* the outcome s_2 is coherently correlated with the outcome a_2. This is a superposition, but it is non-paradoxical and agrees with observation. Entangling the state (1) with the detector transforms the *coherence of the states of S* into the *coherence of the correlations between S and A*, allowing S and A to exhibit definite outcomes while preserving global coherence as demanded by unitary evolution. This is how nature resolves the problem of outcomes.

This analysis shows local observations *must* reveal phase-independent mixtures of definite outcomes, rather than a phase-dependent superposition, because of Einstein's prohibition on superluminal signaling. The coherence of the MS must be invisible to local observers, and yet show up in the global MS in order to preserve the unitary dynamics. The collapse from a superposition (1) of S to local states (mixtures) of the subsystems S and A is a consequence of the MS's nonlocality (an implication of the unity of the quantum) plus special relativity's ban on instant signaling.

3. Discussion

This section discusses the extent to which the results of Section 2 resolve the measurement problem, the relation of this analysis to previous analyses such as the modal interpretation, and two frequent objections to analyses of the present sort, namely improper mixtures and basis ambiguity.

Understanding the MS as a superposition of correlations rather than a superposition of states resolves the problem of definite outcomes. Even though S and A are in a pure state, namely the MS, A itself exhibits an indeterminate mixture of either eigenvalue a_1 or a_2, implying that S exhibits either eigenvalue s_1 or s_2. Thus Schrodinger's cat is either alive or dead, and the nucleus is, correspondingly, either undecayed or decayed.

But this conclusion does not entirely solve the measurement problem because both subsystems remain in the MS. How does one get from the reversible superposition of correlations pictured by the solid and dashed lines in Figure 1 to a single irreversible outcome, namely *either* the solid line *or* the dashed line? The argument surely involves the arbitrarily large separation of *A* and *S*, the macroscopic nature of the measuring apparatus *A*, and decoherence, but this paper claims only to resolve the problem of definite outcomes and not the full measurement problem.

In 1968, Josef Jauch noted that the reduced density operators (5) appear to offer just the right resolution of the measurement problem {26}. At least one other published report {27} has made the same point. Quoting Jauch regarding (5),

> "We see that both states have become mixtures. ...There is no question of any superposition here. ...Moreover, we have a measurement since the events in [*A*] and in [*S*] are correlated."

However, Jauch got it slightly wrong in saying that "both states have become mixtures" because the *state* of both *A* and *S* remains the pure MS, not the mixtures (5). It is straightforward to show {8} that the reduced density operators (5) give correct *expectation values* for any observable of either subsystem, but it is a mistake to conclude that either subsystem is actually in the *state* shown in (5). The states (5) are mixtures, and neither subsystem is in a mixture. The correct statement is that (5) yields the correct measured *values* for each subsystem, even though both subsystems are actually in the pure MS. That is, the MS does not yet imply the mixture (3); it implies, rather, a mixture of eigenvalues:

$$\text{either } s_1 \text{ and } a_1\text{, or } s_2 \text{ and } a_2. \tag{8}$$

This is not the desired final state of affairs (3), since all we can say about the state at this point of the argument is that it is the pure entangled MS. Getting from the observed eigen*values* (8) to the eigen*states* (3) must involve irreversibility, for (3) is trully a mixed *state* (which, unlike a pure state, has an entropy greater than zero), rather than a mixture of observed eigen*values.*

The distinction between (3) and (8) entails a small change in one standard quantum principle. The "eigenvalue-eigenstate link" (ee link) says that a system exhibits a definite eigenvalue if and only if it is in the corresponding eigenstate {28}. The preceding analysis shows we must drop the "only if" part, because the measurement yields definite eigenvalues even though *S* and *A* are not in eigenstates but rather in the MS. This is a trivial revision that neither alters nor contradicts other quantum principles or experiments {8}.

This resolution of the problem of definite outcomes has much in common with the "modal interpretations" of quantum physics {29}{30}{31}. In fact,

> "the central idea of the modal interpretation is to interpret the mathematical state that quantum mechanics associates with a physical system in terms of *properties* possessed by that system {29},"

and we've seen that such an interpretation in terms of properties (values, eigenvalues) rather than states is crucial in understanding the mixtures (5). Modal interpretations are motivated by a property of Hilbert space called the Schmidt bi-orthogonal decomposition theorem. This says that, for any vector $|\psi>$ in a tensor-product Hilbert space formed from two N-dimensional subspaces S and A, there exist orthogonal bases $\{|s_k>\}$ and $\{|a_k>\}$ for S and A such that $|\psi>$ takes the form

$$|\psi> = \Sigma\, c_k\, |s_k>\, |a_k> \tag{9}$$

The two basis sets are unique if the $|c_k|$ are all different. Quoting Dieks:

> I now propose the following physical interpretation of this formal state. The mathematical state description (9) corresponds to a physical situation in which the partial system associated with [*S*], taken by itself, possesses one of the values of the observable associated with the set $\{|s_k>\}$. The probability that the *k*th possible value is actually present is given by $|c_k|^2$ {31}.

But the MS (2) is a special case (for $N=2$) of (9). Thus the modal approach *postulates*, as a new quantum interpretation, the conclusion *derived* above from standard quantum principles. But this paper's resolution of the problem of definite outcomes follows from the standard principles. It is not a new interpretation, nor does it propose significant new postulates (although it alters the ee link). The modal view is often called a "no collapse" interpretation because it rejects von Neumann's collapse postulate {1}. The present paper also rejects the notion that collapse needs a special postulate, but it doesn't reject collapse. Its point is to show that collapse, upon measurement, from an indefinite superposition to a mixture of definite outcomes *follows* from the other quantum principles.

The mixtures (5) are often rejected on the ground that they are "improper" {8}{29}. "Proper mixtures" describe indeterminate outcomes where the uncertainty arises only from human ignorance of the actual state, and are not unique to quantum physics. The same notion arises in thermodynamics where the precise classical state is represented by probabilities because humans cannot feasibly observe or calculate the precise N-body state. In fact, we apply ignorance-based probabilities to a classical situation whenever we describe a flipped coin as having a 50-50 chance of coming up heads or tails.

The mixtures (5) are not "proper" because they don't arise from ignorance of the state. The state is the MS. It's an experimentally-detectable error to suppose that *A* is actually in one of the eigenstates $|a_i>$. Nevertheless, these mixtures tell us correctly what is observed locally at either subsystem: At *A*, for example, we observe either a_1 or a_2--e.g. either an alive cat or a dead cat. Although (5) can be easily misinterpreted, there is nothing "improper" about (5). These mixtures are just what we expect when we reduce a known entangled state of two quanta to obtain predictions for one quantum, namely mixtures expressing the "either/or" of quantum uncertainty. We don't expect them to express *ignorance*, because quantum uncertainty doesn't arise from ignorance. So-called "improper mixtures" represent the randomness inherent in quantum physics even though there is no ignorance, i.e. even though the quantum description is "complete."

A better term for (5) is "local states." The central point of this paper is that the definite values predicted by these local states are correct, even though the subsystems are not in these states. Far from being improper, local states are the formal expression of the quantum randomness that persists even when we know the precise state of a system.

A second objection to (5) is "basis ambiguity" {8}. The argument is that the mixtures (5) are mathematically ambiguous in the degenerate case $|c_1|^2 = |c_2|^2 = 1/2$ because (5) then reduces to $\rho_S = I_S/2$ and $\rho_A = I_A/2$ where I_S and I_A are subspace identity operators. Thus, (5) would take the same form in any basis. For example, (5a) could be written

$$\rho_S = (|r_1> <r_1| + |r_2> <r_2|)/2, \tag{10}$$

where $|r_1> = (|s_1> + |s_2>)/\surd 2$ and $|r_2> = (|s_1> - |s_2>)/\surd 2$. This seems to imply that, immediately following the measurement, *S* exhibits the properties associated with either $|r_1>$ or $|r_2>$ rather than $|s_1>$ or $|s_2>$. The present analysis avoids this criticism because *S* and *A* are not claimed to be in the states (5), and (1) and (2) define the $|s_i>$ and $|a_i>$ unambiguously.

4. Conclusions

When a detector measures a superposed quantum, the unitary Schrodinger evolution leads to a pure coherent entangled measurement state (MS) of the composite detector-plus-quantum system. Both quantum theory and experiment show the MS entails observed outcomes that are mixtures of definite eigenvalues of the quantum and its detector, not indefinite superpositions. A second argument also implies definite outcomes: The observed local states *must* contain no hint of the coherent global MS lest relativity's prohibition on instant signaling be violated; this implies the observed outcomes must be phase-independent mixtures, not superpositions, of eigenvalues.

This resolution of the problem of definite outcomes requires that the standard "ee link" be revised. The ee link states that a quantum system exhibits, upon measurement, a definite eigenvalue of some observable property if and only if it is in an eigenstate of that property. The words "and only if" must be dropped--a trivial revision that neither alters nor contradicts other quantum principles or experiments. This revision implies that we cannot conclude, from observation of a specific eigenvalue, that the observed quantum is in the corresponding eigenstate. The quantum might, for example, be entangled with other quanta.

Nonlocal experiments with entangled photons demonstrate the precise nature of the coherent global MS: It is a superposition only of *correlations* between the detector and the detected quantum, not a superposition of *states* of the detector, or states of the quantum, or composite states of both subsystems. It should be read as "the quantum's first eigenvalue is coherently correlated with the detector's first eigenvalue *and* the quantum's second eigenvalue is coherently correlated with the detector's second eigenvalue." This coherent superposition is non-paradoxical. It's by this shifting of coherence from the superposition state of the quantum to coherent correlations between the quantum and its detector that the composite system's global evolution can remain unitary while both local states collapse, upon entanglement, into incoherent mixtures. This is the way nature achieves definite but indeterminate outcomes while preserving unitary evolution.

Thus quantum physics predicts Schrodinger's cat is either dead or alive, not both. This analysis resolves the problem of outcomes, a key part of the measurement problem, but it does not entirely resolve the measurement problem because the MS remains a coherent pure state superposition of correlations as shown in Figure 1. Hopefully, further analysis will be able to show that this superposition collapses into one or the other of its two branches upon interaction with a many-body macroscopic environment.

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