Thought experiment on the reality of matter wave

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Abstract: A thought experiment regarding the light-wave diffraction by a matter-wave lattice is proposed. Matter-wave diffraction by an optical-lattice is already a well-established experimental technology, therefore, its symmetrical phenomenon, i.e., light-wave diffraction by a matter-wave lattice, does not seem to be unlikely. If the diffracted light could be observed, it would suggest the presence of the matter-wave itself in this three-dimensional space, whereas the orthodox interpretation of quantum mechanics that describes the wave-function as merely a mathematical tool representing the probability of a particle. Therefor this should be a good test to verify the reality of the wave-function.

Keywords: quantum experiment; matter wave; reality of wave function

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1. Introduction

The peculiar phenomenon of wave performance such as the interference or diffraction of matter-waves has been proven by a large number of experiments with electrons, neutron, molecules and even such massive molecules as fullerene [1-10]. These results showed that the matter-wave is equivalent to an electro-magnetic wave in terms of optical phenomenon realized in three-dimensional space through its de Broglie wavelength. Recently, Bragg diffraction of matter waves by an optical lattice has been demonstrated [11-14] and has already been adapted to for use as an important device in atom-optics technology [15, 16]. Bragg diffraction is the typical behavior of a wave, when it encounters such periodical structures. The wave is selectively diffracted to some specified direction allowed by Ewald’s vector relation, between the lattice pitch and the wavelength. This is because the matter-wave performs as a wave that senses the real structure of lattice in this three-dimensional space, which means that the matter-wave-front covers optical lattice plane. It is entirely different from the classical view that the single particle is directly deflected by the optical lattice potentials.

Here, if we consider the symmetry of duality between matter waves and light waves, Bragg diffraction of a light-wave by the standing matter wave cannot be regarded as an unlikely phenomenon. This matter-wave is considered to correspond to the wave function ψ, which is the solution of the Schrödinger equation defined in the configuration space, although both spaces are mathematically equivalent for the one particle Schrödinger equation case. This is not true for the path-integral formalism, but in this paper, the discussion is confined to the Schrödinger equation formula for simplicity. In any case, the wave-front or wave-function of ψ does not represent any real physical quantity that is defined as the *operator* in the standard quantum mechanics.

When we talk about "physical reality", the definition is still vague in the quantum world and this is the subject of intense ongoing debate [17-19], and may not be settled for a long time. Recently, discussions initiated by Pusey *et al*. [20] have aroused much controversy involving the philosopher of science epistemology [21-24]. However, we will not enter into these debates here, we only consider all things from the view point of actual experiment. In that case, we have to think about why we does not say the wave is real, although we have many experimental evidence in past that the matter wave shows the wave nature. And also it is the true that we have not any evidence that matter wave is *not real wave* from past experiment. One practical reason is because most of them is only to observe the resultant behavior of the amplitude ψ as a wave nature.

Here we propose a new type of experiment that is different from the conventional diffraction or interference experiments. That is to verify the light diffraction by a matter wave lattice. This means to use the modulus $\psi\psi^*$ instead of ψ. In the case of a light wave, EE^* is measurable as a real physical quantity, i.e., the intensity of the light. Therefore, the optical lattice means the localized real structure to be sensed by matter wave in this three-dimensional space. If the matter wave is *a real wave* like an electromagnetic wave, its standing wave behaves as the matter wave lattice that could cause the diffraction of light. In other words, it suggests that $\psi\psi^*$ plays some real physical role to influence to the real light wave. This is likely true, if we think about symmetry between light waves and matter waves.

If ψ represents nothing more than a carrier of some mathematical information as predicted by quantum mechanics reveals only some mathematical information carrier as the quantum mechanics, then this experiment would produce only Rayleigh scattering from particles, and the symmetry of duality between light waves and matter waves will be broken. These situations are illustrated schematically in Figure 1.

Moreover, this wave-function has nothing to do with the number of atoms, it reveals a momentum-fixed state, not a position-fixed state. Therefore, the number of atoms becomes entirely uncertain in the interference region because of the uncertainty relation from quantum mechanics. Hence the diffraction could be expected to even in the case of a dilute matter beam of only one particle passing the average one atom in the interference region.

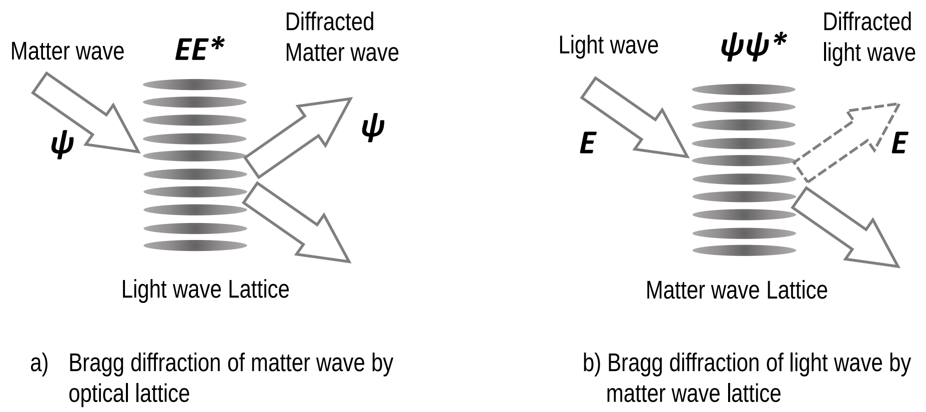

Figure 1. Schematic of matter-wave diffraction by an optical lattice (a); and light-wave diffraction by a matter wave lattice (b). This shows the symmetrical relation of duality between matter waves and light waves. However, case B is questionable from the quantum mechanical view. Proposed experiment will provide some information about the reality of the matter-wave-function.

The standing wave of a matter-wave is expressed as the localized fringe of $\psi\psi^*$ in this three-dimensional space whether it is mathematical or real. Incidentally, from a practical point of view it is difficult to form a standing wave cavity analogous to that for light waves because of the lack of effective mirrors for matter waves. However, the standing matter wave is expected to be formed at the interference region of a Mach-Zehnder interferometer when the beam splitter at the beam crossing position is removed as described in section 3 and we could observe the resultant fringe of the standing matter-wave. A large number of experiments in the past proved this is correct, because we can see the fringes distributed on the screen or some other detector in the form of the accumulated particles. From the analogy with an optical fringe, which is expected to have a three-dimensional structure in this three-dimensional space, it could then be said that the standing matter-wave exists in this three dimensional-space.

If the particles are fluorescent atoms, the localized three-dimensional fringe image of the matter wave could be observed using an optical microscope. The results of Mach-Zehnder interferometry with a Bose-Einstein condensate (BEC) is very impressive and famous demonstration of this standing wave consisting of a light-emitting atom cloud [25]. However, this phenomenon does not mean that the wave function is directly observed, rather only the resultant atom density distribution, which is proportional to the modulus of the wave function $\psi\psi^*$, is observed. Incidentally, there is one very puzzling phenomenon within this BEC interference; the fact that no atoms exist at the node of the fringe, whereas the atom flow passes through this node region. This is a very strange phenomenon from the classical point of view.

2. Theoretical background

The theory of diffraction for matter waves by an optical-lattice has been thoroughly studied in the past and is well summarized by Adams and others [13-16]. First, we review the diffraction of matter waves by an optical-lattice. The basic equation is the orthodox Schrödinger equation, and it is sufficient to consider only the time-independent form. The time-independent Schrödinger equation in position representation is given by

$$\nabla^2\psi(\boldsymbol{r}) + \frac{2m}{\hbar^2}[E - V(\boldsymbol{r})]\psi(\boldsymbol{r}) = 0, \tag{1}$$

where $V(\mathbf{r})$ is the optical potential, which has periodic structure in accordance with the optical lattice. This Schrödinger equation has the same mathematical as the Helmholtz equation of a classical electro-magnetic wave, therefore, the solution of Eq. (1) will be equivalent to the solution of an electro-magnetic wave under some boundary condition. If we consider a $\boldsymbol{k}$-vector, analogues to the wave-vector of classical optics by

$$\boldsymbol{k}(\boldsymbol{r}) = \sqrt{2m[E - V(\boldsymbol{r})]/\hbar^2}\boldsymbol{e}, \tag{2}$$

where $\boldsymbol{e}$ is the propagation vector of wave, then. When V=0, the Schrödinger equation reduces to the standard Helmholtz equation

$$(\nabla^2 + k^2)\psi(\boldsymbol{r}) = 0. \tag{3}$$

One solution of this equation has the following form,

$$\psi(\boldsymbol{r}, t) = \exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}). \tag{4}$$

In the case of an electro-magnetic wave, this is a plane wave and it actually has a wave front with an infinite extent. The analogy between the matter wave and the classical electro-magnetic wave implies that the diffraction of matter wave can be described by the Fresnel-Kirchhoff formula, which is formulated from the Huygens-Fresnel principle in which the wave-front configuration is essential [26]. Numerous diffraction experiments of matter waves proves that the diffraction pattern through some aperture is described by this Fresnel-Kirchhoff formula, but of course, they all involve the case of small wavelength compared to the aperture size. The Fresnel-Kirchhoff formula of a matter wave is described as follows, and shown in Figure 2,

$$\psi_p = -\frac{i\psi}{2\lambda_d}\int_S \frac{\exp[i\boldsymbol{k}\cdot(\boldsymbol{r}+\boldsymbol{r}\prime)]}{rr\prime}[\cos(\boldsymbol{n},\boldsymbol{r}) - \cos(\boldsymbol{n},\boldsymbol{r}')]dS, \tag{5}$$

where λ_d is the de Broglie wavelength (i.e., $\lambda_d = h/mv$), *h* is Planck's constant, *m* is the mass of the particle, and *v* is the velocity of the particle.

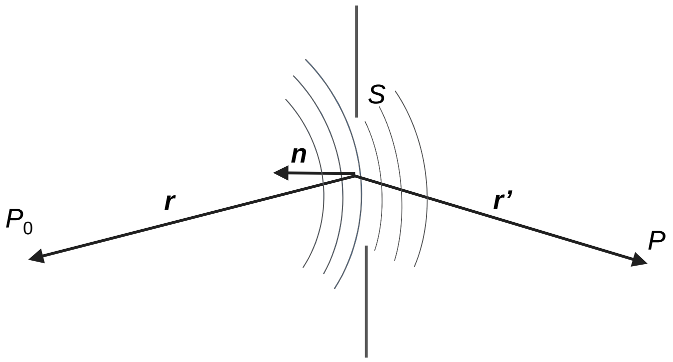

Figure 2. Schematic illustration of Fresnel-Kirchhoff diffraction. Waves originating from a monochromatic point source P_0 are diffracted by an aperture A. The amplitude at the point P is calculated by the superposition by the sum over all the secondary wavelets on the aperture.

It is a well-known fact that the Fresnel-Kirchhoff integral is mathematically equivalent to Feynman's path-integral of quantum mechanics.

In Eq. (1) the element of optical potential *V*(r) is described as the following interaction Hamiltonian,

$$H_{int} = -\boldsymbol{d}\cdot\boldsymbol{A} \ , \tag{6}$$

where ***d*** is the dipole moment of the particle and ***A*** is the vector potential of the light wave. This is a representation of a classical dipole and electric field interaction that provides a potential energy term in the Schrödinger equation when the particle is modeled as a dipole in the case of a simple atom. As a result, it works as the phase grating.

This interaction Hamiltonian has exactly the same form for the following two cases: One case is matter-wave diffraction by an optical lattice; the other case is the diffraction of a light wave by a matter wave. This is true, because the fundamental force between the electric field and the atom is the same. The only difference between two cases is in the macroscopic boundary conditions; in other words, the basic Schrödinger equation describes both states. From this consideration, it is highly probable that light-wave diffraction will be induced by the standing matter wave; this means that the symmetrical duality for light waves and matter waves will hold. The standing matter wave is expressed formally in the same way as the standing light wave only by replacing ***E*** with ψ as follows (see Appendix A),

$$P(r) = \psi(\boldsymbol{r},t)\psi^{\dagger}(\boldsymbol{r},t) = 4{\psi_0}^2\cos^2(px), \tag{7}$$

where *P(r)* is the probability density of the particle at ***r***, $p = k_0\sin\theta$ and θ is the crossing angle of the two beams.

Since the intrinsic relation of Bragg diffraction is revealed by the Ewald vector relation between the grating-pitch vector and the wave-number vector as shown in Figure 3, the diffraction of the light wave will be expected to occur to the direction allowed by this relation, regardless of whether the diffraction is caused by a mechanical grating or *a matter wave lattice*. Thus, this new experiment is entirely analogues to the case of matter-wave diffraction by a standing light wave.

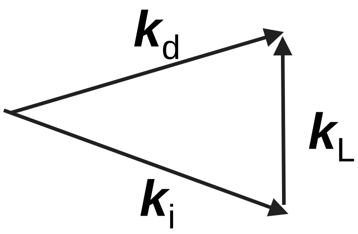

Figure 3. Schematic of the Ewald vector relation. $\boldsymbol{k}_i$ is the wave vector of the incident wave, $\boldsymbol{k}_d$ is that of the Bragg diffracted wave and $\boldsymbol{k}_L$ is the grating vector of the periodic structure.

3. Experimental configuration

Here, we present only a brief sketch of the basic configuration of the experiment, the detailed description of the experimental setup will be skipped in order to focus on the conceptual discussion. The basic idea is to use the interference fringes formed in a Mach-Zehnder interferometer of the matter wave. A mirror or a beam-splitter for the matter wave is realized using an optical-lattice device [9, 15, 16]. In the case of light optics, Mach-Zehnder interferometer is a typical configuration of a two-path interferometer, in which the amplitude of the incident light is divided into two beams that recombine where we can observe the localized standing wave at the interference region. If the light source has an ideal coherence, the localized interference fringes of the standing wave exist over the entire three-dimensional interference region. If we insert a screen into the interference region, we can observe some fringes on the screen, the fringe pattern depends on the relationships between the path differences of the two beams, the angle of two beams, and the set of the angle of the screen; therefore, we can recognize the reality of localized interference, which proves the reality of light wave. Usually the localized fringe that occurs when two travelling waves with same wave number, travelling in opposite directions, encounter each other is called a standing wave; but in general, it is called a localized standing wave even with an arbitrary angle, as described in Appendix A.

In the case of a matter wave, the occurrence of wave-like phenomena should be considered just as in the case of a light wave in this three-dimensional space. Therefore, the standing wave of a matter wave should be formed at the region of interference of a Mach-Zehnder interferometer, and we can observe the localized fringe of the matter wave. A large number of experiments in the past have proved that this is correct, because we can see the fringes distributed on the screen or some other detector in the form of accumulated particles. From the analogy with optical fringes, it can be said that the standing wave of the matter wave is likely to exist in this three-dimensional space.

Figure 4(a) illustrates entire schematic configuration of the Mach-Zehnder interferometer for this experiment, and Figure 4(b) shows the parts of the observing optics. In Figure 4(b), only the Bragg diffraction case is illustrated, but Raman-Nath diffraction will also occur if the thickness of the matter-wave lattice is thin.

In order to realize this experiment, several important points should be taken into account as follows. First we should chose the wavelength of observing laser as short as possible, because the matter wave-length is very short compared with light wavelength. From the practical stand point, the 193nm light from an ArF laser will be shortest wavelength in good coherence and high power, thus, the standing wave pitch of the matter wave must be over this laser wavelength to fulfill the Bragg diffraction condition under practical apparatus configuration (see Appendix A).

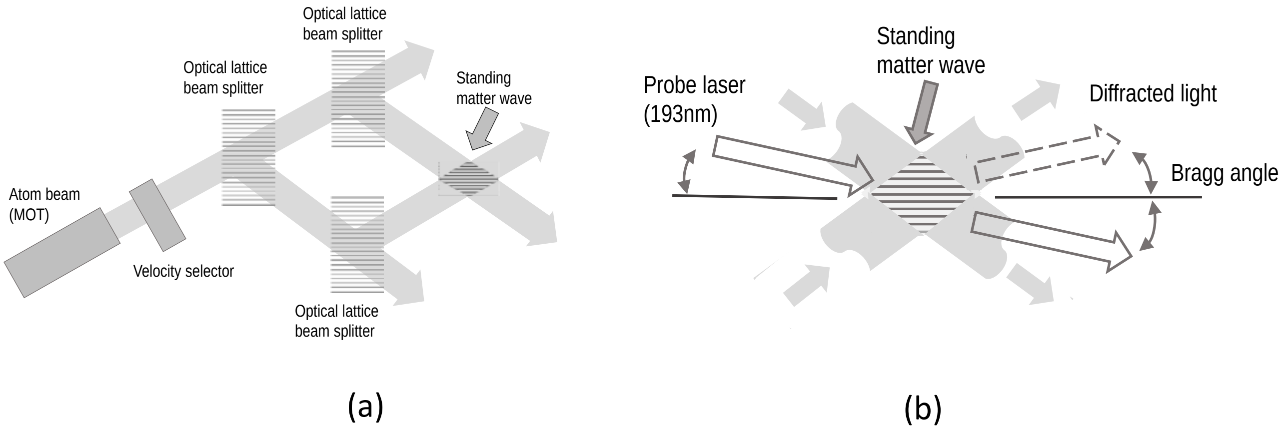

Figure 4. Schematic drawing of the experimental configuration. (a) Mach-Zehnder interferometer for this experiment using very cold atoms from a magnet-optical trap (MOT). The second beam splitter of the standard Mach-Zehender interferometer configuration is removed to form the localized fringe. (b) Observing optics portion with Bragg diffraction generated by the standing wave of interference.

The de Broglie wavelength λ_d of the atomic beam from the oven or discharge vessel used in past experiments is the order of picometers for Ne, Ar, Na, Rb, and so on. This is three or more orders of magnitude smaller than the required λ_d that is needed to form an appropriate pitch to diffract the laser wave. Hence, the very cold atom beam is indispensable; it could be supplied from a magnetic optical trap (MOT) or BEC, whose velocity is in the range from less than 1 m/s to several cm/s [27, 28]. Sodium atom from MOT will provide a slow atom beam with a velocity less than 25 cm/s; therefore, λ_d over several tens of nm could be obtained. The velocity distribution of these atom beams should be arranged to be narrow enough to show good contrast fringes. The velocity broadening is decreased to approximately 5% of λ_d with the full width at half maximum using the mechanical velocity selector, and the beam splitter of an optical lattice itself is also a good selector of velocity [29, 30], furthermore, the new multiplexed velocity selection technique [31] will be feasible to obtain a sharp spectrum; thus, this combination is expected to satisfy the coherence requirement to form a good standing wave. One example of an angular management of experiment set up is as follows: When the crossing angle of two beams of the Mach-Zehnder interferometer is 40 degree, the pitch of standing wave will be approximately 200nm. Therefore, the Bragg condition for 193 nm will be satisfied, if the ArF laser beam arrives with a 30-degree incidence (see Appendix B).

A possible alternative to the Mach-Zehnder interferometer for forming the standing wave would be to use an intensity-modulated beam [32-34] of a very low-velocity matter-wave. If we can use a matter-wave beam with a velocity of 25 cm/s, then a high frequency modulation of 1.25 MHz would provide a 2-μm-pitch fringe of sinusoidal intensity distribution along the traveling direction of the wave (see Appendix B). Thus we can obtain a good matter wave lattice in which the number of fringes is very large (i.e., 500 fringes for 1 mm beam length). Actually, this is a form of moving grating, but the direction of the diffracted light is not changed, because the direction of diffraction is defined by the Fourier transformation in space.

A one more alternative, a BEC interference fringe [25] can be considered as one kind of standing matter wave. If this BEC fringe is usable, it could provide a bright diffraction light because of its dense cluster of atoms. However, the issue is how to distinguish the matter-wave lattice from the classical dielectric crystal-lattice. This is discussed later in section 4.

4. Discussion

4-1 From the point of view of the orthodox quantum mechanics interpretation

If the wave function is only a mathematical inventory for the probability density of particle existence in three-dimensional space, which is the conventional orthodox interpretation, then the standing wave of the matter wave is not really localized in this space like an optical lattice, but only reveals the distribution density of atoms. The interaction Hamiltonian between the light and the dipole is described by Eq. (6) from classical electro-magnetic theory. That means that the dipole corresponds to a localized particle, not to a non-localized substantial entity distributed in three-dimensional space. The charge-distributed hypothesis of an electron, which was traditionally proposed by Schrödinger, has been re-examined by Gao [32] who concluded that the electron cloud is not distributed in three-dimensional space, but is the resultant property of the ergodic motion. The dipole consists of a pair of two opposite charges, this may be more problematic, so the reality of a dipole cloud is a more unlikely concept. In other words, based on the above discussion from the orthodox interpretation of quantum mechanics, the standing matter-wave will not cause the light diffraction, instead, it will be interpreted to represent only the fringe of the distribution of atoms as shown in the interference experiment of BEC. From this discussion it might be said that the *real existence* of the wave function will be unlikely, and this means that the symmetry of the duality between matter waves and light waves will be broken

4-2 From the point of view of the reality of wave-front

Notwithstanding the above discussion in section 4-1, the existence of a wave front in three-dimensional space should be thought to be indispensable, if we look at the results of matter-wave experiment with diffraction or interference. Beginning with electrons and various other matter waves (e.g., neutron, He, Ne, Ar, Rb, Na and massive molecules such as fullerene), every particle shows the same diffraction or interference behavior as a classical light wave. This means that the Huygens principle from classical optics still dominates the matter-wave behavior in real three-dimensional space. The Huygens principle is formulated as the Fresnel-Kirchhoff diffraction formula as shown in section 2. From this formula, we can clearly understand the fact that the new wave front far from some aperture is configured by the entire wave front that covers the aperture. This implies that the information of every point on the new wave front is integrated with all the information of the wave front on the aperture structure through the Fresnel-Kirchhoff formula. Therefore, a matter wave is also expected to have some wave front or some sort of reality corresponding to a wave front. From the viewpoint of the path-integral formula of quantum mechanics, the phase of a particle path is equivalent to the wave front, but this path is neither a real trajectory of a particle nor a real wavelet of a matter wave, therefore it does not really exist in this three-dimensional space.

If something provides a phase shift to one of the matter-wave beams, it brings about a shift of the interference fringes by overlapping the phase-shifted wave to the original wave. A beautiful photo of the experiment of the Aharonov-Bohm effect by Tonomura [33] showed this fact, moreover, matter-wave holography [34] indicated the existence of the wave front. These matter-wave interference experiments shows vividly the existence of a “wave-front” in this three-dimensional space.

The presence of the wave front in three-dimensional space would indicate the presence of the original wave itself. The fact that “light is a wave” is proven by comparing many experimental results of interference or diffraction with the calculated results of Fresnel-Kirchhoff, which is derived from the fact that light is the electromagnetic wave solution of Maxwell’s equation.

In the case of matter waves, whereas the time-independent Schrödinger equation of one particle has the same form as the Helmholtz equation and its solution also formally satisfies the Fresnel-Kirchhoff formula like an electromagnetic wave in three-dimensional space, the Schrödinger equation is still defined in another space and the wave front of the wave-function is not defined in three-dimensional space; its modulus merely provides the probability of the particles. If that is the case, there remains questions such as "What does sense the structure of the aperture like as a real wave-front?" or "Is there any real quantity equivalent to a wave front corresponding to the wave-function?" These points remain ambiguous.

Most past experiments in atom optics were performed to show the direct wave nature; therefore, they have been explained by a simple assumption of the wave nature of particle without asking about the presence of a wave front. However, in this proposed experiment, if we observe the diffracted light, the resultant intensity distribution of the matter wave lattice should be regarded as the some real physical quantity such as the intensity of light, since the light wave is diffracted only by the real structure. Therefore, the matter wave lattice should be regarded as some sort of presence of real. Thus, the proposed experiment seems to be a good test of whether the wave function of matter waves really exists or not.

4-3 Matter-wave lattice with intensity modulation

Intensity modulation is mathematically equivalent to the standing wave as shown in Appendix B; it merely takes the form of a travelling standing wave because of the small difference between the two wavelengths that are superposed. Therefore, the foregoing discussion concerning the reality of the matter wave holds in the same manner. Hence, as a matter of fact, the diffraction of light will occur if a beam-let really exists in three-dimensional space. According to a prior the experiment [32-34], it has been observed that the separation of the de Broglie wavelength occurred as a result of the intensity modulation of a matter wave. Just as in the previous discussion, this fact means that the beam-let senses the real modulation. That is to say, in the space domain, the wave-front senses the real structure of the entire aperture, and in the time domain the beam-let senses real modulation for the entire time frame, i.e., from the infinite past to the infinite future. This fact is a consequence of the Fourier transformation in time.

If ψ is assumed to be a mere probability wave, the same question arises, i.e., what is intended to detect the modulation of the beam-let in the three-dimensional space-time? (The intensity modulation is referred as "time lattice" in corresponding to "space lattice".) Therefore, the above discussion leads to the same perspective as the case of a wave front in space. These fact suggests the existence of ψ or something that senses the actual three-dimensional space-time structure in corresponding to ψ.

4-4 Matter-wave lattice with BEC

In the case of BEC, we can expect to observe the diffraction of the light wave from the matter-wave fringe as shown in section 3. Since the BEC cloud is modeled as a coherent wave cloud, in which the wave functions of a large number of atoms are superposed coherently, then the interference fringe could provide a well-defined lattice like a light lattice. If so, we can expect strong diffracted light due to its large number of atoms.

On the other hand, if we consider the Clausius-Mossotti formula that describes the relation between the optical dielectric ratio and atom density from classical electro-magnetism [35]. In fact, the fringe of BEC interference shows merely the distribution of atom cluster, if the wave-function of BEC indicates only the probability. However, in this case we also might say that diffraction will occur from the classical consideration of electro-magnetism using the Clausius-Mossotti relation about the atomic gas of BEC. Therefore, it might be difficult to decide whether the observed light diffraction is caused by the real presence of a matter wave lattice or merely by the classical dielectric gas of atoms.

5. Summary

We have proposed a thought experiment concerning the diffraction of a light wave by a matter-wave lattice, in accordance with the fact that the matter wave is diffracted by the light lattice, and taking into account the view point of symmetry of the duality between light waves and matter waves. We also found that this experiment might provide some information about testing the reality of the wave function in this three-dimensional space.

If we illuminate a matter-wave lattice by a light wave, there will be two possibilities: either we can obtain light diffraction or we cannot. In other words, the proposed experiment will provide information whether we can sense the modulus of wave function by a light wave or not, thus the results will show the reality of the wave function of a matter wave.

From the standard interpretation of quantum mechanics, it is true that the standing wave of a matter wave exists as the probability distribution of a particle, and the presence of dense and coarse distributions of atoms is real, but the fringes is not the standing wave of the wave function itself, so it is not likely to be possible to observe the diffracted light from the matter-wave lattice.

On the other hand, the wave-front of a matter wave in three-dimensional space is very likely to exist like as an electromagnetic wave from a large number of diffraction or interference experiments on matter waves, that is to say, there is some possibility of some sort of physical reality about the wave-function of a matter wave because the diffraction of a matter wave should obey the Fresnel-Kirchhoff formula just as light optics do. Moreover, the existence of a beam-let similar to an electro-magnetic wave is likely to be possible from the intensity modulation experiment of matter waves. From these experimental configurations, it might be very reasonable to image the physical presence of the wave function or some kind of real presence corresponding to the wave function. These considerations suggest that the localized standing matter-wave exists as a real physical presence. This assumption will be confirmed, if the light diffraction from the matter-wave lattice is observed.

Therefore we believe that the proposed simple experiment should be a very good test to examine whether the wave function exists as something associated with some sort of physical presence in three-dimensional space-time or merely as mathematical tool.

Appendix A. Standing waves of light waves and matter waves

The standing wave in the case of an electro-magnetic wave is formulated as follows [36]:
Two plane waves with the same frequency ω that travelling in the z direction are written as

$$u_1(\boldsymbol{r}_1, t) = u_0 \exp i(\boldsymbol{k}_1 \cdot \boldsymbol{r}_1 - \omega t), \tag{A1}$$

$$u_2(\boldsymbol{r}_2, t) = u_0 \exp i(\boldsymbol{k}_2 \cdot \boldsymbol{r}_2 - \omega t), \tag{A2}$$

where u_1 and u_2 are the amplitude of each wave, here the electromagnetic field is expressed as a scalar field for simplicity, and $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ are the wave vector of each wave. These are written as follows in Cartesian coordinate as shown in Figure A1,

$$u_1(\boldsymbol{r}_1, t) = u_0 \exp i(px + qz - \omega t), \tag{A3}$$

$$u_1(\boldsymbol{r}_2, t) = u_0 \exp i(-px + qz - \omega t), \tag{A4}$$

where p ,q are $p = k_0 \sin\theta$, $q = k_0 \cos\theta$, and 2θ is the intersection angle of the two beams.
When the two beams are superposed,

$$u(\boldsymbol{r},t) = u_1(\boldsymbol{r}_1,t) + u_2(\boldsymbol{r}_2,t) = 2u_0\cos(px)\exp i(qz - \omega t), \quad \text{(A5)}$$

and the intensity can be written as,

$$I(r) = u(\boldsymbol{r},t)u^*(\boldsymbol{r},t) = {u_0}^2\cos^2(px). \quad \text{(A6)}$$

Thus, the pitch of the standing wave d is $\lambda/2cos\theta$.

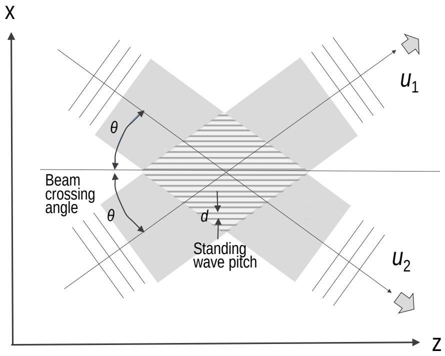

Figure A1. Interference of two waves forms the standing wave of intensity as a lattice. The pitch of the lattice is determined by the beam-crossing angle. In the case of two counter-propagating waves, it forms the usual standing wave.

In the case of two beam interference of matter waves, E is replaced by ψ,

$$\psi_1(\boldsymbol{r}_1,t) = \psi_0 \exp i(\boldsymbol{k}_1 \cdot \boldsymbol{r}_1 - \omega t), \quad \text{(A7)}$$

$$\psi_2(\boldsymbol{r}_2,t) = \psi_0 \exp i(\boldsymbol{k}_2 \cdot \boldsymbol{r}_2 - \omega t); \quad \text{(A8)}$$

these are written as follows in Cartesian coordinates as shown in Figure A1,

$$\psi_1(\boldsymbol{r}_1,t) = \psi_0 \exp i(px + qz - \omega t), \quad \text{(A9)}$$

$$\psi_2(\boldsymbol{r}_2,t) = \psi_0 \exp i(-px + qz - \omega t). \quad \text{(A10)}$$

When the two beams are superposed,

$$\psi(\boldsymbol{r},t) = \psi_1(\boldsymbol{r}_1,t) + \psi_2(\boldsymbol{r}_2,t) = 2\psi_0\cos(px)\exp i(qz - \omega t), \quad \text{(A11)}$$

and

$$P(r) = \psi(\boldsymbol{r},t)\psi^\dagger(\boldsymbol{r},t) = 4{\psi_0}^2\cos^2(px), \quad \text{(A12)}$$

where P means the modulus of ψ, which is the probability density of particle existence.

Appendix B. Intensity modulation of light waves and matter waves

The superposition of two beams of light waves that have slightly different frequencies is equivalent to the intensity modulation of a single-wavelength light beam as described below, the situation is also the same with matter waves, because these are connected by a simple Fourier transformation relation.

If u_1 and u_2 are light wave field with frequencies of ω_1 and ω_2 that differ by $2\Delta\omega$, the superposition of the two beams provides an intensity beat with a frequency of $2\Delta\omega$; this is entirely equivalent to the expression of the intensity modulation for a light beam as follows and as shown in Figure B1.

$$u_1(\boldsymbol{r},t) = u_0 \exp i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega_1 t), \tag{B1}$$

$$u_2(\boldsymbol{r},t) = u_0 \exp i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega_2 t), \tag{B2}$$

where $\omega_1 = \omega_0 + \Delta\omega$ and $\omega_2 = \omega_0 - \Delta\omega$.
When two beams are superposed,

$$u(\boldsymbol{r},t) = u_1(\boldsymbol{r},t) + u_2(\boldsymbol{r},t), \tag{B3}$$

and its intensity is written as,

$$I(r) = u(\boldsymbol{r},t)u^*(\boldsymbol{r},t) = 2{u_0}^2(1+\cos[(\omega_1-\omega_2)t]) = 2{u_0}^2(1+\cos 2\Delta\omega t). \tag{B4}$$

Whereas the techniques to generate these states, i.e., superposition and modulation, are different, they are exactly the same state. If we measure the intensity-modulated light beam with a spectrometer, we will obtain two spectral lines differing by $2\Delta\omega$. In addition, if we measure the output of an optical detector for the two superposed beams with different $2\Delta\omega$, we obtain the beat signal of $2\Delta\omega$. In the case of matter waves, the discussion is exactly the same as follows,

$$\psi_1(\boldsymbol{r},t) = \psi_0 \exp i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega_1 t), \tag{B5}$$

$$\psi_2(\boldsymbol{r},t) = \psi_0 \exp i(\boldsymbol{k}\cdot\boldsymbol{r} - \omega_2 t), \tag{B6}$$

$$P(r) = \psi(\boldsymbol{r},t)\psi^\dagger(\boldsymbol{r},t) = 2{\psi_0}^2(1+\cos[(\omega_1-\omega_2)t]) = 2{\psi_0}^2(1+\cos 2\Delta\omega t), \tag{B7}$$

where P is the probability density of particle.

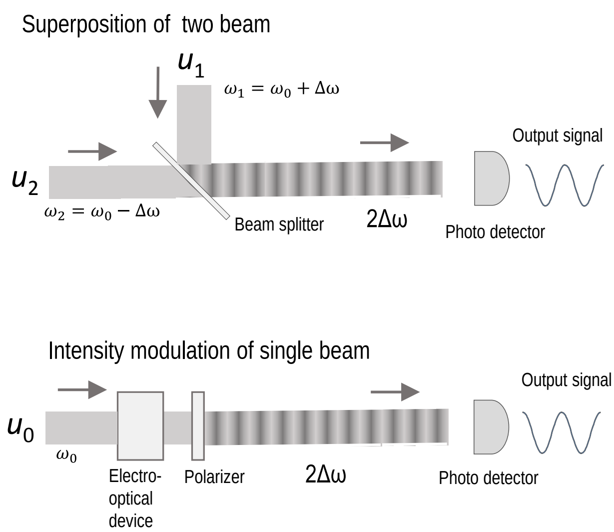

Figure B1. (a) Intensity beat produced by superposing two coherent laser beams with two different frequencies where $\omega_1 = \omega_0 + \Delta\omega$ and $\omega_2 = \omega_0 - \Delta\omega$. (b) Intensity modulation by $2\Delta\omega$ of single coherent laser beam. The resultant states of the light beam is the same.

The essential point is that this is simply the same Fourier expansion for both cases: namely the classical light-wave case and the quantum matter-wave case. However, the matter-wave case raises a question about a phenomenon that is puzzling from the classical point of view, because any particle that passes through the modulator does not sense any energy or momentum from the modulator or from any other particle, but the whole *continuous* intensity lattice structure of the beam-let produces the energy-shifted particle. i.e., some particle changes to $\omega_0 + \Delta\omega$ and some other particle changes to $\omega_0 - \Delta\omega$ without any exchange of energy with other particles or with the modulator. In any case, this phenomenon is symmetrically equivalent in spatial the domain also; i.e., the grating produces two shifted wave vectors from one wave vector of the beam in the usual diffraction grating. However, these phenomena are true in quantum mechanical world

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