# A PROPOSED RESOLUTION OF THE DELAYED-CHOICE PARADOX

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ABSTRACT. This paper proposes a resolution of the delayed-choice paradox of the Conventional Theory of quantum mechanics, where a particle seems to know what will happen in the future, and changes its present behavior accordingly. A comparison of the Conventional Theory with an Advanced Theory and a Symmetrical Theory of the same delayed-choice experiment suggests that the apparent paradox is caused by an incorrect assumption about the interaction of a wavefunction with a beam-splitter. Once this assumption is corrected, there is no delayed-choice paradox in any of the three theories.

## 1. INTRODUCTION

One of the great challenges of modern physics is to resolve the conceptual paradoxes in the foundations of quantum mechanics [1]. Some of these paradoxes concern time. For example, in 1926 Lewis proposed a delayed-choice thought-experiment which appeared to show a temporal paradox in the Conventional Theory of quantum mechanics [2, 3]. He considered a double-slit interference experiment using a single photon from a distant star. A thousand years after the photon has left the star, but just before it reaches the two slits (A and B) on Earth, we randomly choose to either keep both slits A and B open, or close slit A only, or close slit B only. We repeat this experiment for a large number of single photons, to obtain an ensemble of experimental results. In the sub-ensemble where we chose to keep both slits open, we see an interference pattern, implying each photon took both paths from the star. In the sub-ensemble where we chose to keep only one slit open, we do not see an interference pattern, implying each photon took only one path from the star. Lewis concluded that "...in some manner the atom in the source $S$ can foretell before it emits its quantum of light whether one or both of the slits $A$ and $B$ are going to be open" [2]. How can an atom in a distant star know what we will choose to do on Earth, a thousand years in the future? This is the delayed-choice paradox. Weizsäcker and Wheeler later rediscovered and elaborated on Lewis's thought-experiment [4, 5, 6, 7]. Delayed-choice experiments with photons, neutrons, and atoms have confirmed this paradoxical behavior [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19].

## 2. THE MACH-ZEHNDER INTERFEROMETER (MZI)

We will analyze the delayed-choice experiment using the MZI shown in Fig. 1. In a classical physics analysis, the sources $S1$ and $S2$ emit light waves, the beam-splitters $B1$ and $B2$ split
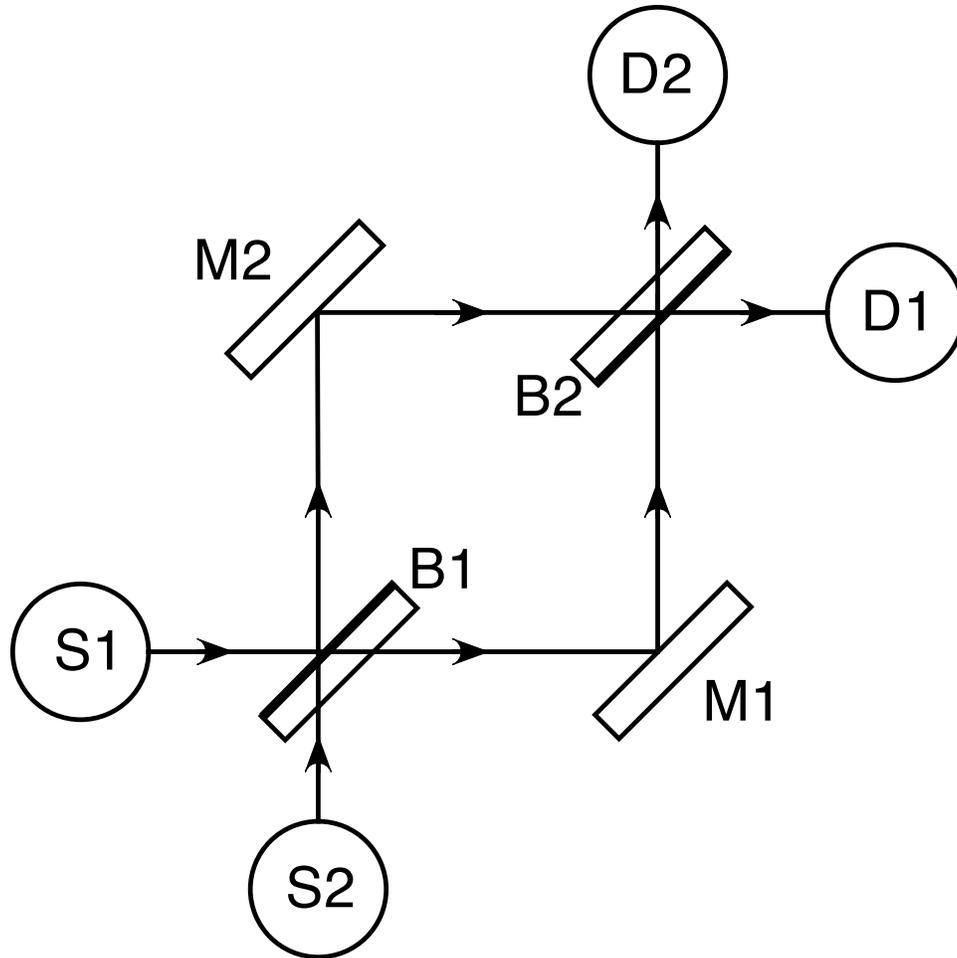
FIGURE 1. The Mach-Zehnder interferometer (MZI). $S1$ and $S2$ are sources, $B1$ and $B2$ are beam-splitters, $M1$ and $M2$ are mirrors, and $D1$ and $D2$ are detectors. When the interferometer arms are the same length, every wave emitted from $S1$ will be detected in $D1$, and every wave emitted from $S2$ will be detected in $D2$. In the quantum limit, $S1$ and $S2$ emit single particles.

incoming waves into a reflected part and a transmitted part, the mirrors $M1$ and $M2$ reflect waves, and the detectors $D1$ and $D2$ detect waves. The beam-splitters are composed of a dielectric coating on glass, and reflect half of every incident wave, transmitting the other half. Waves reflected at the air/dielectric interface are shifted in phase by $\pi$, while waves reflected at the glass/dielectric interface have no phase shift. Waves transmitted through the beam-splitters have no phase shift. Waves reflected from the mirrors are shifted in phase by $\pi$. If the path lengths inside the MZI are all equal, waves from $S1$ will always go to $D1$, and waves from $S2$ will always go to $D2$. This is due to constructive and destructive interference when the waves recombine at $B2$. In a quantum physics analysis, $S1$ and $S2$ emit single particles. What happens to a single particle in the MZI will be discussed next.
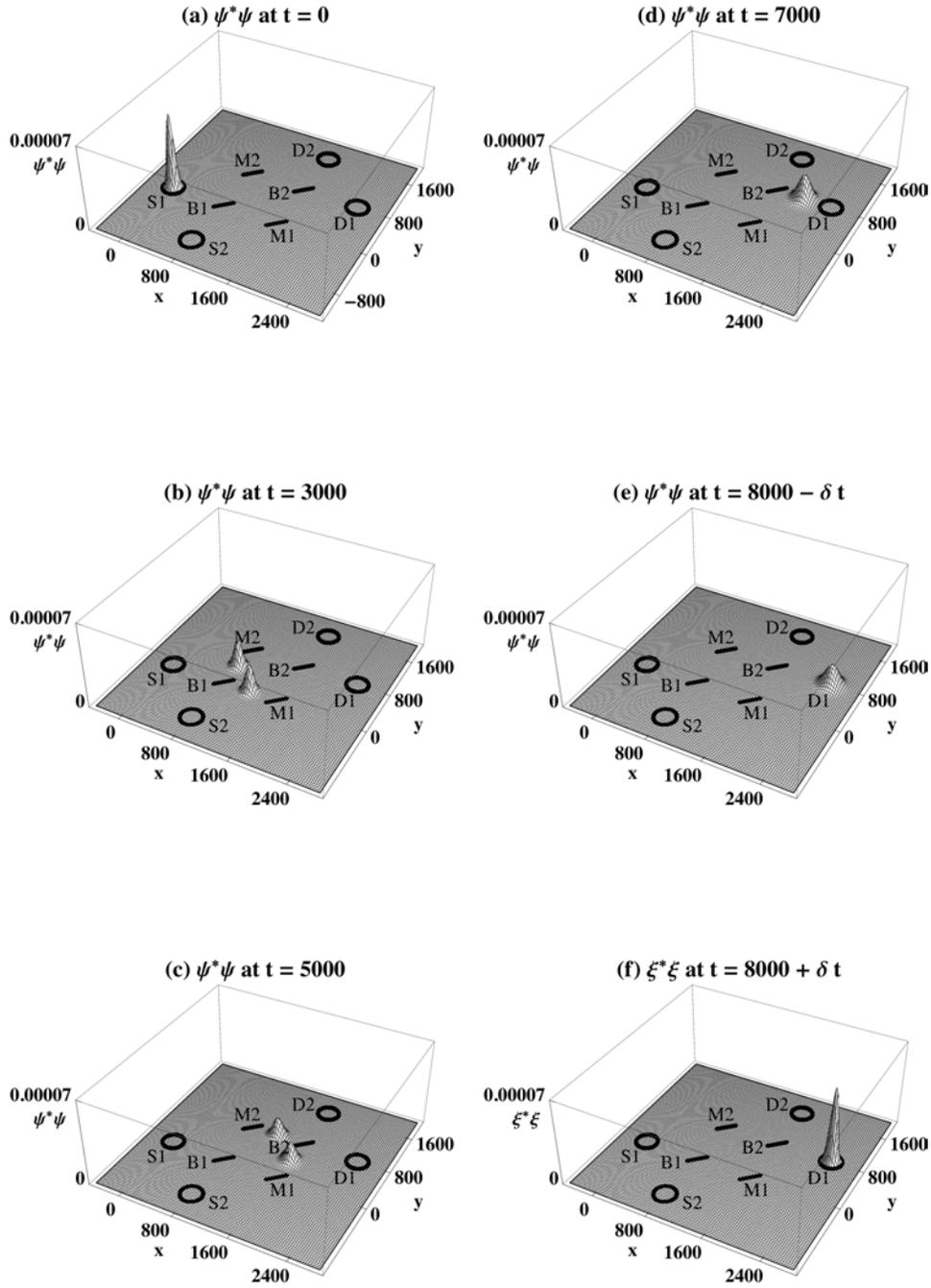
FIGURE 2. The Conventional Theory (CT) of the basic MZI experiment, with a single particle emitted at $S1$. (a) $\psi$ is localized inside $S1$. (b) $\psi$ is split in half by $B1$. (c) The two halves are reflected by $M1$ and $M2$. (d) The re-combined $\psi$ interferes constructively towards $D1$ and destructively towards $D2$. (e) $\psi$ arrives at $D1$, but is not localized inside $D1$. (f) Upon measurement, $\psi$ collapses to a different wavefunction $\xi$, localized inside $D1$. A second measurement immediately afterwards would give the same wavefunction $\xi$.

## 3. The Conventional Theory (CT) of the MZI experiment

The CT postulates that a single free particle with mass $m$ is described by a retarded wavefunction $\psi(\vec{r}, t)$, which satisfies the initial conditions and evolves forwards in time according to the Schrödinger equation:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi. \tag{1}$$

The Schrödinger equation only has solutions which evolve forwards in time, and henceforth will be referred to as the retarded Schrödinger equation (RSE). The CT is a retarded theory of quantum mechanics. We will use natural units and assume $\psi(\vec{r}, t)$ is a traveling gaussian with standard deviation $\sigma = 50$, momentum $k_x = 0.4$, and mass $m = 1$. Figure 2(a) shows the particle's CT probability density $\psi^*\psi$ at the time of emission. $\psi$ is localized inside $S1$. Figure 2(b) shows $\psi$ after being split in half by $B1$. Figure 2(c) shows $\psi$ after the two halves have reflected from $M1$ and $M2$. Figure 2(d) shows $\psi$ after the two halves have recombined at $B2$. The two halves interfere constructively towards $D1$ and destructively towards $D2$. Figure 2(e) shows $\psi$ when it arrives at $D1$, just before detection. $\psi$ is not localized inside $D1$. Figure 2(f) shows what happens upon measurement of the particle at $D1$: $\psi$ collapses instantaneously, indeterministically, and irreversibly into the different wavefunction $\xi$, which is localized inside $D1$. A second measurement, immediately following the first measurement, will show that the final wavefunction is $\xi$. Wavefunction collapse is a postulate of the CT, and is required to obtain agreement between the CT predictions and experimental results.

## 4. The CT Delayed-Choice Paradox

There are four possible ensembles of completed MZI experiments: (1) a particle is emitted from $S1$ and detected in $D1$; (2) a particle is emitted from $S2$ and detected in $D2$; (3) a particle is emitted from $S1$ and detected in $D2$; and (4) a particle is emitted from $S2$ and detected in $D1$.

First, consider the basic experiment (BE) shown in Fig. 1, where both beam-splitters are in place for the entire experiment. Experiments show that only ensembles 1 and 2 occur. Lewis and Wheeler concluded that each particle must be split in half by $B1$, one half taking the upper path and the other half taking the lower path, these two halves then recombining at $B2$ to give interference.

Next, consider a modified experiment (ME) where $B2$ is removed for the entire experiment. Experiments show that ensembles 1, 2, 3, and 4 occur with equal probability. Lewis and Wheeler concluded that: in ensemble 1, each particle must have taken only the upper path; in ensemble 2, each particle must have taken only the lower path; in ensemble 3, each particle must have taken only the lower path; and in ensemble 4, each particle must have taken only the upper path. None of the particles could have taken both paths simultaneously, so there is no interference.

Finally, consider a delayed-choice experiment (CE) where $B2$ is removed before each particle is emitted. We expect each particle to take either the upper path or the lower path with equal probability, as in the ME. At time $t = 5000$, each particle has already chosen either the

upper or the lower path, and has been reflected by either $M1$ or $M2$, but has not yet reached the location where $B2$ was. At this time we spontaneously choose to reinsert $B2$. If the particle is in the upper path, it will either reflect from $B2$ and go to $D2$, or transmit through $B2$ and go to $D1$, with equal probability. If the particle is in the lower path, it will either reflect from $B2$ and go to $D1$, or transmit through $B2$ and go to $D2$, with equal probability. So we expect that ensembles 1, 2, 3, and 4 will occur with equal probability. However, experiments show that only ensembles 1 and 2 occur! There is interference, implying that each particle must have taken both paths. If we had instead spontaneously chosen to not reinsert $B2$, experiments show that ensembles 1, 2, 3, and 4 will occur with equal probability, implying that each particle took only one path. How could the particle know what we would choose to do in the future, and change its behavior accordingly? Wheeler concluded that "...we have a strange inversion of the normal order of time" [7]. This is the delayed-choice paradox of the CT.

## 5. The Advanced Theory (AT) of the MZI experiment

Penrose pointed out that experiments such as the MZI experiment can be explained equally well by an AT of quantum mechanics [20]. The AT postulates that a single free particle with mass $m$ is described by an advanced wavefunction $\varphi^*(\vec{r}, t)$, which satisfies the final conditions and evolves backwards in time according to the advanced Schrödinger equation (ASE):

$$-i\hbar\frac{\partial \varphi^*}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\varphi^*. \tag{2}$$

The ASE only has solutions which evolve backwards in time. We will use natural units and assume $\varphi^*(\vec{r}, t)$ is a traveling gaussian with standard deviation $\sigma = 50$, momentum $k_x = 0.4$, and mass $m = 1$. Figure 3(a) shows the particle's AT probability density $\varphi^*\varphi$ at the time of detection. $\varphi^*$ is localized inside $D1$. Figure 3(b) shows $\varphi^*$ after being split in half by $B2$. Figure 3(c) shows $\varphi^*$ after the two halves have reflected from $M1$ and $M2$. Figure 3(d) shows $\varphi^*$ after the two halves have recombined at $B1$. The two halves interfere constructively towards $S1$ and destructively towards $S2$. Figure 3(e) shows $\varphi^*$ when it arrives at $S1$, just before preparation. $\varphi^*$ is not localized inside $S1$. Preparation of a quantum particle is assumed to be the time-reverse of detection of that same quantum particle. For example, the source could be an atom in the ground state of a parabolic potential, where the potential quickly drops to zero to emit the atom, while the detector could be a similar parabolic potential, quickly raised from zero to capture the atom. Figure 3(f) shows what happens upon preparation of the particle at $S1$: $\varphi^*$ collapses instantaneously, indeterministically, and irreversibly into the different wavefunction $\zeta^*$, which is localized inside $S1$. A second measurement, immediately previous to the first measurement, will show that the initial wavefunction is $\zeta^*$. Wavefunction collapse is a postulate of the AT, and is required to obtain agreement between the AT predictions and experimental results.

## 6. The AT Advanced-Choice Paradox

First, consider an advanced basic experiment (ABE) shown in Fig. 1, but with the direction of all arrows reversed. Experiments show that only ensembles 1 and 2 occur. Lewis and
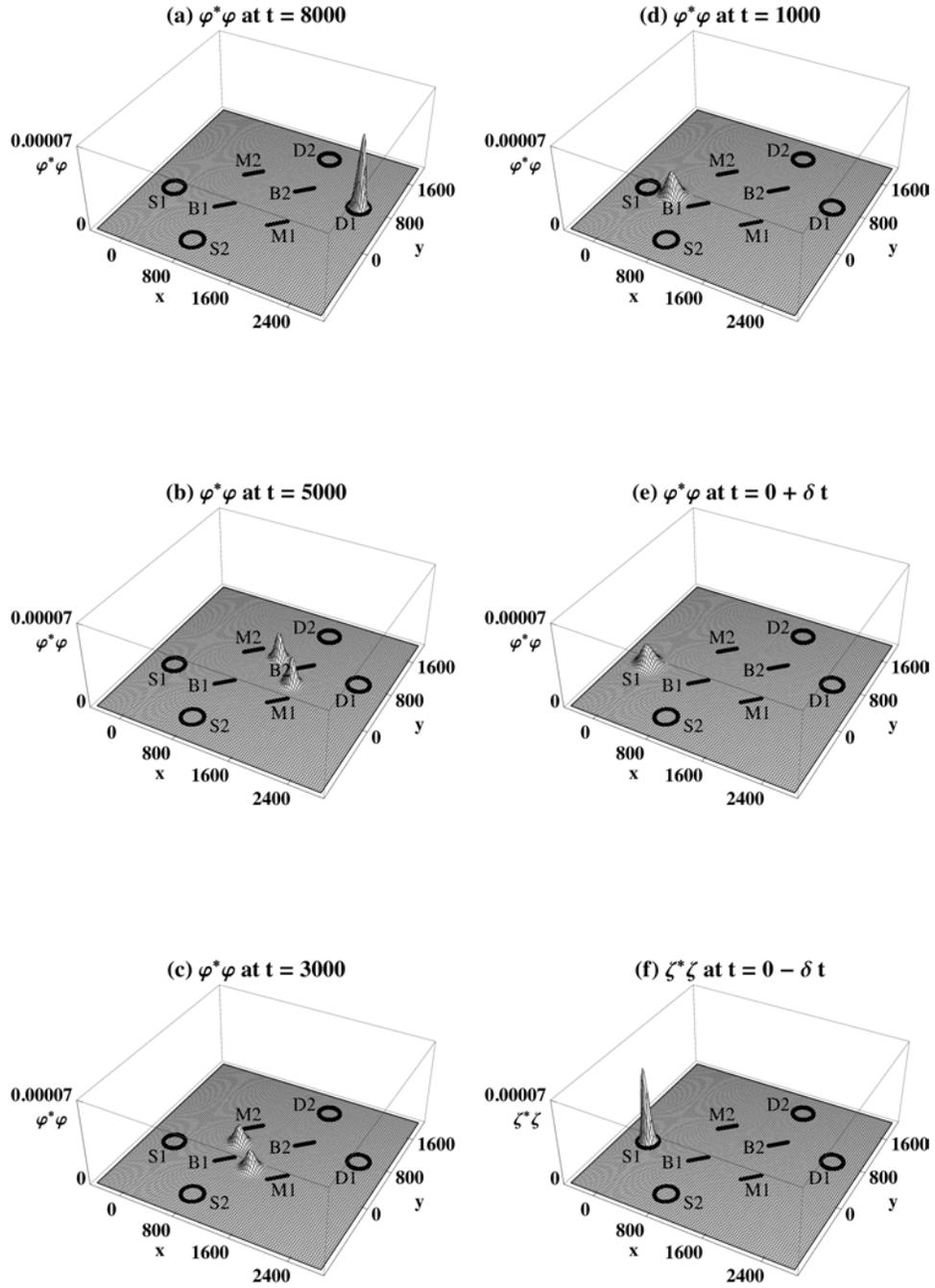
FIGURE 3. The Advanced Theory (AT) of the same MZI experiment, with a single particle detected at $D1$. (a) $\varphi^*$ is localized inside $D1$. (b) $\varphi^*$ is split in half by $B2$. (c) The two halves are reflected by $M1$ and $M2$. (d) The recombined $\varphi^*$ interferes constructively towards $S1$ and destructively towards $S2$. (e) $\varphi^*$ arrives at $S1$, but is not localized inside $S1$. (f) Upon preparation, $\varphi^*$ collapses to a different wavefunction $\zeta^*$, localized inside $S1$. A second measurement immediately before would give the same wavefunction $\zeta^*$.

Wheeler would have concluded that each particle must be split in half by $B2$, one half taking the upper path and the other half taking the lower path, these two halves then recombining at $B1$ to give interference.

Next, consider an advanced modified experiment (AME) where $B1$ is removed for the entire experiment. Experiments show that ensembles 1, 2, 3, and 4 occur with equal probability. Lewis and Wheeler would have concluded that: in ensemble 1, each particle must have taken only the lower path; in ensemble 2, each particle must have taken only the upper path; in ensemble 3, each particle must have taken only the lower path; and in ensemble 4, each particle must have taken only the upper path. None of the particles could have taken both paths, so there is no interference.

Finally, consider an advanced-choice experiment (ACE) where $B1$ is not present between $t = 8000$ and $t = 3000$. We expect each particle to take either the upper path or the lower path with equal probability, as in the AME. At time $t = 3000$, the particle has already chosen either the upper or the lower path, and has been reflected by either $M1$ or $M2$, but has not yet reached the location where $B1$ was. At this time we spontaneously choose to reinsert $B1$. If the particle is in the upper path, it will either reflect from $B1$ and go to $S1$, or transmit through $B1$ and go to $S2$, with equal probability. If the particle is in the lower path, it will either reflect from $B1$ and go to $S2$, or transmit through $B1$ and go to $S1$, with equal probability. So we expect that ensembles 1, 2, 3, and 4 will occur with equal probability. However, experiments show that only ensembles 1 and 2 occur! There is interference, implying that each particle must have taken both paths simultaneously. If we had instead spontaneously chosen to not reinsert $B1$ at $t = 3000$, experiments show that ensembles 1, 2, 3, and 4 will occur with equal probability, implying that each particle only took one path. How could the particle, traveling backwards in time from the future, know what we would choose to do in the past, and change its behavior accordingly? This is the advanced-choice paradox of the AT.

## 7. The Symmetrical Theory (ST) of the MZI experiment

It is also possible to explain the MZI experiments by the ST of quantum mechanics described in [21, 22, 23]. The history of various symmetrical theories of quantum mechanics, and their relations to each other, were also described in these papers. The CT and AT implicitly assume that quantum mechanics is a theory about particles, while the ST explicitly assumes that quantum mechanics is a theory about transitions. The CT and AT postulate that a particle is described by one boundary condition and one wavefunction, while the ST postulates that the transition of a particle is described by two boundary conditions and the algebraic product of two wavefunctions: a retarded wavefunction $\psi(\vec{r}, t)$ that obeys the retarded Schrödinger equation and satisfies only the initial boundary condition; and an advanced wavefunction $\varphi^*(\vec{r}, t)$ that obeys the advanced Schrödinger equation and satisfies only the final boundary condition. The CT and AT postulate that the wavefunction collapses instantaneously, indeterministically, and irreversibly into a different wavefunction at one of the boundary conditions, while the ST postulates that no wavefunctions ever collapse. Consequently, the CT and AT have intrinsic arrows of time and are indeterministic, while the ST has no intrinsic arrow of time and is deterministic.
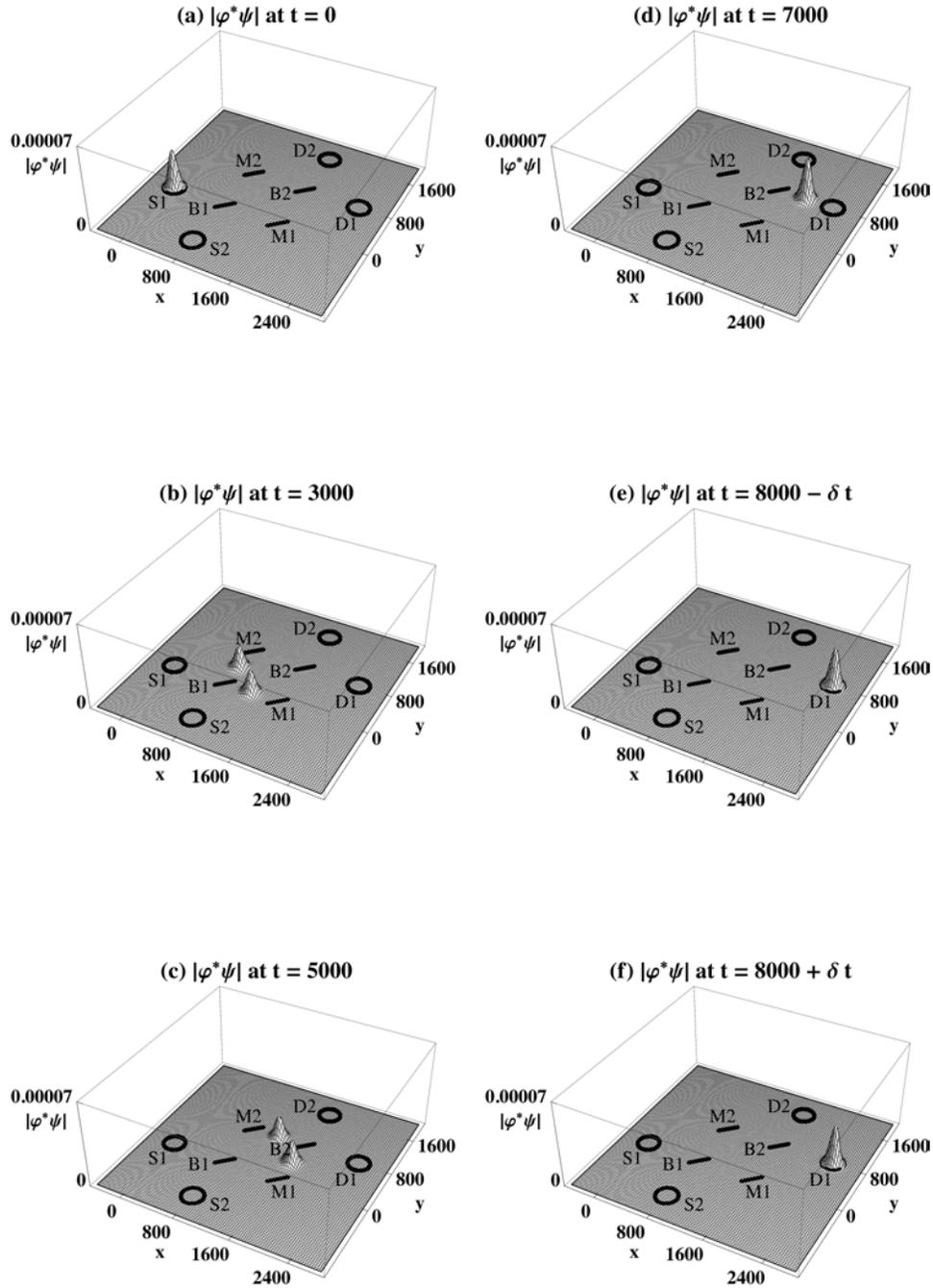
FIGURE 4. The Symmetrical Theory (ST) of the same MZI experiment. (a) $\varphi^*\psi$ is localized inside $S1$. (b) $\varphi^*\psi$ is split in half by $B1$. (c) The two halves are reflected by $M1$ and $M2$. (d) The recombined $\varphi^*\psi$ interferes constructively towards $D1$ and destructively towards $D2$. (e) $\varphi^*\psi$ arrives at $D1$, and is localized inside $D1$. (f) A second measurement immediately afterward would give the same $\varphi^*\psi$: there is no wavefunction collapse upon measurement.

Figure 4 shows the ST explanation of the basic MZI experiment, which we will call the SBE. The product wavefunction $\varphi^*\psi$ is symmetrical in time, and could be viewed as traveling either forwards or backwards in time between the two boundary conditions. We will use the forwards in time viewpoint and the same advanced and retarded wavefunctions described earlier. Figure 4(a) shows $\varphi^*\psi$ at the time $S1$ emits a particle. It is localized inside $S1$. Figure 4(b) shows $\varphi^*\psi$ after being split in half by $B1$. Figure 4(c) shows $\varphi^*\psi$ after the two halves have reflected from $M1$ and $M2$. Figure 4(d) shows $\varphi^*\psi$ after the two halves recombine at $B2$. The two halves interfere constructively towards $D1$ and destructively towards $D2$. Figure 4(e) shows $\varphi^*\psi$ when it arrives at $D1$, just before detection. Figure 4(f) shows $\varphi^*\psi$ just after detection. A second measurement, immediately following the first measurement, will show the same $\varphi^*\psi$. There is no need to postulate wavefunction collapse to explain the localization inside $D1$ upon measurement, since $\varphi^*\psi$ is already localized inside $D1$ before measurement. Similarly, if $S2$ emits a particle, it will always go to $D2$. A backwards in time viewpoint gives the same results.

## 8. Resolving the Paradoxes

First, consider the ST analysis of the delayed-choice experiment (CE), where we choose to have $B2$ not present between $t = 0$ and $t = 5000$, and present between $t = 5000$ and $t = 8000$. In this situation, the advanced wavefunction will take both paths from the detector to the source for all four ensembles. For ensemble 1, the retarded wavefunction will take only the upper path from $S1$ to $D1$, so the product wavefunction will be nonzero only on the upper path. For ensemble 2, the retarded wavefunction will take only the lower path from $S2$ to $D2$, so the product wavefunction will be nonzero only on the lower path. For ensemble 3, the retarded wavefunction will take only the lower path from $S1$ to $D2$, so the product wavefunction will be nonzero only on the lower path. For ensemble 4, the retarded wavefunction will take only the upper path from $S2$ to $D1$, so the product wavefunction will be nonzero only on the upper path.

Next, consider a ST analysis of the advanced-choice experiment (ACE), where we choose to have $B1$ present between $t = 0$ and $t = 3000$, and not present between $t = 3000$ and $t = 8000$. The retarded wavefunction will take both paths from the source to the detector for all four ensembles. For ensemble 1, the advanced wavefunction will take only the lower path from $D1$ to $S1$, so the product wavefunction will be nonzero only on the lower path. For ensemble 2, the advanced wavefunction will take only the upper path from $D2$ to $S2$, so the product wavefunction will be nonzero only on the upper path. For ensemble 3, the advanced wavefunction will take only the lower path from $D2$ to $S1$, so the product wavefunction will be nonzero only on the lower path. For ensemble 4, the advanced wavefunction will take only the upper path from $D1$ to $S2$, so the product wavefunction will be nonzero only on the upper path.

The same delayed-choice and advanced-choice paradoxes appear, since the ensembles 3 and 4 never occur. But there are also two new inconsistencies: for ensemble 1, the ST analysis of the delayed-choice experiment predicts a nonzero product wavefunction only on the upper path, while the ST analysis of the advanced-choice experiment predicts a nonzero product wavefunction only on the lower path; and for ensemble 2, the ST analysis of the delayed-choice experiment predicts a nonzero product wavefunction only on the lower path, while the

ST analysis of the advanced-choice experiment predicts a nonzero product wavefunction only on the upper path. The product wavefunction is time-symmetric, and an advanced-choice experiment is a delayed-choice experiment running backwards in time. Experiments with the same boundary conditions should then have the same product wavefunctions.

These two inconsistencies can be fixed by assuming that when an incoming wavefunction hits the dielectric-coated side of a 50/50 beam-splitter, one half of the wavefunction is *always* transmitted with zero phase shift, while the other half of the wavefunction is *always* reflected with a $\pi$ phase shift. Also, when an incoming wavefunction hits the glass side of a 50/50 beam-splitter, one half of the wavefunction is *always* transmitted with zero phase shift, while the other half of the wavefunction is *always* reflected with zero phase shift. These assumptions also fix the delayed-choice and advanced-choice paradoxes of the ST, CT, and AT. For the delayed and advanced experiments described above, this assumption predicts that ensembles 1 and 2 will have nonzero product wavefunctions on both paths, and ensembles 3 and 4 will have zero product wavefunctions on both paths. This agrees with experiment. For the alternative choices, where one or the other of the beam-splitters is never present, each of the 4 ensembles will have a nonzero product wavefunction on one of the paths, with equal probability. This also agrees with experiment. We conclude that a beam-splitter *always* splits an incoming wavefunction into two parts, contrary to the assumptions of Lewis and Wheeler, and there are no paradoxes in the CT, AT, or ST.

Ellerman came to this same conclusion about a beam-splitter, by a different line of reasoning, by analyzing just the CT delayed-choice experiment [24]. He proposed that Lewis and Wheeler mistook the creation of a superposition state at the first beam-splitter for a measurement that collapses the retarded wavefunction. Let us assume that $B1$ always splits the particle's wavefunction into a quantum superposition of two parts: one part traveling along the upper path and the other part traveling along the lower path. When $B2$ is not present, the part that takes the upper path will always arrive at $D1$, while the part that takes the lower path will always arrive at $D2$. When a particle is detected in one of the detectors, the wavefunction collapses to being 100% in that detector and 0% in the other detector. Since the two parts have the same magnitude, the collapse is equally likely to occur in either one of the detectors. When $B2$ is present, the part that took the upper path and the part that took the lower path always interfere at $B2$, such that a wavefunction from $S1$ will always go to $D1$, while a wavefunction from $S2$ will always go to $D2$. These predictions agree with experiment, and have no delayed-choice paradox.

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