A matter wave thought experiment concerning Galilean transformations

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Abstract: The Schrödinger equation is invariant under Galilean transformations. Therefore, a matter wave should undergo the Doppler effect in the same manner as a classical wave and it should be observed when the measurement apparatus, i.e., the interferometer, moves with some velocity in the laboratory reference frame. Similarly, wave-like characteristics can be expected to appear if the measurement apparatus moves with the same velocity relative to the original particle, despite the fact that the particle is at rest in the laboratory reference frame. This paper provides a proposal for a simple thought experiment related to the reference frame dependence of the wave-particle duality using current matter wave optics technology.

Keywords: quantum experiment; matter wave; Galilean transformation; reference frame

1. Introduction

It is an established fact that nature exhibits wave-particle duality in the quantum world. Moreover, it is a fact that quantum mechanics provides almost perfect predictions about most phenomena in the micro-world. Despite these facts, no interpretation can provide a satisfactory description of method the classical world connects to the quantum world. Every particle, electron, neutron, and even atom has been proven to exhibit a wavelike nature. Furthermore, it is has been shown that even a very large molecule, e.g., fullerene C_{60}, exhibits a very clear interference pattern [1,2]. Recently, Arndt *et al*., physicists at the University of Vienna have demonstrated an interference experiment using fullerene, and a number of experiments concerning matter waves have been performed; in fact, this field of study is now referred to as "matter wave optics" [3–5]. These large molecules can be directly observed as truly localized classical particles using a transmission electron microscope (TEM) or an atomic force microscope (AFM) [6–8]. These amazing experimental facts, however, reawaken the old question : "what is wave-particle duality?" [9].

It is a well-known historical fact that the argument concerning this duality was initiated by the famous Bohr-Einstein debate, and has been continued by many physicists and philosophers. Accompanying this argument, various interpretations have been proposed [10 –14]. Nevertheless, it is difficult to determine any satisfactory interpretation that explains the bridge between the classical and the quantum worlds. Needless to say quantum mechanics is a complete and self-consistent theory for predicting most physical phenomena. The problem occurs when we step over the boundary between quantum mechanical and classical spaces while maintaining a classical worldview. In fact, only the statistical probability interpretation of the modulus of wave function is the bridge between the wave function and the classical particle. If we enter this intermixing region, various quantum mysteries appear.

As regards the space in which the equations of motion are determined, we see that classical mechanics is defined in three-dimensional space, whereas quantum mechanics is defined in configuration space; however, this configuration space is equivalent to three-dimensional space only in the case of the one body problem. Accordingly, both spaces are interconnected by the relation of de Broglie formula relation for one particle (i.e., $\lambda_d = h/mv$, where λ_d is the de Broglie wavelength, h is Planck's constant, m is the mass of the particle, and v is the velocity of the particle in some reference frame). In the classical mechanics space, the equation of motion describes a localized particle's behavior, whereas in the quantum mechanics space, the wave equation describes a wave's behavior in the form of a wave function, thus it is difficult to determine the relation between two spaces via one-to-one correspondence. Incidentally, both equations are invariant under Galilean transformations in each space. Therefore, comparing the behaviors of the particle and the wave under the Galilean transformations can be expected to provide new information about the underlying duality.

Here I would like to propose a simple experiment concerning Galilean transformations that concerned with the way in which these spaces interconnect with each other. More specifically, the aim of this paper is to study how the duality changes when it is observed from different inertial frames using the particle which looks like a clearly classical appearance, such as C_{60} ; in other words, an experiment is proposed in which the Doppler effect of matter waves can be observed. This involves a very simple discussion concerning Galilean transformations of the Schrödinger equation. Nevertheless, we will soon see that this experiment introduces one sharp paradox; that is, a particle at rest in some reference frame exhibits a wave-like nature if it is observed from a different inertial frame.

In Section 2, the Galilean transformation of the Schrödinger equation is reviewed. The details of this discussion are based on Ballentine [15], and are further discussed in Brown & Holland [16]. In Section 3, the experiment concerning the Doppler effect for matter waves is proposed based on the Vienna group's experiment. In Section 4, as an extended case of this Doppler effect, an experiment involving a particle at rest and a moving grating set is proposed using the atoms of a Bose-Einstein condensate (BEC). In Section 5, certain issues of these thought experiments are discussed.

2. Schrödinger equation invariance under Galilean transformation

In this section, an overview of the Galilean transformation of the Schrödinger equation is given following Ballentine. We can see that the Schrödinger equation is invariant under a Galilean transformation between two reference frames, and the corresponding wave functions are differ only by a phase change. We can see this phase change corresponds to the laws of conservation of momentum and energy of the particle in a classical mechanics between two inertial frames, as shown later.

Figure 1 shows a schematic diagram of a particle and a measurement apparatus; reference frame *F* is attached to the laboratory, and hereafter we call this the laboratory reference frame. Another frame, *F'*, is moving with some velocity relative to the laboratory, and we hereafter call this the moving reference frame. For simplicity, both frames are assumed to be the inertial frames. The Doppler effect for a matter wave corresponds to the Galilean transformation between these two reference frames and this wavelike behavior of the particle is measured by an interferometer. The motion of the particle between both inertial frames is also determined using the same Galilean transformation in classical mechanics.

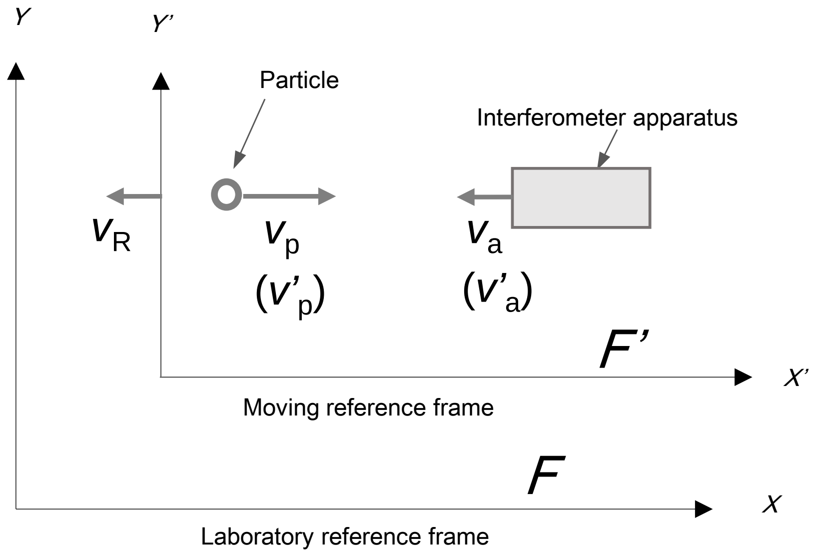

Figure 1. Schematic configuration of Doppler effect experiment for matter waves. In the laboratory reference frame, *F*, a particle moves at a velocity v_p and an interferometer for observing matter waves moves at a velocity v_a. Another inertial frame, *F'*, moves with velocity v_R. For simplicity, *F* and *F'* are assumed to both be inertial frames. The velocities of the particle and the interferometer are v'_p and v'_a respectively, with respect to *F'*. From the quantum mechanics perspective, *F'* is always attached to the interferometer. If $v_a = 0$, this case reduces to the standard observation of the wave-like nature of a particle provided by an interferometer at rest in the laboratory. If v_a is non-zero, the Doppler effect of the matter waves can be observed by the interferometer. The transformation of both frames is described by a Galilean transformation. With this configuration both the classical equation of motion and the Schrödinger equation are invariant under the Galilean transformation.

The Galilean transformation of the Schrödinger equation with this configuration is described as follows. The Schrödinger equation for a moving particle is

$$i\hbar\frac{\partial}{\partial t}\psi(r,t) = -\frac{\hbar^2}{2m}\Delta\psi(r,t)\,. \tag{1}$$

Its general solution is written as a wave function

$$\psi(r,t) = Ae^{i(kr-\omega t)}, \tag{2}$$

where

$$k = p/\hbar = mv/\hbar, \tag{3}$$

and

$$\omega = E/\hbar = mv^2/2\hbar. \tag{4}$$

In these equations, k and ω are the wave vector and angular frequency of the wave function, respectively, and p and E are the momentum and energy of the particle, respectively. Further,

$$\lambda_d = 2\pi\hbar/p = 2\pi/k = 2\pi\hbar/mv. \tag{5}$$

If we view the Schrödinger equation from *F*', which is moving with velocity v' relative to *F*, the new Schrödinger equation (and associated wave function) *F'* can be expressed as

$$i\hbar\frac{\partial}{\partial t}\psi'(r,t) = -\frac{\hbar^2}{2m}\Delta\psi'(r',t'). \tag{6}$$

For simplicity the formula is hereafter treated in one spatial dimension,

$$\psi'(x',t') = e^{i(kx'-\omega t')}. \tag{7}$$

From the definition of the Galilean transformation, we have

$$x = x' + v't' \text{ and } t = t', \tag{8}$$

and, from the probability density conservation relation we have

$$|\psi(x,t)|^2 = |\psi'(x',t')|^2. \tag{9}$$

Therefore, the wave function in *F'* can be expressed as

$$\psi(x,t) = e^{if(x,t)}\psi'(x',t'), \tag{10}$$

where the function $f(x,t)$ can be written as

$$f(x,t) = \frac{m}{2\hbar}(v'x - v'^2 t/2). \tag{11}$$

Hence, the wave function in *F'* can be expressed as

$$\begin{aligned}\psi(x,t) &= e^{if(x,t)}\psi'(x-v't,t) \\ &= \exp\left[\frac{i}{\hbar}(\hbar k + mv')x - \frac{i}{\hbar}\frac{(\hbar k + mv')^2}{2m}t\right].\end{aligned} \tag{12}$$

Here we introduce k' and ω', where

$$k' = p'/\hbar = m(v+v')/\hbar, \tag{15}$$

$$\omega' = E'/\hbar = p'^2/2m = m(v+v')^2/2\hbar . \quad (16)$$

Hence, the relations of the wave vectors k, k' and the angular frequencies ω, ω' are equivalent to those of the p and E of a particle in classical mechanics between two inertial frames. That is, comparing Equations (15) and (16) with Equations (3) and (4). However the de Broglie wavelength itself is not invariant under the Galilean transformation, which differs from the case of a classical wave.

Here, v and v' are defined as $v = v_p$ (the particle velocity) and $v' = v_a$ (the interferometer velocity) according to Figure 1, so that Equation (12) can be rewritten as

$$\psi(x,t) = e^{if(x,t)}\psi'(x - v't, t)$$
$$= \exp\left[\frac{i}{\hbar}(mv_p + mv_a)x - \frac{i}{\hbar}\frac{(mv_p + mv_a)^2}{2m}t\right]. \quad (17)$$

From this equation, all wave functions are equivalent, if $v_p + v_a$ is a constant. This indicates that the wave function does not depend on the velocity of the particle in F, but depends on the relative velocity between the particle and the measurement apparatus (i.e., the interferometer) v_r only. Here, F' is always attached to the interferometer which can measure the new, transformed wave function. In other words, only one observer standing on the interferometer's reference frame determines the wave-like nature of the particle in the quantum mechanics framework.

On the other hand, in the case of a classical particle, we can observe the particle from any moving reference frame at the same time. The equations of motion from the observers in each inertial frame are simply related via the classical Galilean transformation. Therefore, we can see that various relations among the reference frames exist that depend on the reference frame occupied by the observer. The typical situations are summarized in the following five cases. Here, v_0, v_1, and v_2 indicate real numbers.

Case 1. Here, $v_p = v_0$ and $v_a = 0$ with respect to F. This case yields no Doppler effect and the measured wavelength corresponds simply to the λ_d of a particle with velocity v_0. In this case, the considered interferometer reference frame is identical to F, and the Schrödinger equation is also described within F.

Case 2. The particle velocity is the same as above (i.e., $v_p = v_0$), but the interferometer unit moves with a velocity of $v_a = v_1$ with respect to F. This case yields the basic Doppler effect. The measured wavelength will exhibits a Doppler shift given by $\lambda' = \lambda - \Delta\lambda$, from Equations (5) and (15). In this case, the Galilean transformation of the Schrödinger equation is performed from F to F'.

Case 3. If the observer occupies F', the particle velocity v'_p and the interferometer velocity v'_a can defined as $v'_p = v_0 - v_2$ and $v'_a = v_1 + v_2$, respectively, from the observer's perspective. However this case has no bearing on the Schrödinger equation, because v_r does not change (see Equation (17)).

Case 4. From case 3, if $v_1 = 0$, this is identical to case 1, except the observer occupies the moving reference frame with velocity v_2. Nevertheless, v_r is equal to that of case 1 (i.e., the relative velocity is v_0), and λ_d equals that associated with v_0.

Case 5. This is an extension of case 3, wherein $v_1 = 0$, and $v_2 = v_0$. In this case, the observer moves with the same velocity as the particle (i.e., the moving reference frame is attached to the particle). Therefore, the particle appears to be at rest and the interferometer moves at v_0, although the interferometer is at rest in F. Nevertheless, $v_r = v_0$, and, thus, the λ_d observed by the interferometer should be identical that of case 1. These cases are summarized in Table 1 and a schematic drawing of this table is given in Figure A1.

Case	Velocity of moving reference frame to laboratory v_R	Particle velocity			Interferometer velocity		Relative velocity v_r between particle and interferometer
		v_p *Observed in laboratory reference frame*	v'_p Observed in moving reference frame		v_a *Observed in laboratory reference frame*	v'_a Observed in moving reference frame	
1	$v_R = 0$	v_0	–		0	–	v_0
2	$v_R = 0$	v_0	–		$-v_1$	–	$v_0 + v_1$
3	$v_R = v_2$	v_0	$v_0 - v_2$		$-v_1$	$-(v_1 + v_2)$	$v_0 + v_1$
4	$v_R = v_2$	v_0	$v_0 - v_2$		0	$-v_2$	v_0
5	$v_R = v_2 = v_0$	v_0	0		0	$-v_0$	v_0

Table1. Typical cases of velocity relations between the particle, interferometer, observer, and various reference frames from the relations illustrated in Figure 1. Two reference frames are considered: the laboratory reference frame and another that moves with a velocity v_R with respect to the laboratory reference frame. In the case of the Schrödinger equation, the relative velocity only is meaningful; here the phrase "relative velocity v_r" refers to the velocity between the particle and the interferometer. There are many possible cases in the classical mechanics framework, depending on the reference frame occupied by the observer. If the observer occupies the moving reference frame that moves with velocity v_p, the particle is at rest and the interferometer is moving with this velocity.

No issues arise if we consider the motion with respect to either classical mechanics, or quantum mechanics only. However, a certain mysterious aspect arises when we try to connect these frameworks together. In particular, case 5 represents the specified mysterious situation, in which the existence of the wave-particle nature depends on the reference frame occupied by the observer. This mystery originates from the same conventional paradoxes (e.g., double slit interference and delayed choice experiment) associated with duality; this is because the Schrödinger equation does not provide any straightforward description of the connection between particles and waves.

3. Thought experiment concerning matter waves Doppler effect

In this section, a realistic thought experiment concerning the Doppler effect for matter waves is proposed. A conceptual drawing of the experimental setup is shown in Figure 2. The basic configuration is in accordance with the experiment on matter waves conducted by the Vienna group [4,5]. However, a crucial difference exist as regards the implementation of the moving interferometer unit. Specifically this unit moves with a very accurate and constant velocity. When the interferometer moves toward the particle, the Doppler effect results in a shorter λ_d , and vice versa. If v is the stage's velocity and v_0 is the particle's velocity in laboratory reference frame, the wavelength shift is proportional to v_0 $(1 + v/v_0)$ for the forward motion of the interferometer stage and to v_0 $(1 - v/v_0)$ for the backward motion of the interferometer stage.

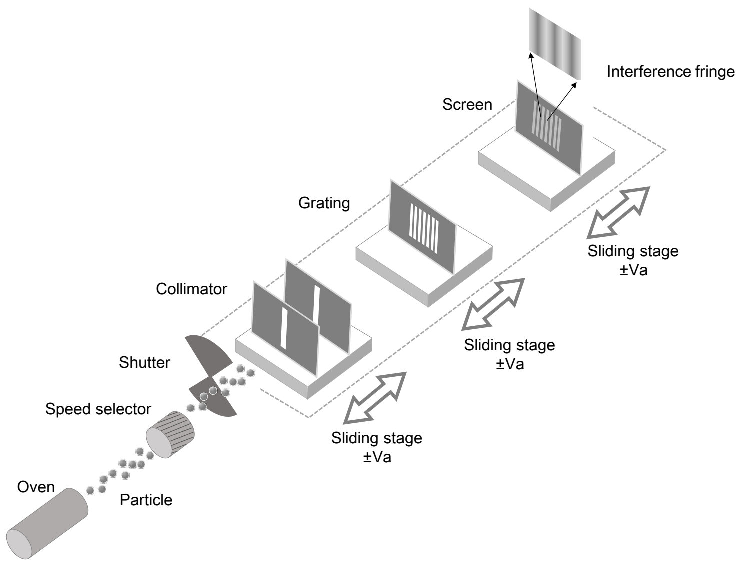

Figure 2 Schematic illustration of the experimental setup for the measurement of the Doppler effect for matter waves. This illustrates a conceptual experimental configuration using a conventional far-field-type interferometer, based on the successful setup of the University of Vienna group. The velocity selector causes the velocity distribution of the particles to narrow in order to satisfy the coherence condition. The shutter selects the particle for either backward or forward stage motion. The detector is of particle-adsorption-type and consists of a plane plate only, and the molecule fringe pattern that accumulates on the plate is measured using a fluorescence microscope or an atomic force microscope (AFM).

The details of this experimental setup are described as follows. The fullerene source is an oven (at approximately 700 K), and the average velocity of fullerene molecules ranges from 100 to 200 m/s. The corresponding λ_d is approximately 3–6 pm, and the velocity broadening is decreased (i.e., to approximately 5% of the full width at half maximum (FWHM)) using the velocity selector [5]. The molecular beam from the oven is collimated into a parallel beam by two slits in order to provide sufficient spatial coherence for the interferometer. The grating consists of a 100nm pitch SiNx membrane fabricated using microlithography; its first-order diffraction peak is located approximately 50 μm from the center of the screen at a distance of 1.2 m from the grating. A 10% shift in the wavelength corresponds to a 5 μm shift in the diffraction peak at the screen, which may be sufficient for detecting the Doppler effect of matter waves. The main problem with this set-up concerns the large size of the interferometer apparatus unit; therefore, it is difficult to place this unit in a vacuum chamber in a laboratory when it is required to move with a velocity of 5 or 10 m/s. The near-field interferometer (i.e. the Talbot-Lau interferometer (TLI)), that was used by the Vienna group [17–22] is very compact. However, it is not appropriate for wavelength-shift observations (such as those involved in measuring the Doppler effect) because tuning of the different Talbot lengths is necessary to clarify the peak wavelength shift. Thus, the far-field interferometer provides a simpler solution for observing the wavelength shift. On the other hand, it is difficult to downsize the apparatus for easy implementation in a vacuum chamber because the grating pitch of 100 nm is close to the limiting value of the fabrication techniques involved. Furthermore, the influence of the van der Waals forces from the slit wall on the particles cannot be neglected if the slit width becomes narrower than several tenth of a nm [2,19] , as this causes degradation of the matter wave coherence. The most realistic approach therefore involves setting the three-parts collimator, grating, and screen on three separate sliding stages and co-driving them very accurately.

Regarding the detector, the ionization method requires large tools, so an adsorbing-type detector, in which the interference pattern is piled and measured using fluorescent microscopy or AFM, is practically preferable. Thus, the fullerene particle should be replaced by a fluorescent molecule, such as phthalocyanine [18,19,22]. In this way, the detector unit becomes compact and is therefore easy to integrate onto a moving stage. Furthermore, a shutter should be implemented to select the molecular beam for either backward or forward stage motion.

A moving stage with a speed of over 10 m/s is necessary to clearly observe the wavelength shift caused by the Doppler effect, because the FWHM of the wavelength distribution for matter waves in the above configuration is approximately 5% of the peak wavelength [2]. The state-of-the-art technology used in semiconductor exposure tools, known as a scanner, could present one realistic solution to this issue. The reticle stage of the current volume production exposure tool moves at more than 5 m/s through at distance of more than 200 mm, with very high accuracy. An air bearing stage and a linear motor with a laser interferometer are the key technological components of this high-speed moving stage, which has already been realized in a vacuum [23,24]. Therefore, if we could apply this technology to the experiment under consideration here, we could, in principle, conduct measurements of the Doppler effect for matter waves.

Such a moving stage, however, is rather expensive and quite complex. One alternative is to use a molecular beam with a narrower velocity distribution. If we use twin velocity selectors [25,26], the distribution should become narrower; (e.g., down to 0.25% of the peak wavelength). For this stage, only a 0.25% shift of the wavelength is required to sense the Doppler shift, which means that a velocity of only 25 cm/s is required for the moving stage. A stage with these parameters that can be placed in a vacuum is not unusual as regards to current standards. The detriment, however, is a longer measurement time. More specifically, a 20 times longer measurement period would be required to yield the same signal-to-noise ratio for the interference fringes as compared to the original single selector. However, according to Stibor *et al.* [19], the detection efficiency is significantly improved through the application of their new concept, which uses fluorescence. Further, the accumulation time required to provide a clear interference pattern for one velocity selector becomes roughly 1 h. Therefore, in the case of the above twin velocity selector, approximately 20 h would be required, which is not an impractical length of time for this experiment.

4. Particle at rest and moving grating set

The actual experimental set up of case 5 is quite different from that described in Section 3.

This is because the particle should be at rest in F for case 5. At present, laser cooling and magnetic-optical trap (MOT) devices have been realized for many kinds of atoms, and BEC technology for several kinds of atoms has also been established, in which atomic velocity is less than a few centimeters per second. Such atoms could be said to be almost at rest relative to an observation stage with a speed of several meters per second. However, applying this technology to a large molecule, such as fullerene, poses a substantial challenge. Therefore, we should use an atom instead of fullerene, because the discussion of fullerene in case 5 experiment continue to hold in that case. This is based on the fact that the image of an atom as a particle can be observed via AFM or TEM, if the atom is trapped in a particular potential, such as the surface of some crystal. Needless to say, many atomic interference experiments in the past have demonstrated the wave-like nature of atoms. Accordingly, the experimental setup configuration for case 5 is described as follows.

In 1995, Ketterl and coworkers investigated the BEC of Na atoms. In their article, the velocity distribution of the BEC atoms was obtained using the time of flight (TOF) method [27,28]. This atomic cloud revealed a very small velocity distribution of less than a few centimeters per second; thus we can expect to realize simple experimental configurations using BEC atoms. In the case of the Na atom, λ_d will be approximately 25 nm when the velocity is 50 cm/s, and the FWHM of the velocity distribution is less than 5% of the wavelength. Thus, the moving stage provides sufficient speed at 50 cm/s, and the similar stage, presented in Section 3 suffices for this experiment. This wavelength broadening can allow us to observe a clear diffraction fringe on the screen in the same manner as the experiment discussed in section 3.

The λ_d of the Na atom with 50 cm/s velocity is significantly longer than that of fullerene. Hence, the TLI system becomes easy to fabricate [20–22]. If we choose a 20 μm pitch grating, the Talbot length is 16 mm and a very compact interference apparatus unit can be realized. A few examples of small TLI are shown in Appendix B. We can therefore expect to implement such small interferometers in the BEC experimental setup. Figure 3 illustrates a diagram of this setup. The detection method is also of the surface-adsorption type and the resultant accumulated diffraction fringe pattern can be easily observed on the screen using a fluorescence optical microscope. However, we must articulate the mechanics of the moving grating system that were incorporated in the complicated MOT structure [29] in order to use the BEC. Nevertheless, it is possible to fabricate a grating system on a reduced scale of several millimeters, if a smaller grating pitch is adopted. The advantage of using the BEC is that the accumulation of atoms for a single running of the grating unit is sufficient, simply because the BEC cluster has a large number of atoms.

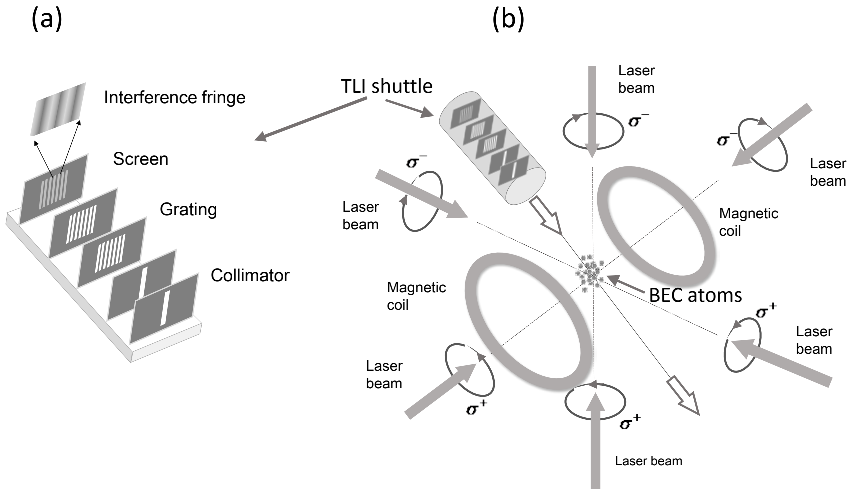

Figure 3. Schematic experimental setup used to detect wave nature of particles at rest in laboratory reference frame using BEC atoms. (a) Configuration using the Talbot-Lau interferometer (TLI) apparatus, based on the successful setup of the University of Vienna group. (b) Schematic illustration of the MOT implementing the TLI. The basic MOT configuration consists of an anti-Helmholtz coil with current I and three orthogonal, circularly polarized laser pairs of σ^{+}, σ^{-}. The BEC region requires a very high vacuum and is usually set in a glass cell. The TLI should be very compact, and this small TLI shuttle passes through the BEC atom cloud. Because the BEC has a large number of atoms within a region of a few hundred micrometers, a single running of the shuttle allows a sufficiently number of atoms to be adsorbed on detector plate surface; hence, the interference fringe of the atom-waves can be observed with good contrast using a fluorescence microscope.

One important point to consider is the nature of BEC source, for which most of the waves associated with each atom are coherent from the quantum mechanical point of view. Thus, the BEC cloud is considered to be equivalent to a certain plane wave from the optics perspective. On the other hand, from past experimental results for the interference between two BEC clouds, we know that the λ_d corresponding to the BEC is maintained as that of a single atom [30]. Thus the interference pattern obtained using the BEC is also expected to be identical to that of an independent particle with the same velocity, as they have the same wavelength. The van Cittert-Zernike theorem gives the relation between the coherence of a light source and the distribution of incoherent point sources [31]; however, it is not clear that the point sources could correspond to the independent particles in the case of matter waves. This is because this simply represents the problem of wave-particle duality: “under which circumstances does a particle come to behave as a wave?”

One other important point to consider is that we must deactivate the laser and magnetic trap when the grating begins to run, because any interaction with the particle disturbs free particle state. It is true that some discussion about weak measurement has been conducted but, here, the Schrödinger equation (1) inherently describes a free particle.

5 Remarks and Discussion

In Section 2, a general formula for the Galilean transformation of the Schrödinger equation (and the associated wave function) was reviewed based on the work of Ballentine [15]. Accordingly, we investigated the reference frame dependency of the behavior of the quantum mechanical wave-function and classical particle motion. The Doppler effect of a matter wave can be observed from *F'*, which the measurement apparatus (i.e., the interferometer) occupies, and it does not depend on the velocity of the particle in *F*. Rather it depends on v_r only. In the case of a classical particle, the velocity of the particle depends on the reference frame in which the observer occupies.

From *F'*, various particle velocities became possible, although they are entirely equivalent to each other relative to the reference frame attached to the interferometer. In other words, only v_r can produce the wave-like behavior. From this point of view, we can say that a particle that is at rest in some reference frame can behave as a wave from a different moving reference frame.

In Section 3, the Doppler effect experiment for matter waves was proposed. The focus of this experiment is similar to studies of the conventional Doppler effect for classical waves, but it has not yet been performed. It is not difficult to realize such an experiment if we use the present matter wave optics technology. The predicted results are not surprising, however, because the matter wave travels exactly like a classical wave in this three-dimensional space. In contrast, in Section 4, a sharp paradox arose in that the classical particle at rest behaves as a wave if it is observed from a moving reference frame. However, this fact does not contradict the principle of quantum mechanics, as described in Section 2. Therefore, the experiment in Section 4 is outwardly different to that of Section 3, but we can discover they are smoothly connected from the perspective of the reference frame argument presented in Section 2.

Nevertheless, this gives rise to a new question: how does this static particle obtain these wave-like properties relative to *F'*? In terms of a wave, this has nothing to do with the existence of the observer in *F*; the wave solution of the Schrödinger equation is determined only and uniquely by the structure of the boundary conditions as a stationary state. In other words, only the configuration of the particle and interferometer apparatus determines the wave function. However, the wave function does not provide any particle information, as is well known. More specifically, information regarding the mechanism for transitioning from particle-like behavior is absent from this equation. Furthermore, if we continue to observe the particle in *F*, the wave-like nature does not appear, according to the principles of quantum mechanics. Rather, this mysterious behavior arises when we attempt to connect the classical and quantum worlds via a Galilean transformation, as shown in Section 2. This discussion is in fact equivalent to the discussion of the collapse of the wave function, which can be said to be the inverse process of the emergence of the wave-like properties of a particle discussed in this article. This situation simply arise when the classical and quantum spaces are intermixed, as mentioned in section 1.

According to the conventional understanding of quantum mechanics, the wave-particle duality follows the complementarity principle of Bohr. Therefore, the dependency of wave-particle duality on the reference frame may be included in the complementarity principle, but this argument is beyond the scope of this paper.

Appendix A. Illustration of Galilean transformation for classical particle and quantum wave function

The Galilean transformation of the Schrödinger equation is a very simple relation; however it arises in a complex scenario when we attempt to connect a state described by wave functions with a classical particle, as discussed in Section 2. Figure A1 schematically illustrates these relations.

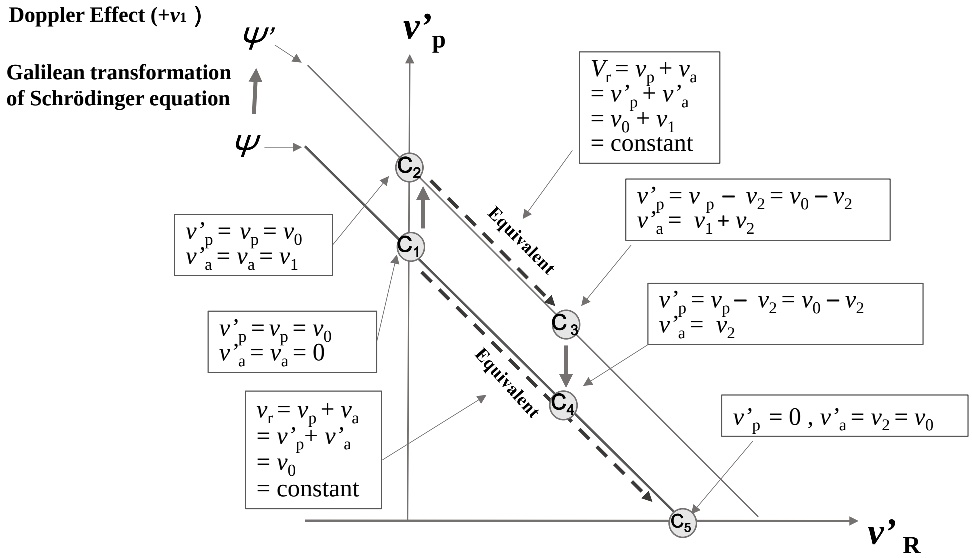

Figure A1. Schematic illustration of Table 1. The vertical and horizontal axes indicate the particle velocity in the moving reference frame and the velocity of the moving reference frame, respectively. C_N refers to case N (=1,2,3,4, or 5) from Section 2. The diagonal line signifies the same relative velocity between the particle and the interferometer; it therefore corresponds to one quantum state modeled by the wave function $\psi(x,t)$ from Equation (17). The Galilean transformation of the Schrödinger equation corresponds to a parallel shift of the diagonal line labeled as $\psi(x,t)$ to $\psi'(x',t')$. In the case of the classical particle, the velocity of the particle depends on the reference frame occupied by the observer. In case 5, the particle is at rest and the interferometer moves with velocity v_0 in the laboratory reference frame F; this is equivalent to case 1. (Rigorously, the two diagonal lines are not on the same $v'_p - v'_R$ plane.)

Appendix B. Small TLI configuration

A small TLI is required for the experiment in Section 4. Table B1 shows some possible values for achieving quite a small TLI configuration.

TLI grating pitch	De Broglie wavelength (Na atom velocity)		
	5 nm (1 m/s)	25 nm (50 cm/s)	50 nm (25 cm/s)
20 μm	80 mm	16 mm	8 mm
15 μm	45 mm	9 mm	4.5 mm
10 μm	20 mm	4 mm	2 mm
5 μm	5 mm	1 mm	0.5 mm

Table B1. Table of Talbot-Lau lengths. The Talbot-Lau length L is determined by the equation $L= d^2/\lambda$, where d is the grating pitch and λ is the wavelength [21].

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