On the Analysis of Bell's 1964 Paper by Wiseman, Cavalcanti, and Rieffel

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Abstract

In a recent series of papers Wiseman, Cavalcanti, and Rieffel have outlined and contrasted the operationalist and realist views about what we now call Bell's theorem. They also assert that Bell presented these two different versions at different times. A careful examination of the historic 1964 paper and of the writings of Bell and others that preceded it shows clearly that their interpretation of that paper is incorrect.

1 Introduction

Just over 50 years ago Bell wrote the paper[1] in which he derived the following result:

> "In a theory in which parameters are added to quantum mechanics to determine the results of individual measurements, without changing the statistical predictions, there must be a mechanism whereby the setting of one measurement device can influence the reading of another instrument, however remote. Moreover, the signal involved must propagate instantaneously, so that such a theory could not be Lorentz invariant."

A half century later, there is still contentious debate about the implications of Bell's work.

In an effort to improve communication between two 'camps' of researchers in quantum foundations, Wiseman, Cavalcanti, and Rieffel (WCR) have described two different ways in which Bell's result can be obtained[2, 3, 4][1]. They argue that one method of derivation is favored by 'operationalists', and the other by 'realists'. They also make the much more controversial historical claim that Bell, himself, presented these two different versions at different times. In their view, the earlier (1964) paper

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[1]The first paper cited was authored solely by Wiseman, the second by Wisemann and Cavalcanti, and the third, which was a response to Norsen's comment[5], was authored by Wiseman and Rieffel. The authors deal with a number of issues related to the operationalist-realist controversy. This essay is focused soley on the question of what Bell demonstrated in his 1964 paper

contains the operationalist version, and the realist version was not published until 1976[6]. Norsen has written a vigorous dissent[5] from their interpretation of the original (1964) article.

Although Norsen has already made a strong case, the historic importance of Bell's 1964 contribution makes it worth presenting some additional evidence for his point of view.

Let us begin by outlining the two derivations described by WCR. The background assumption for both of the derivations is that the quantum statistical predictions (QSP) are correct. Given this premise there are two ways in which to derive Bell's result. One of them is to assume: (a) that the proposed theory (θ) implies that the setting of one of the measurement instruments does not change the total probability of the outcomes detected by the other instrument; (b) that the proposed theory is deterministic. This method reflects the operationalist viewpoint. The other (realist) derivation assumes that the proposed theory implies that all physical processes propagate continuously through space within the forward light cone. In other words, it does not allow action-at-a-distance. Together with the background assumption (QSP), this *implies* the determinism that is assumed in the other approach.

WCR formalize premise (a) of the operationalist method as:
$P_\theta(B|a, b, c, \lambda) = P_\theta(B|b, c, \lambda)$, where P is the function that assigns probabilities to various outcomes, θ is the theory by which the probabilities are calculated, B represents the outcome of the second measurement, a and b represent the settings of the first and second measurement instruments, and c and λ represent all other (possibly hidden) variables that might be relevant. The additional assumption in the realist approach entails restrictions on the *joint* probability of outcomes, A and B, for the experimental arrangement that is involved. These restrictions can be formalized as: $P_\theta(A, B|a, b, c, \lambda) = P_\theta(A|a, c, \lambda)P_\theta(B|b, c, \lambda)$. This expression does not exhaust the meaning of the realist assumption, but it is sufficient to derive Bell's result.

Both the operationalist and the realist formal expressions assert the independence of the probability of the result of a measurement from the setting of a distant instrument. However,the weaker operationalist assumption involves only the probability of outcome B *averaged* over all possible outcomes of the other measurement, while the realist assumption imposes constraints on pairs of measurement outcomes.

In order to clearly distinguish the two sets of premises, care must be taken in choosing the terminology. The formalization of the operationalist assumption is often termed 'parameter independence' (PI), and that is how I will label it here. Wiseman, Cavalcanti, and Rieffel call it 'locality' (the term used by Bell in his 1964 paper), but the meaning of 'locality' is precisely what is at issue in the debate over the historical claim about what Bell actually proved in 1964. For the realist assumption, in later papers Bell adopted the phrase, 'local causality', but this can also cause confusion since he used the term 'causality' as a synonym for 'determinism' in earlier works, including the 1964 article. It is important to note that the realist assumption of continuous propagation does *not*, by itself, imply determinism. Although this logical point is straightforward, it is easy to lose sight of, since the continuous propagation of physical processes is so often described by deterministic equations. In later works Bell took pains to make this point explicit. To help keep the distinction clear we can phrase the realist concept in the negative as 'no action-at-a-distance'. So it will be abbreviated as NAD. The separate assumption (or implication) of determinism will be abbreviated as DET. The contradiction that Bell derived in section 4 of his paper will be indicated as XX.

So the shorthand representations of the two derivations are:
Operationalist: $QSP + PI + DET \implies XX$;
Realist: $QSP + NAD \implies DET$; $QSP + NAD + DET \implies XX$.

Since $PI + DET \implies NAD$, and $NAD \implies PI$, and $QSP + NAD \implies DET$, it is easy to see that the two sets of premises are logically equivalent. So there does not appear to be any serious dispute about whether Bell's result *could* be derived in different ways. The different derivations of the result can be viewed as two different theorems. Serious disagreement sets in with the claim that what Bell actually presented in 1964 was the operationalist derivation, and, hence, that both of these theorems are *Bell's* theorems.

I will argue here that the realist version of the argument is the one that Bell actually presented in 1964. To understand what Bell was doing in [1] consider the general structure of his argument. As indicated above, the realist version has two stages:
(a) $QSP + NAD \implies DET$;
(b) $QSP + NAD + DET \implies XX$.
The first of these is the central argument of the EPR paper[7] as understood by Bell in 1964.[2] Bell took the *validity* of this argument as being both well established and familiar to his audience. By conjoining the consequence (determinism) of (a) with the two premises and deriving a contradiction, he was able to show that this version of the EPR argument is *unsound*. In other words, either QSP or NAD is false; either quantum theory is incorrect, or there are real physical effects that are propagated outside the light cone.

In fact, Bell's demonstration is a conventional proof by contradiction. (His argument is contained mainly in section 4, entitled "Contradiction".) Consider that in virtually every proof of this sort the author states the premises, demonstrates the conflict, and concludes that one of the premises must be false. Immediately after the passage quoted above in which he states that NAD is inconsistent with QSP, he raises the possibility that QSP is wrong:

> "Moreover, the signal involved must propagate instantaneously, so that

[2]The EPR argument uses the more general premise of no disturbance of one subsystem by a measurement on the other. The more general premise is, of course, implied by the assumption of no action-at-a-distance, and it appears that nearly everyone understood this. For additional background, the reader is referred to the original article[7], Bohr's reply to it[8], Fine's Stanford Encyclopedia article[9], Einstein's later writings[10, 11], and the 1957 article by Bohm and Aharonov[12].

> such a theory could not be Lorentz invariant.
>
> Of course, the situation is different if the quantum mechanical predictions are of limited validity."

If, as Wiseman, Cavalcanti, and Rieffel assert, the premises are PI and determinism, why is it that Bell does not clearly state that one of *these* premises must be false?

In the next two sections I will present the case for the realist interpretation of Bell's 1964 paper in more detail.

2 What Bell Meant by 'Locality'

The key point in dispute centers on what Bell meant by the term 'locality'. Bell, himself, stated later quite clearly that the argument presented in his 1964 paper was the realist version. In a 1981 essay[13](p. 143)[3] he says:

> "It is important to note that to the limited degree to which *determinism* plays a role in the EPR argument, it is not assumed but *inferred*. What is held sacred is the principle of 'local causality' - or 'no action at a distance'."

A few sentences later, in a footnote, he says:

> "My own first paper on this subject[*] starts with a summary of the EPR argument *from locality* to deterministic hidden variables. But the commentators have almost universally reported that it begins with deterministic hidden variables"

In [2] (p.17) Wiseman insists that Bell's explanation of what he meant was mistaken:

> In any case, it seems that once Bell had explicitly defined LC[[i.e., the realist notion of locality]], he wished all previous localistic notions he had used, in particular the notion of locality as per Definition 9, to be forgotten. Moreover, after a few years he became convinced that it was the notion of LC that he had in mind all along."

In defending this interpretation of Bell's notion of locality, Wiseman and Rieffel also speculate[4](pp.5-6) that:

> "the word 'locality' did not enter the physics lexicon until Bell's 1964 paper."

[3]For Bell's papers that are reprinted in **Speakable and Unspeakable in Quantum Mechanics**, revised edition (2004), all page references here are to that edition.

Since the authors are inclined to discount the explanations that Bell offered later, it is necessary to examine the use of the terms 'local' and 'locality' by Bell and others that either preceded Bell's 1964 paper, or were contemporary with it. Let us recall that, before his publications on quantum foundations, Bell was highly regarded for his work in quantum field theory. In fact, his first major publication was a proof of the PCT theorem[14], which was almost simultaneous with that of Lüders[15].[4] So let us first compare Wiseman's explication of the term, 'local theory' (presented in 'Definition 9' referred to in his passage cited above), with that which Bell gave in his *1955* paper.

> Wiseman: "A theory θ is local, i.e. satisfies locality (L), iff $P_\theta(B|a, b, c, \lambda = P_\theta(B|b, c, \lambda)$, (2)plus the corresponding equation $P_\theta(A|a, b, c, \lambda) = P_\theta(A|a, c, \lambda)$ for Alice."

> Bell (1955): "A 'local' theory is one whose basic equations impose relations between the variables *and their derivatives* at each space-time point separately,..." (emphasis added)

Although Bell's 1955 paper concerns quantum field theory, his definition of 'local theory' occurs in Section 3, entitled, CLASSICAL (*C*-NUMBER) FIELDS. The reference to the derivatives clearly carries the implication of *continuous propagation* through space, which is the essence of the no-action-at-a-distance assumption.

The concept of 'local field' and its connotation of continuous propagation was very prominent in the physics community in which Bell worked. This is made evident by the discussion in the very widely used (1965) quantum field theory text authored by Bjorken and Drell[17], *Relativistic Quantum Fields*.[5] The first section(11.1) of their book is entitled *Implications of a Description in Terms of Local Fields*. The section opens with the following statement:

> "Before continuing and exploring the consequences of applying the quantization procedure to classical fields which satisfy wave equations, it is perhaps worthwhile to discuss the implications of such a program. The first is that we are led to a theory with differential wave propagation. The field functions are continuous functions of continuous parameters **x** and t, and the changes in the fields at a point **x** are determined by the properties of the fields infinitesimally close to the point **x**."

Compare this to Bell's 1990 formulation of local causality[19]:

[4]For a very nice summary of Bell's career see *The Depth and Breadth of John Bells Physics* by Jackiw and Shimony[16].

[5]This book was published in 1965 and was a companion to their earlier volume, *Relativistic Quantum Mechanics*[18] which had been published a year earlier. So this passage was almost certainly written before the publication of Bell's paper in December, 1964.

> "The direct causes (and effects) are near by, and even the indirect causes (and effects) are no further away than are permitted by the velocity of light." (Bell 1990)

As already noted, in practice the notion of continuous propagation through space is usually linked with determinism, but these concepts are logically distinct. The fact that Bell (in 1964) did not fully anticipate that others would fail to clearly see this distinction is not a reason to attribute their misunderstanding to him.

Given the relationship of the 1964 paper to the EPR argument it is also worth comparing a 1948 passage from Einstein[10]:

> "The following idea characterises the relative independence of objects far apart in space, A and B: external influence on A has no direct influence on B; this is known as the Principle of Local Action, which is used consistently only in field theory."

(It must be noted that in an alternate translation by Born the term 'Local Action' is rendered as 'contiguity'.)

The argument in favor of Bell's later account of his 1964 paper, as opposed to Wiseman's, can be further strengthened by analyzing some passages from the paper Wiseman very aptly describes as a prequel[6] to "On the Einstein-Podolsky-Rosen paradox". In his paper on hidden variables[20] Bell had shown that the requirements imposed by von Neumann[21] to rule out the possibility of adding hidden variables to quantum theory were arbitrary and unreasonable. In the last section of the paper he proposed a requirement that he considered much more natural, and in which he described the *motivation for considering theories of deterministic hidden variables.* Near the beginning of this section entitled "Locality and separability" he says:

> "... there *are* features which can reasonably be desired in a hidden variable scheme. The hidden variables should surely have some spacial significance and should evolve in time according to prescribed laws. These are prejudices, but it is just this possibility of interpolating some (preferably causal) space-time picture, between preparation of and measurements on states, that makes the quest for hidden variables interesting to the unsophisticated[*reference to the Einstein work cited three times in the opening paragraphs of the subsequent paper*]."

The phrases, "spacial significance" and "interpolating some... space-time picture between", are clearly meant to convey the idea that "Locality" (the title of the section) includes in an essential way the notion that all physical processes propagate continuously through space. The term "unsophisticated" appears to be an ironic reference to Einstein and his resistance to the "orthodox" interpretation promoted by Bohr[8, 22] and Heisenberg[23, 24].

[6]The paper on hidden variables was written prior to the EPR paper, but published later.

In fact, the final sentence in the quotation can be read as a very brief summary of the EPR argument. The possibility of maintaining a picture in which physical processes propagate continuously within the light cone is what leads "unsophisticated" people like Einstein (and Bell himself) to consider a theory of hidden variables, since without such variables one is forced to accept action-at-a-distance. Note the parenthetic phrase, "*preferably* causal". 'Causal' is being used here, as in the subsequent, paper as a synonym for 'deterministic'. The qualifier, "preferably" indicates that Bell does not insist on a deterministic account; it is just that it is the only way to save Einstein locality and still reproduce the perfect correlations discussed by EPR.

It is only after explaining why *Einstein's principle of locality* (i.e., no action-at-a-distance) requires deterministic hidden variables that Bell goes on to discuss Bohm's theory[25], and to characterize it as "grossly non-local". Later in the section he clearly foreshadows his argument in [1].

> "...in general, the wave function is not factorable...the disposition of one piece of apparatus affects the results obtained with a distant piece. In fact the Einstein-Podolsky-Rosen paradox is resolved in the way which Einstein would have liked least [reference to the same Einstein passage referred to earlier]".

There are three notable points in this brief passage. First, he explains how the nonlocality originates in the nonfactorability of the wave function. This leads immediately to the nonfactorability of the probabilities, which is central to the argument in [1]. Second, he cites the effect of the setting of the measurement instrument as the unambiguous experimental signature of the nonlocality. This issue will be examined in more detail in the next section. Third, and most important for the realist/operationalist argument, is the way in which Bell emphasizes the violation of Einstein locality as the most striking consequence of Bohm's theory. This emphasis anticipates the primary conclusion of the subsequent paper in which he shows that *any* theory reproducing the quantum statistical predictions must also violate this realist version of locality:

> "Moreover, the signal involved must propagate instantaneously, so that such a theory could not be Lorentz invariant."

Given the weight of the evidence that Bell was correct in his later assertion that the realist notion of locality was the key premise of his 1964 proof, why do WCR insist so strongly on a conflicting interpretation? Their principal arguments are addressed in the next section.

3 Bell's Unambiguous Criterion for Violating 'Locality'

The principal evidence for Wiseman's claim that Bell *defined* 'locality' as PI, $P_\theta(B|a,b,c,\lambda) = P_\theta(B|b,c,\lambda)$, consists of four passages from [1]. (He also offers a number of additional arguments that will be reviewed later.)

(1) "It is the requirement of locality, or more precisely that the result of a measurement on one system be unaffected by operations on a distant system with which it has interacted in the past, that creates the essential difficulty." (p.14)

(2) "Now we make the hypothesis[*], and it seems one at least worth considering, that if the two measurements are made at places remote from one another the orientation of one magnet does not influence the result obtained with the other." (p.14/15)

(3) "The vital assumption[*] is that the result B for particle 2 does not depend on the setting **a** of the magnet for particle 1, nor A on **b**." (p.15)

(4) "In a theory in which parameters are added to quantum mechanics to determine the results of individual measurements, without changing the statistical predictions, there must be a mechanism whereby the setting of one measurement device can influence the reading of another instrument, however remote." (p.20)

Note first that nowhere does Bell use the term, 'define' or a close equivalent, or write down any expression that resembles PI. (The phrase in the first passage, "or more precisely that..." is used to point out a particular consequence of locality - not to present a definition.) So Wiseman's claim is based on his insistence that the terms, '(un)affected', 'influence', and 'depend' *must* be understood as implying that the *total* probability of an outcome of a measurement made on one branch of an entangled system is altered by changes in the *setting* of the measurement apparatus that acts on the other branch. This interpretation rests essentially on the fact that, in all of these passages, Bell refers to the dependence of the outcome of the second measurement *exclusively* on the setting of the first instrument, rather than on *both* the setting of the instrument and the outcome. In Wiseman's view the fact that Bell never explicitly mentions dependence on the outcome of the distant measurement, means that one cannot take the expression involving joint probabilities, $P_\theta(A, B|a, b, c, \lambda) = P_\theta(A|a, c, \lambda)P_\theta(B|b, c, \lambda)$, as being implied by the notion of locality. So we are left with PI: $P_\theta(B|a, b, c, \lambda) = P_\theta(B|b, c, \lambda)$.

One can answer Wiseman's principal argument by explaining the reason for Bell's phrasing. The first thing to note is that the first three passages (which occur early in the paper) are phrased in the negative. Local theories, according to Bell, must *not* allow the setting of the first apparatus to influence the outcome of the second measurement. In the fourth passage, which forms part of his conclusion, Bell is stating that any proposed local (and, *hence*, deterministic) theory consistent with QSP fails this test. Therefore, such theories are impossible.

The reason that Bell focuses solely on the setting of the apparatus is that it is the one aspect of the experimental arrangements envisioned that can be placed *unambiguously* outside the past light cone of the second measurement. To see this, recall that there were two theories with which Bell was thoroughly familiar that could reproduce all of the statistical predictions of quantum theory. The first of these was orthodox quantum mechanics; the other was Bohmian theory[25]. In orthodox quantum mechanics the outcome of a measurement on one of a pair of entangled systems is influenced by two factors that are not necessarily connected to anything in the past light cone of that measurement outcome: (1) the setting of the distant instrument *and* (2) the outcome of the distant measurement.[7] In Bohm's theory, however, it is *only* the setting of the instrument that can be placed strictly outside the past light cone of the second measurement. The trajectory of the first particle (and, hence, the deterministic result of the measurement) is heavily influenced by the previous interaction that generated the entanglement with the second system, and this interaction is clearly *within* the past light cone of the second measurement. So inclusion of possible influences of the *result* of the first measurement would have involved a complicated mix of factors affecting the second outcome and might have detracted from the clarity of Bell's result. Thus, it was entirely natural, or even essential, for Bell, in searching for a clear-cut test to demonstrate the nonlocality of quantum theory, to frame the issue in terms of the influence exerted by the setting of the first measurement apparatus. This point can be driven home by considering the closing paragraph of the 1964 paper in which he describes specific experimental tests.

> "Of course, the situation is different if the quantum mechanical predictions are of limited validity. Conceivably they might apply only to experiments in which the settings of the instruments are made sufficiently in advance to allow them to reach some mutual rapport by exchange of signals with velocity less than or equal to that of light. In that connection, experiments of the type proposed by Bohm and Aharonov[*], in which the settings are changed during the flight of the particles, are crucial."

In further support of this explanation for his choice of phrasing, recall that Bell was hugely influenced by Bohm's theory[25]. It was Bohm's theory (apparently) that first convinced him that von Neumann's no-hidden variable theorem[21] was seriously flawed, and, after recognizing the nonlocal nature of the theory, he had spent a great deal of effort in trying to construct a version without this problematic feature. It was largely this effort that led him to his 1964 theorem. With this background, it is entirely understandable that he emphasized how the *setting* of one measurement instrument influences the outcome of the distant measurement, since this is the one

[7] "Orthodox" quantum mechanics can be understood in either of two ways, but this statement holds true in both of them. In the "von Neumann" version the wave function, regarded as a genuine physical entity, undergoes a collapse that spans a spacelike interval. Viewed in this way, the state to which the wave function collapses (and, hence, the outcome of the second measurement) is *nonlocally influenced* by both the setting of the measurement instrument, which reduces the number of possible resultant states from infinity to two, and the outcome of the first measurement, which determines which of the two possibilities is realized. In the operational or "Bohr-Heisenberg" version there is nothing physical for the measurement setting, by itself, to influence. The only meaningful influences are on experimental outcomes, and these are influenced (nonlocally) by both the setting of the first instrument and by the outcome of the first measurement.

clearly identifiable feature that is not influenced by events in the past light cone of the distant measurement.

As Wiseman readily acknowledges, Bell was a confirmed realist. Given the explanation for why Bell chose the specific phrasing that he did, it is very implausible to force a strictly operationalist interpretation of terms such as 'influence' and 'locality' onto his statements.

So the first three passages cited above should be read, not as a definition of 'locality', but as a completely unambiguous criterion for ascertaining whether the principle of locality is violated. The final passage is the statement that any theory consistent with QSP violates this criterion.

In his comment Norsen[5] has also argued that Bell's phrasing amounts to stating a criterion for violating locality, rather than offering a definition. He has made a number of other compelling points that are worth reviewing here. These concern Bell's brief introductory section and the first two paragraphs of his section 2 (Formulation). The first three passages cited above are all contained in this portion of the paper. As Norsen points out, in all three of these passages Bell refers to a principle enunciated by Einstein regarding the issues of locality and separability[11]:

> "But on one supposition we should, in my opinion, absolutely hold fast: the real factual situation of the system S_2 is independent of what is done with the system S_1, which is spatially separated from the former."

These references are indicated by the footnotes ([*]) in the passages. The footnote for the first passage modifies a use of 'locality' just prior to the text quoted. In the third paper[4] Wiseman and Rieffel use the term "interruption" for these references, and describes them as appeals to authority. They argue that it would have been better to omit them in order to "improve the grammatical and scientific clarity of the sentence". But they cannot rewrite Bell's paper in order to eliminate portions that conflict with their interpretation of it. Bell referred to the same quotation from Einstein three times in the first page and a half of his paper, directly qualifying the three critical passages that have been cited above. Recall from the earlier discussion that there were *two other references to the same passage* in the prequel[20] in the final section entitled "Locality and separability". Obviously, Bell was trying to convey to the reader the critical connotations of his notion of locality.

Norsen also points out that Wiseman's interpretation of 'locality' as PI is inconsistent with Bell's statement at the beginning of the paper that additional (i.e.., hidden) variables were needed to *restore* locality to quantum theory. (Maudlin has also made this point as reported by Wiseman in [2]). PI is a central feature of quantum mechanics; it is what prevents superluminal signaling within the theory. It is very difficult to believe that Bell was unaware of such a basic property of quantum theory, or that he thought that Einstein was unaware of it.

To a large extent, the dispute about what Bell proved in 1964 centers on the adequacy of his recapitulation of the EPR argument in the opening paragraph of his second section, "Formulation". Because it is crucial to the issue at hand it is worth quoting in full.

> "With the example advocated by Bohm and Aharonov[*], the EPR argument is the following. Consider a pair of spin one-half particles formed somehow in the singlet state and moving freely in opposite directions. Measurements can be made, say by Stern-Gerlach magnets, on selected components of the spins σ_1 and σ_2. If measurement of the component $\sigma_1 \cdot a$, where a is some unit vector, yields the value +1 then, according to quantum mechanics, measurement of $\sigma_2 \cdot a$ must yield the value -1 and vice versa. Now we make the hypothesis[[reference to Einstein quotation]], and it seems one at least worth considering, that if the two measurements are made at places remote from one another the orientation of one magnet does not influence the result obtained with the other. Since we can predict in advance the result of measuring any chosen component of σ_2, by previously measuring the same component of σ_1, it follows that the result of any such measurement must actually be predetermined. Since the initial quantum mechanical wave function does not determine the result of an individual measurement, this predetermination implies the possibility of a more complete specification of the state."

The general form of the argument has already been briefly summarized above. Given the statistical predictions of quantum theory, there are systems with physical quantities that are not determined by the theory, but which can be precisely ascertained by making measurements on entangled partner systems that are spacelike separated. If we assume that there is no action-at-a-distance then these quantities must be determined by events in the common past of both systems. Since quantum theory does not provide specific values for all of these quantities it is incomplete. (A complete theory would yield values for all such quantities, and would, therefore, be deterministic.)

For the reasons given above Wiseman insists that Bell is not using Einstein's notion of locality here (no action-at-a-distance), but rather the much weaker assumption of parameter independence. Therefore, Wiseman concludes that Bell is mistaken in assuming that determinism follows from the stated premises. In other words, despite the fact that he states that he is summarizing the EPR argument, Bell, without fully realizing it, substitutes a weaker premise for Einstein's concept of locality, and then, without fully realizing it, makes an assumption of determinism rather than simply restating an established result.

Without a truly compelling reason for accepting Wiseman's interpretation of 'locality', his reading of this paragraph appears exceptionally strained and artificial. We have already seen that his reason for attributing the operationalist meaning of 'locality' to Bell evaporates on closer examination, and also that Bell had on several previous occasions used the terms 'local' and 'locality' as essentially synonymous with

Einstein's principle of no action-at-a-distance. But, since Norsen has also stated that this very brief discussion could benefit from a more general description of 'locality', it will be helpful to view Bell's short summary of EPR against the most relevant background.

As Bell states, he was working from the version of the EPR "paradox" that had been presented by Bohm and Aharonov[12] in 1957 (only seven years before his paper). The closing paragraphs of their introductory section summarize the EPR argument and apply it to their proposed experimental arrangement. This straightforward exposition should make it clear why Bell did not feel compelled to repeat it at greater length.

> "One could perhaps suppose that there is some hidden interaction between B and A, or between B and the measuring apparatus, [[which measures A]] which explains the above behavior. Such an interaction would, at the very least, be outside the scope of current quantum theory. Moreover, it would have to be instantaneous, because the orientation of the measuring apparatus could very quickly be changed, and the spin of B would have to respond immediately to the change. Such an immediate interaction between distant systems would not in general be consistent with the theory of relativity.
>
> This result constitutes the essence of the paradox of Einstein, Rosen, and Podolsky."

This passage clearly describes the apparent conflict between the predictions of quantum theory and the principle of no action-at-a-distance. It also makes obvious the concern that violations of this principle would occur if the *setting* of one of the measurement instruments (which could be changed while the entangled particles were in flight) affected the distant measurement.

Before addressing his remaining arguments, let us summarize the case concerning Wiseman's claim that Bell *defined* 'locality' as PI. First, in the 1964 paper Bell never used the term 'define' in connection with locality or wrote down any expression resembling PI. Wiseman's primary evidence for his assertion consists solely in his own purely operational interpretation of the terms, '(un)affected', 'influence', and 'depend', and in the fact that Bell restricted his discussion of nonlocal effects to those involving the setting of the measuring instruments, rather than measurement outcomes. I have argued that Bell's restrictive language was careful and well-motivated. It was intended to maintain the clarity of his argument by referring only to those factors that were unambiguously outside the past light cone of problematic measurement outcomes. Furthermore, Bell was certainly not an operationalist, and given the explanation for his restrictive language, there is no reason to adopt a purely operationalist interpretation of very general terms such as 'influence' and 'affect'. Acceptance of Wiseman's assertion forces one into an extremely tortured reading of Bell's summary of the EPR argument. It also forces one to ignore Bell's repeated references to the passage from Einstein regarding his version of locality. Bell referred to this passage three times in the first page and a half of his article (qualifying all three of the passages that Wiseman cites as evidence), in addition to the two references that he had made in the closing section of the previous paper that serves as a prequel to "On the Einstein-Podolsky-Rosen paradox". Bell apparently saw himself as addressing an audience that was familiar with the EPR argument, and that had an understanding of 'locality' that was consonant with that of Einstein. The term, 'local field' was in standard use, and carried the clear connotation of continuous propagation through space. And Bell makes it clear in the prequel, in the section entitled 'Locality and separability' that it is the "possibility of interpolating some (preferably causal) space-time picture, between preparation of and measurements on states" that generates the interest in hidden variable theories. Clearly, Bell was employing Einstein's concept of locality.

We can now turn to the other arguments offered by WCR (summarized on page 10 of [2] and repeated in [4]). In reference to the interpretation just offered Wiseman says:

> "To me, the advantages of this reading are demonstrably outweighed by its many disadvantages: i) it does not explain why Bell would, in 1964, state his result *four times* as requiring two assumptions, locality and determinism, and not once as requiring onlythe assumption of locality; ii) it does not explain why in his first subsequent paper on the topic of hidden variables [*], after seven years to think about how best to explain his result, he still states it (somewhat redundantly) as being that no local deterministic hidden-variable theory can reproduce all the experimental predictions of quantum mechanics [*]; iii) it does not explain why Bell would, in 1964, define locality *four times* in terms of independence from the remote setting, as per Definition 9, andnever any other way; iv) it does not explain why Bell would state the conclusion of the supposedly crucial first part of his theorem as being merely that it implies the possibility of a more complete specification of the state.; v) it does not explain why Bell would place this supposedly crucial first part *prior* to the mathematical formulation of his result, and not mention it anywhere else in the paper."

The third point has been dealt with extensively above. Let us consider point (i). The four passages that Wiseman is referring to are:

> "In this note that idea [[causality and locality]] will be formulated mathematically, and shown to be incompatible with the statistical predictions of quantum mechanics." (p.14)

> "This is characteristic, according to the result to be proved here, of any such theory [[like Bohm's hidden variable theory]] which reproduces exactly the quantum mechanical predictions." (p.14)

> "the statistical predictions of quantum mechanics are incompatible with separable predetermination."(p.20)

> "In a theory in which parameters are added to quantum mechanics to determine the results of individual measurements, without changing the statistical predictions, there must be a mechanism whereby the setting of one measurement device can influence the reading of another instrument, however remote. " (p.20)

The first of these passages is the third sentence of "On the Einstein-Podolsky-Rosen paradox". The two opening sentences that precede it are:

> "The paradox of Einstein, Podolsky, and Rosen[*] was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables. These additional variables were to restore to the theory causality and locality[reference to Einstein's 1949 passage]."

Again, Bell is taking the EPR argument as an established result. That argument proceeded from the no-disturbance assumption implied by Einstein locality (NAD), and the limited set of quantum predictions that involve perfect correlations between outcomes of spacelike-separated entangled systems. It *concluded* that a hidden-variable (deterministic) theory was required. Locality (i.e., Einstein locality) and causality (i.e., determinism) are *premises* of Bell's argument; Wiseman fails to distinguish between premises and assumptions. The first of these premises, Einstein locality, is an assumption, but the second, determinism, is a property that follows from (Einstein) locality and the other assumptions of the EPR argument. This point can be equally well applied to the other three passages mentioned by Wiseman.

The same answer can be directed to Wiseman's point (ii). But, in addition, the paper that Wiseman cites[26] includes statements that clearly contradict the interpretation that he is attempting to construct. In it Bell says (p. 36)[8]:

> "But our notion of locality requires that A does not depend on $\widehat{b}$ nor B on $\widehat{a}$. We then ask if the mean value $E(\widehat{ab})$ of the product AB, i.e.,
> $E(\widehat{ab}) = \int(d\lambda\rho(\lambda)A(\widehat{a},\lambda)B(\widehat{b},\lambda))$ (5)
> can equal the quantum mechanical prediction."

Note that this expression involving the joint probability, which is identical to the formulation that Bell used in "On the Einstein-Podolsky-Rosen paradox", is presented as a consequence of *locality* - not of locality and determinism. Bell then immediately presents a slight generalization of this expression: $E(\widehat{ab})$ of the product AB, i.e.,

[8] I have made a slight change in Bell's notation. Bell uses $'P'$ for expectation values, which I have changed to $'E'$ in order to avoid confusion with the expressions for probability.

” $E(\widehat{a}\widehat{b}) = \int(d\lambda\rho(\lambda)\overline{A}(\widehat{a},\lambda)\overline{B}(\widehat{b},\lambda))$ (6)”

where the overlines on A and B are intended to indicate averages over possible variations in the values of A and B. These variations might be attributable to hidden variables specifically affecting the measurement instruments, OR as Bell adds in footnote 10:

> ”Clearly (6) is appropriate also for *indeterminism* with a certain local character.”

So Bell states explicitly that the expression involving joint probabilities follows from the assumption of locality without any assumption of determinism. This makes sense for Einstein's notion of locality, but not for PI.

Wiseman's fourth point appears to be completely lacking in substance. It refers to Bell's brief recapitulation of the EPR argument in the opening paragraph of Section 2. This paragraph was quoted in full above. For some reason, Wiseman fails to see that the conclusion of Bell's EPR description consists of two sentences - not just one. In the penultimate sentence Bell states very clearly that the need for a deterministic theory *”follows”* from the assumptions of no action-at-a-distance and the perfect correlations predicted by quantum theory in the type of experiment that he discusses. In the final sentence he notes the EPR conclusion that such a theory would involve a more complete specification of the state. The inference to determinism is an essential premise of Bell's argument. Bell closes this recapitulation by describing the hope of 'completing' the theory in a manner that allows one to save Einstein locality because this is what his subsequent demonstration will show to be impossible.

Regarding point v) Wiseman apparently believes that in order to show that the quantum statistical predictions conflict with Einstein locality, Bell would have needed to present an explicit logical formalization of the *premises* of the EPR argument, rather than simply incorporating the EPR *results* into his mathematical formulation. As already discussed, Bell assumed that his audience was familiar with the EPR argument. He also assumed that the failure of standard quantum theory to account for the perfect correlations discussed in EPR without violating the intuitive idea of no action-at-a-distance was sufficiently obvious that a more precise formalization of Einstein locality was not required. It was because of this fairly obvious clash between the intuitive notion and the quantum predictions that both Bell and Bohm-Aharonov used the the term *”paradox”* in their titles in reference to the EPR argument.

4 Summary

Wiseman, Cavalcanti, and Rieffel deal with a number of differences between operationalist and realist perspectives, and, in particular, the various meanings that can be attached to the term 'locality'. These issues are well worth exploring, but the only question that is being addressed in this comment concerns what Bell demonstrated in his 1964 paper.

The realist interpretation of Bell's argument was outlined in the introduction:
(a) (EPR argument) $QSP + NAD \implies DET$;
(b) (Bell's argument based on EPR) $QSP + NAD + DET \implies XX$.
The operationalist version of Bell's argument was represented as:
(b') $QSP + PI + DET \implies XX$.

Since (a), the EPR argument, was regarded as an established result, Bell did not attempt to formalize it. What was "formulated mathematically" was a *consequence of the conjunction* of the premises of (b) or (b'). Since the conjunctions of the premises of (b) and (b') are (essentially) logically equivalent, this does not tell us whether Bell's use of the term 'locality' should be formalized as parameter independence or as no action-at-a-distance. The fact that determinism (DET) is used as a *premise* in both versions fully explains why Bell refers to it in stating his conclusion. So the reference to it cannot be used in support of the operationalist interpretration. In fact, as Norsen has pointed out, the fact that the paper is entitled "On the Einstein-Podolsky-Rosen paradox" argues strongly that Bell was taking determinism as a consequence of the EPR argument - not as an independent assumption.

So the case for interpreting 'locality' as parameter independence turns entirely on the fact that Bell restricted his discussion of possible nonlocal effects to potential influences by the *setting of a measurement instrument* on a distant measurement outcome. The insistence that nonlocal effects must take into account both the setting of an instrument and the result of the measurement made with that instrument appears to result from a failure to see that standard quantum theory was not the *only* theory considered by Bell that was capable of reproducing the quantum statistical predictions. In Bohm's theory, which Bell had studied in great depth, the result of a measurement depends on a combination of factors, some of which are inside the past light cone of the distant measurement, and some of which are outside. The only event that can be placed clearly outside the past light cone is the setting of the instrument.[9]

So there is no real basis for interpreting Bell's use of 'locality' in operationalist terms. In contrast, the case for reading 'locality' as no action-at-distance is very strong. Bell's opening remarks that an extension of quantum theory would "*restore* to the theory ...locality", make perfect sense, as does his recapitulation of the EPR argument (particularly against the background of the Bohm-Aharonov discussion). One does not need to argue away the three references to Einstein's 1949 passage that qualify his discussions of locality. His prior use of the terms 'local' and 'locality' (which was consistent with the contemporary understanding) carried the clear connotation of continuous propagation through space. Finally, we should consider the manner in which Bell concluded his analysis of the EPR argument - by emphasizing the violation of *Einstein locality.*

[9]The assumption that the setting *can* be securely placed outside the past light cone of the second measurement depends on the denial of *superdeterminism.* This is a point that Bell, later[19], readily acknowledged.

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