Towards an Objective Physics of Bell Non-Locality: Palatial Twistor Theory

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Abstract In 1964, John Stewart Bell famously demonstrated that the laws of standard quantum mechanics demand a physical world that cannot be described entirely according to local laws. The present article argues that this non-locality must be gravitationally related, as it comes about only with quantum state reduction, this being claimed a gravitational effect. A new formalism for curved space-times, *palatial* twistor theory is outlined, which appears now to be able to accommodate gravitation fully, providing a non-local description of the physical world.

1. Non-locality and quantum state reduction

Whereas *quantum entanglement* is a normal consequence of the unitary (Schrödinger) evolution **U**, of a multi-particle quantum system, this evolution is nevertheless *local* in the sense that it is described as the continuous (local) evolution in the relevant configuration space. What the EPR situations considered by John Stewart Bell **[1]** demonstrated was than when widely separated quantum *measurements*, of appropriate kinds, are performed on such entangled states (a current record for distance separation being 143km, for entangled photon pairs **[2]**), the results of such measurements (which may be probabilistic or of a yes/no character) cannot be mimicked by any local realistic model. Thus, in trying to propose a mathematical modelling of what is going on *realistically* in the physical world, one would need to face up to the actual physical process involved in quantum *state reduction* **R**, which is an essential feature of quantum measurement.

For many years, my own position on the state-reduction issue has been that **R** would have to involve a fundamental mathematical extension of current quantum theory, giving a description of something *objectively taking place* "out there" in the physical world (**OR**: *objective* reduction), rather than **R** being the effect of some kind of "interpretation" of the standard unitary quantum formalism. Moreover, the extended theory should be able to describe the *one* physical world that we all experience, rather than some sort of co-existing superposition of vast numbers of alternatives. To be more specific about my own viewpoint, it has long been my position that such necessary *deviation* from unitarity **U** in this **OR** activity would result from a correct melding of the quantum formalism with that of *general relativity*. Unlike standard attempts at a quantum gravity theory, this would entail some accommodation on the part of *both* quantum mechanics *and* gravitational theory, involving some kind of "gravitization" of quantum mechanics **[3]**, in addition to the conventional viewpoint of there being some needed quantum modification of the classical picture of space-time.

In my opinion, there are several reasons for believing that standard **U**-evolution cannot remain completely true when the effects of Einstein's general relativity become significantly involved. For example, there is the so-called "black-hole information paradox". According to Hawking's 1976 analysis **[4]**, information would be lost as an aspect of black-hole evaporation, this entailing a deviation from standard unitary evolution **U**. Although Hawking subsequently reversed his opinion (see **[5]**), I believe that his original argument is the more cogent one (and, in my view, information loss is also a clear implication of an examination of the appropriate conformal diagrams; see **[6]**, especially Fig.30.14), so that **U** in must in any case be violated in such extreme gravitational situations. Otherwise, one is led to unpleasant and improbable-sounding conclusions such as "firewalls" instead of horizons **[7]**. Moreover, the very curious way in which the Big Bang must have been an extraordinarily special initial state, as implied by the second law of thermodynamics, also sheds doubt on the conventional presumption that that event can have been the result of a standard quantum-gravity type evolution, since *gravitational* degrees of freedom were enormously suppressed at the Big Bang, this applying *only* to gravitation, the matter and radiation degrees of freedom having apparently been almost completely thermalized (see **[6]**, Chapter 27).

In addition to these very, large-scale phenomena, fundamental issues are raised when quantum superpositions of even very tiny gravitational fields are involved **[8]**, **[9]**. It turns out that Einstein's foundational *principle of equivalence* (between a local gravitational field and an acceleration) is in conflict with the standard linear **U**-evolution even in the case of the Newtonian limit of general relativity. For it can be shown that such superpositions are, strictly speaking *illegal*, according to the quantum formalism, as the accelerating frames relevant to the two components of such a superposition refer to *different vacua* **[3]**. The specific type of scheme that I have been promoting, in order to handle this conflict, suggests a particular scale for an **OR**-type process. All this agrees, fairly closely, with an earlier proposal introduced by Lajos Diósi **[10]** (see also **[11]**) but now with clear motivations from foundational principles of general relativity, most particularly the Galilean limit of the equivalence principle (see **[3]**). According to this proposal, a macroscopic quantum superposition between two quantum states **A** and **B**, in the (Newtonian) $c=\infty$ limit would undergo a spontaneous reduction (**OR**) of the superposed state to one of **A** *or* **B**, in a timescale $\tau = \sim \hbar/E_G$, where E_G is the gravitational self-energy of the mass distribution (positive in some regions and negative in others) of the *difference* between the distributions in **A** and **B** separately **[3]**, **[8]**, **[9]**. Here we assume that each of **A** and **B** would, on its own, have been completely *stationary*. In space-time terms, we have a quantum superposition of two different space-time geometries that persists only for a time $\sim\tau$, where the total space-time separation (superposition) period, before reduction takes place, would be of order *unity*, in Planckian units **[3]**, the separation of the space-time geometries being given in terms of a symplectic measure that can be explicitly given in the linearized limit **[8]**.

2. Twistor non-locality and its basic algebra

Among the principle motivational ideas behind the original introduction of *twistor theory* **[12]** was the feeling that one should seek a description of the physical world that would be fundamentally *non-local*. I had hit upon twistor theory's initial notions in late 1963, before I had had the advantage of knowing John Bell's remarkable demonstration that the effects of conventional quantum mechanics cannot actually be explained in terms of a local realistic model. Yet, I had already felt that there were some reasonably persuasive indications from the earlier work of Einstein, Podolski and Rosen (EPR) **[13],** and David Bohm **[14]**, that some kind of spatial non-locality must be true of the real world, although we were certainly not *forced* into this viewpoint until Bell's famous result appeared in 1964.

The kind of non-locality exhibited in the original twistor viewpoint was, however, rather limited. The idea was that space-time points should not be thought of as primitive entities, but *secondary* to the non-local notion of a space-time *light ray*—henceforth referred to simply a *ray*—namely a complete null geodesic in the *classically* viewed space-time manifold ℏ . Clearly a ray *is* a *non*-local entity, representing the entire history of an idealized freely moving classical massless (and spinless) particle. The 5-real-dimensional space ℏ𝒩 of all such rays in ℙ is Hausdorff, provided that we can assume that ℙ is globally hyperbolic **[15]**. Any point **r** of ℙ may be identified within ℙ𝒩 (with perhaps some ambiguity, for certain very special ℙ s) as the sphere **R** of rays that pass through **r**. This S^2 (basically the "celestial sphere" of an observe at **r**) has the structure of a *conformal* sphere, and one of the defining aspects of twistor theory was to try to regard **R** as a *Riemann sphere* (complex projective line) within the structure of ℙ𝒩.

However, for a general ℙ , the real 5-manifold ℙ𝒩 does not possess anything directly of the nature of the *complex* structure that would enable such an interpretation of the 2-sphere **R** to be inherited from an ambient complex structure, of some sort, within ℙ𝒩 itself. Nevertheless, the 6-manifold 𝕡𝒩 of *momentum-scaled* rays is actually a *symplectic* manifold, where a momentum scaling assigns a (space-time) *null* convector **p** at each point of the ray, written in index form as p_a, which points along the ray in the future direction and is parallel-propagated along it. The symplectic structure of 𝕡𝒩 is defined by the closed symplectic 2-form $\mathbf{\Sigma}$, and a natural symplectic potential 1-form $\mathbf{\Phi}$, given by

$$\mathbf{\Sigma} = \mathrm{d}p_a \wedge \mathrm{d}x^a = \mathrm{d}\mathbf{\Phi}, \quad \text{where} \quad \mathbf{\Phi} = p_a\, \mathrm{d}x^a,$$

(abstract indices being used throughout **[16]**). In conventional coordinate notation, "$\mathrm{d}x^a$" would simply stand for the coordinate 1-form basis; in abstract indices $\mathrm{d}x^a$ is just a "Kronecker delta" translating the abstract index on "p_a" to a conventional 1-form notation. We shall be seeing in §3 that, in some sense, it is the process of *canonical quantization* (in the guise of geometric quantization **[17]**), when applied to the symplectic structure $\mathbf{\Sigma}$ (via its associated Poisson-bracket structure $\mathbf{\Sigma}^{-1}$), that gives quantum significance to this Riemann sphere interpretation. But we shall find that there are considerable subtleties about such a quantization procedure which, as it turns out, cannot be applied globally in the appropriate way to the whole 6-space 𝕡𝒩, when ℙ is conformally curved. The global inconsistency of this quantization process is the key to a new notion in twistor theory known as *palatial twistor theory* **[18]**, which will be described in outline in §6 and which offers some hope for a fundamentally non-local description of general curved 4-dimensional space-times.

When ℙ is *conformally flat*, and most particularly in the case ℙ =𝕄 where 𝕄 is Minkowski 4-space, we do not need to appeal to quantization procedures, and we find that the required complex structure is *already* provided by classical theory, when looked at in the appropriate way. This interpretation is explicit, as we shall see below, and (at least locally) ℙ𝒩 can then be identified as a 5-dimensional real hypersurface

$\mathbb{PN}$ in a complex space $\mathbb{PT}$ (which is a complex projective 3-space, $\mathbb{CC}^3$) referred to as *projective twistor space*. Thus, $\mathbb{CN}$ becomes what is referred to as a *CR-manifold* (Cauchy-Riemann or complex-real manifold **[19]**).

The real 5-space $\mathbb{CN}$ divides $\mathbb{CT}$ into two halves $\mathbb{CT}^+$ and $\mathbb{CT}^-$. In each case, the prefix "$\mathbb{C}$" refers to "projective" (i.e. all non-zero complex multiples being projected to a single point), and there is a *non*-projective version, Œspectively, e , $\mathbb{T}^+$, and $\mathbb{T}^-$ of each (see Fig.1). The space $\mathbb{T}$, referred to simply as "twistor space" (or sometimes as "*non*-projective twistor space") is a 4-dimensional complex vector space (zero included) with *pseudo-Hermitian metric form*, of split signature (+,+,–,–). The sub-regions $\mathbb{T}^+$, $\mathbb{T}^-$, and $\mathbb{N}$, of $\mathbb{T}$, are defined, respectively, by the metric form taking positive, negative and zero values. A *twistor* **Z**, sometimes written in abstract-index form as Z^α is an element of $\mathbb{T}$, and in standard coordinates (Z^0,Z^1,Z^2,Z^3) for $\mathbb{T}$, the metric form is

$$||\mathbf{Z}|| \;=\; Z^0\overline{Z^2} + Z^1\overline{Z^3} + Z^2\overline{Z^0} + Z^3\overline{Z^1} \;=\; Z^\alpha\bar{Z}_\alpha \;=\; \mathbf{Z}\bullet\bar{\mathbf{Z}}$$

where the complex conjugate $\bar{\mathbf{Z}}$ of the twistor **Z** is a *dual* twistor, i.e. element of the dual twistor space $\mathbb{T}^*$, and written in abstract-index form as $\bar{Z}_\alpha$, and in standard coordinates, its components are $(\bar{Z}_0,\bar{Z}_1,\bar{Z}_2,\bar{Z}_3) = (\overline{Z^2},\overline{Z^3},\overline{Z^0},\overline{Z^1})$.

If $||\mathbf{Z}||=0$, we call **Z** a *null* twistor, and it represents a *ray* in $\mathbb{M}$. To see this explicitly, we need the fundamental *incidence relation* between $\mathbb{M}$ and $\mathbb{T}$ given by

$$\begin{pmatrix} Z^0 \\ Z^1 \end{pmatrix} \;=\; \frac{\mathrm{i}}{\sqrt{2}}\begin{pmatrix} t+z & x+\mathrm{i}y \\ x-\mathrm{i}y & t-z \end{pmatrix}\begin{pmatrix} Z^2 \\ Z^3 \end{pmatrix}$$

where (t,x,y,z) are standard Minkowski space-time coordinates for $\mathbb{M}$ (with c=1) for a point **r**, either in $\mathbb{M}$ or its *complexification* $\mathbb{CM}$. When **Z** is a *null* twistor, the real points **r** which are incident with **Z** (i.e. whose coordinates satisfy the incidence relation with Z^α) are simply the points that constitute a *ray* **z** in $\mathbb{M}$ (or at infinity, in the conformal compctification $\mathbb{M}^\#$ of $\mathbb{M}$, if $Z^2=Z^3=0$). When **Z** is not null, then there are no real points incident with **Z**, but there are *complex* solutions for (t,x,y,z), giving points of $\mathbb{CM}$. We may see, from the incidence relation, that the rays determined by the null twistors **Y** and **Z** *intersect* (possibly at infinity, in $\mathbb{M}^\#$), if and only if $\bar{\mathbf{Z}}\bullet\mathbf{Y}$ ($=\bar{Z}_\alpha Y^\alpha$) vanishes. If we fix **r** as a point of $\mathbb{CM}$, then the solutions for **Z** of the incidence relation give us a complex projective line (a Riemann sphere) in the projective 3-space $\mathbb{PT}$. If **r** is a real point (point of $\mathbb{M}$, or even of $\mathbb{M}^\#$), then this Riemann sphere lies in $\mathbb{PN}$, in accordance with the above comments concerning $\mathbb{P}\mathcal{N}$. For more details, see **[20]**, **[21]**, **[22]**.

3. Spinor parts and physical interpretation of a twistor

The 4 components of a twistor Z^α, in a standard frame, may be identified as the pairs of components of two *2-spinors* ω^A and $\omega_{A'}$, according to

$$Z^0 = \omega^0,\;\; Z^1 = \omega^1,\;\; Z^2 = \omega_{0'},\;\; Z^3 = \omega_{A'},$$

so we can write

$$Z^{\alpha} = (\omega^{A}, \pi_{A'}) \text{ and } \bar{Z}_{\alpha} = (\bar{\pi}_{A}, \bar{\omega}^{A'})$$

or more simply $\mathbf{Z}=(\boldsymbol{\omega},\boldsymbol{\pi})$ and $\bar{\mathbf{Z}}=(\bar{\boldsymbol{\pi}},\bar{\boldsymbol{\omega}})$. The incidence relation with a space-time point **r** becomes

$$\omega^{A} = \mathrm{i}r^{AA'}\pi_{A'}, \quad \text{which we can write } \boldsymbol{\omega} = \mathrm{i}\mathbf{r}\bullet\boldsymbol{\pi},$$

where we are now regarding "**r**" as standing for the position vector of that point with respect to a given origin point **O**. Under a change of origin $\mathbf{O}\mapsto\mathbf{Q}$, where q^a is the position vector $\overrightarrow{\mathbf{OQ}}$, so that the position vector **r** must undergo $r^a\mapsto r^a-q^a$, the spinor parts of **Z** must accordingly undergo

$$\omega^{A} \mapsto \omega^{A} - \mathrm{i}\, q^{AA'}\pi_{A'}, \qquad \pi_{A'} \mapsto \pi_{A'},$$

this preserving the incidence relation $\boldsymbol{\omega}=\mathrm{i}\mathbf{r}\bullet\boldsymbol{\pi}$. For a dual twistor **W**, with $W_{\alpha}=(\pi_A,\mu^{A'})$, we correspondingly have

$$\pi_{A} \mapsto \lambda_{A}, \qquad \mu^{A'} \mapsto \mu^{A'} + \mathrm{i}\, q^{AA'}\lambda_{A}.$$

The inner product of a twistor $\mathbf{Z}=\boldsymbol{\pi})$ with a dual twistor $\mathbf{W}=(\boldsymbol{\lambda},\boldsymbol{\mu})$ (i.e. $W_{\alpha}=(\lambda_A,\mu^{A'})$) is

$$\mathbf{W}\bullet\mathbf{Z} = \lambda_{A}\,\omega^{A} + \mu^{A'}\,\pi_{A'} = \boldsymbol{\lambda}\bullet\boldsymbol{\omega} + \boldsymbol{\mu}\bullet\boldsymbol{\pi}.$$

In terms of $\boldsymbol{\omega}$ and $\boldsymbol{\pi}$, a *physical* interpretation of a twistor $\mathbf{Z}=(\boldsymbol{\omega},\boldsymbol{\pi})$, can be provided, up to the phase freedom $\mathbf{Z}\mapsto e^{\mathrm{i}\theta}\mathbf{Z}$ (θ real), even in the *non*-null case. The twistor, up to this phase freedom, gives us the 4-momentum **p** (p_a in index form) and 6-angular momentum M^{ab} of a massless particle with spin:

$$p_{a} = \pi_{A'}\bar{\pi}_{A}, \qquad M^{ab} = \mathrm{i}\omega^{(A}\bar{\pi}^{B)}\,\varepsilon^{A'B'} - i\bar{\omega}^{(A'}\pi^{B')}\,\varepsilon^{AB}$$

(where I use abstract indices, so that $a=AA'$, $b=BB'$, etc., the skew ε "metric" spinors defining the space-time metric via $g_{ab}=\varepsilon_{AB}\varepsilon_{A'B'}$; see **[16]** §3.1, **[22]** §6.3). Provided that $\boldsymbol{\pi}\neq0$, this p_a and M^{ab} automatically satisfy all the conditions

$$p_{a}p^{a} = 0, \quad p_{0} > 0, \quad M^{(ab)} = 0, \quad \tfrac{1}{2}\varepsilon_{abcd}p^{b}M^{cd} = sp_{a}$$

required for a free massless particle. Conversely, the twistor **Z** (with $\pi_{A'}\neq0$) is determined, uniquely up to a phase multiplier $e^{\mathrm{i}\theta}$, by p_a and M^{ab}, subject to these conditions. (When $\boldsymbol{\pi}=0$, we get a limiting situation, where the particle is at infinity, but with a zero momentum and non-zero angular momentum.) We shall be seeing shortly that the *phase* also has a key geometrical role to play in the geometric (pre)-quantization procedure mentioned in §2, which is relevant to the *palatial* twistor theory sketched in §6.

The quantity s is the *helicity* of the particle, i.e. its *spin*, but with a *sign*, positive for right-handed and negative for left-handed. The helicity s finds the very simple (and fundamental) twistor expression

$$2s \;=\; \omega^A\bar{\pi}_A + \omega_{A'}\bar{\omega}^{A'} \;=\; Z^\alpha\,\bar{Z}_\alpha \;=\; \|\mathbf{Z}\|.$$

When s=0, the complete interpretation of **Z,** up to phase, is the momentum-scaled ray it determines (possibly at infinity), as given earlier. But when $s\neq0$, there are no such real points, and there is no real world-line that can be associated with **Z** in a Poincaré-invariant way (although there is a spatially non-local real interpretation that can be given in terms of a twisting configuration referred to as a "Robinson congruence" **[22]**, §6.2, from which the term "twistor" was originally derived). There is also an interpretation of **Z** in terms of the family of *complex* points **x** satisfying the incidence relation $\boldsymbol{\omega}$=i$\mathbf{x}\bullet\boldsymbol{\pi}$, these constituting what is referred to as an α–plane in $\mathbb{CM}$ (or $\mathbb{CM}^\#$, if we include points **x** at complex infinity).

In curved space-time $\mathcal{M}$, these notions are not so well defined. Most particularly, the concept of an α–plane disappears for a complexified (real analytic) space-time $\mathbb{C}\mathcal{M}$ for which $\mathcal{M}$ is not conformally flat. Nonetheless, we can go some small way towards defining a hypothetical twistor space $\mathcal{T}$ for $\mathcal{M}$ (analogous to $\mathbb{T}$ defined for $\mathbb{M}$), in that the "non-projective" 7-space $\mathcal{N}$ can be defined from $\mathcal{M}\mathcal{M}\mathcal{M}\mathcal{M}$ $\mathcal{M}$t$\mathcal{M}$ding$\mathcal{M}$h$\mathcal{M}$$\mathbf{p}$–scaling of rays that gave us the 6-space $\mathbb{p}\mathcal{N}$ of §2 to a "$\boldsymbol{\pi}$-scaling" for rays γ. Thus, the 7-space $\mathcal{N}$ is a circle-bundle over $\mathcal{N}$, where the circle is simply the *phase freedom* $\omega_{A'}\mapsto e^{i\theta}\omega_{A'}$ (θ real) referred to earlier, to give us a e*-family of $\boldsymbol{\pi}$-*scaled* rays Γ for each ray γ in e , where p_a=$\pi_{A'}\bar{\pi}_A$, as above, the 2-spinor $\pi_{A'}$ being taken parallel-propagated along each γ. Yet, we do *not* get a canonically defined full complex twistor space $\mathcal{T}$ for a conformally curved e .

As noted above, this circle bundle is, in fact, just what is needed for the procedure of *geometric quantization*, when applied to the symplectic 6-manifold $\mathbb{p}\mathcal{N}$ (see **[17]**). What is first required for this (in the preliminary procedure of "pre-quantization") is a circle-bundle *connection* for which the curvature is ($\hbar$ times) the symplectic 2-form $\boldsymbol{\Sigma}$ (of §2). This connection is directly given by the 1-form i$\hbar\boldsymbol{\Phi}$, where $\boldsymbol{\Phi}$ is the symplectic potential referred to in §2. We find that to proceed to a full *quantization* procedure, we run into issues of non-uniqueness, these being actually *central* to the non-locality that comes about in palatial twistor theory, for which a tentative description will be given in §6

4. Local twistors and the Einstein Λ-equations

Despite these ambiguities, there is, however, a *local* notion of a twistor (*not* requiring s=0), defined at *each point* **q** of any ħ and which also can be carried over to each entire *ray* γ in ħ , i.e. to each point of the associated ray space of $\mathbb{P}\mathcal{N}$, and whence to each point Γ of the $\boldsymbol{\pi}$-scaled-ray space $\mathcal{N}$. This provides us with a flat twistor space $\mathbb{T}_{\mathbf{q}}$, canonically and conformally invariantly defined for each $\mathbf{q}\in\mathcal{M}$, and also such a space$\mathcal{M}\mathbb{T}_\gamma$, for each $\gamma\in\mathbb{P}\mathcal{N}$, where $\mathbb{T}_\gamma$ may be interpreted as a kind of complex "pseudo-tangent space" to $\mathcal{N}$ at each corresponding point $\Gamma \in \mathcal{N}$. These are obtained via the notions of *local twistor*, and *local twistor transport* (**[22]** §6.9).

A local twistor is a quantity $Z^{\alpha}=(\omega^{A},\omega_{A'})$, defined at a point **q** of the space-time , which transforms as

$$\pi^{A} \mapsto \pi^{A}, \qquad \pi_{A'} \mapsto \pi_{A'} + \mathrm{i}\, \pi^{A}\, \Omega^{-1}\nabla_{AA'}\Omega,$$

under a conformal rescaling of 's metric, according to $g_{ab} \mapsto \Omega^{2} g_{ab}$ (Ω being a smooth positive-valued function on). To get an exact correspondence with the twistor concept introduced in §2, we must think of π^{A} (and $\pi_{A'}$) as not being defined with respect to a *fixed* origin point **O**, as in §3, but now taken with respect to a *variable* point **q**∈ . Recall that in $\mathbb{M}$, when the origin **O** is displaced to a *general* point **q** (with position vector q^{a} with respect to **O**) in $\mathbb{M}$, the twistor $(\pi^{A},\pi_{A'})$ defined with respect to **O** becomes $(\pi^{A}-\mathrm{i}q^{AA'}\pi_{A'},\pi_{A'})$ with respect to **q**. The local twistor perspective on this is that $(\pi^{A},\pi_{A'})$, defined at **O**, when carried to **q** by local twistor transport, becomes $(\ {}^{A}-\mathrm{i}q^{AA'}\pi_{A'},\pi_{A'})$ at **q**. This enables us to extend this concept, in a conformally invariant way, to a general . When $=\mathbb{M}$ (or, indeed, when is any simply-connected conformally flat space-time) the notion of local twistor transport is *path-independent*, so that the local twistor concept extends to a global one, but this is not true in general.

The definition of *local twistor transport*, along a smooth curve γ in with tangent vector t^{a}, is

$$t^{a}\nabla_{a}\pi^{B} = -\mathrm{i}\, t^{BB'}\pi_{B'}, \qquad t^{a}\nabla_{\boldsymbol{a}}\pi_{B'} = -\mathrm{i}\, t^{AA'}P_{AA'BB'}\,\pi^{B},$$

where

$$P_{ab} = \tfrac{1}{12}Rg_{ab} - \tfrac{1}{2}R_{ab}, \quad \text{with} \quad R_{ac} = R_{abc}{}^{b}.$$

(sign conventions as in **[16]**, **[22]**). Taking γ to be a *ray*—which is simply-connected, with topology (by 's global hyperbolicity)—we use local twistor transport to propagate $(\boldsymbol{\omega},\boldsymbol{\pi})$ uniquely all along γ, thereby providing us with our canonical twistor space $\mathbb{T}_{\gamma}$, assigned to γ. Correspondingly, we shall have spaces $\mathbb{PT}_{\gamma}$, $\mathbb{N}_{\gamma}$, and $\mathbb{PN}_{\gamma}$, just as in §2. When $\mathbb{N}$ is *conformally flat* (and simply-connected), these spaces are all independent of the choice of any curve γ connecting a pair of points in $\mathbb{N}$, owing to the integrability of local twistor transport, so the local twistor spaces are all canonically indentical and may be referred to simply as spaces $\mathbb{T}$, $\mathbb{PT}$, $\mathbb{N}$, and $\mathbb{PN}$, respectively, but this does not hold if $\mathbb{N}$ is conformally curved.

We must raise the question of the relation between each $\mathbb{NN}_{\gamma}$ and the global space $\mathbb{N}\mathcal{N}$ of rays in a general $\mathcal{M}$. Within each $\mathbb{T}_{\gamma}$, for a ray γ, this ray, when $\boldsymbol{\pi}$-scaled to Γ, can itself be unambiguously represented by $(0,\pi_{A'})$ all along γ, this being unchanged by local twistor transport along γ (since $t^{AA'} \propto \bar{\pi}^{A}\pi^{A'}$ and $\pi^{A'}\pi_{A'}=0$). When $\mathcal{M}$ is *confomally flat* (and simply-connected), the integrability of local twistor transport allows us to achieve this globally for the whole of $\mathbb{P}\mathcal{N}$, where a $\boldsymbol{\pi}$-scaled ray η in $\mathcal{M}$

that meets γ in a point **q** would be represented at **q** by the local twistor $(0,\eta_{A'})$, in both $\mathbb{T}_\gamma$ and $\mathbb{T}_\eta$, where $\eta_{A'}$ provides the direction and $\boldsymbol{\pi}$-scaling for η. In fact the spaces $\mathbb{N}_\gamma - \{0\}$ are all canonically isomorphic with each other, and (locally) with $\mathcal{N}$ itself, so it makes sense to identify $\mathcal{N} \cup \{0\}$ with each $\mathbb{N}_\gamma$. (at least locally). However, this close association does not apply when $\mathbb{N}$ is *not* confomally flat.

When $\mathbb{N}$ is $\mathbb{M}$ or *de Sitter* 4-space $\mathbb{D}$ with (positive) cosmological constant Λ (or *anti*-de Sitter space if $\Lambda<0$), it *is* confomally flat, and the local twistor spaces can all be identified, as can their vector-space tensor algebras. More particularly, they have a specific structure defined by anti-symmetric 2-valent twistors referred to as *infinity twistors* **[22]**, which fix the *metric* structure of the space-time. These are $I^{\alpha\beta}$ and $I_{\alpha\beta}$, taken to be both complex conjugates and *duals* of one another:

$$I_{\alpha\beta} = \overline{I^{\alpha\beta}}, \quad I^{\alpha\beta} = \overline{I_{\alpha\beta}},$$
$$I_{\alpha\beta} = \tfrac{1}{2}\,\varepsilon_{\alpha\beta\rho\sigma}\, I^{\rho\sigma}, \quad I^{\alpha\beta} = \tfrac{1}{2}\,\varepsilon^{\alpha\beta\rho\sigma}\, I_{\rho\sigma},$$

where $\varepsilon_{\alpha\beta\rho\sigma}$ and $\varepsilon^{\alpha\beta\rho\sigma}$ are Levi–Civita twistors, fixed by their anti-symmetry and $\varepsilon_{0123}=1=\varepsilon^{0123}$ in standard twistor coordinates. In standard 2-spinor descriptions (§3), we have:

$$I_{\alpha\beta} = \begin{pmatrix} \frac{\Lambda}{6}\varepsilon_{AB} & 0 \\ 0 & \varepsilon^{A'B'} \end{pmatrix}, \qquad I^{\alpha\beta} = \begin{pmatrix} \varepsilon^{AB} & 0 \\ 0 & \frac{\Lambda}{6}\varepsilon_{A'B'} \end{pmatrix}.$$

For de Sitter space $\mathbb{D}$, the infinity twistors provide a *complex symplectic structure* (not to be confused with the *real* symplectic structure of §2) defined by the 2-form:

$$\boldsymbol{\mathcal{I}} = I_{\alpha\beta}\, dZ^\alpha \wedge dZ^\beta; \quad d\boldsymbol{\mathcal{I}} = 0.$$

Also there is a *symplectic potential* 1-form

$$\boldsymbol{\mathcal{J}} = I_{\alpha\beta}\, Z^\alpha dZ^\beta, \qquad \text{where} \quad \boldsymbol{\mathcal{I}} = d\boldsymbol{\mathcal{J}}.$$

When $\Lambda=0$, this symplectic structure becomes degenerate, the matrices for $I_{\alpha\beta}$ and $I^{\alpha\beta}$ becoming singular. When $\Lambda\neq0$, they are essentially inverses of one another:

$$I_{\alpha\beta}\, I^{\beta\gamma} = -\frac{\Lambda}{6}\,\delta_\alpha^\gamma.$$

but annihilate each other if $\Lambda=0$. For given Λ, the structure afforded to the twistor space $\mathbb{T}$, by $I_{\alpha\beta}$ (or equivalently $I^{\alpha\beta}$, where $\varepsilon_{\alpha\beta\rho\sigma}$ and $\varepsilon^{\alpha\beta\rho\sigma}$ are assumed given) will be called its ***I**-structure* (or $\mathbf{I}_\Lambda$-structure).

A significant feature of local twistor transport is that the satisfaction of Einstein's Λ-vacuuum equations $R_{ac}=\Lambda g_{ab}$ is *equivalent* to the fact that $I_{\alpha\beta}$ (or $I^{\alpha\beta}$) is *constant* under local twistor transport. (See **[22]** p.376 for the case $\Lambda=0$; when $\Lambda\neq0$ this fact can be directly established; see also **[23]**.) Moreover, the local twistors $\varepsilon_{\alpha\beta\rho\sigma}$ and

$\varepsilon^{\alpha\beta\rho\sigma}$ can be seen to be *automatically* local-twistor constant, independently of the Einstein equations. Accordingly, the Einstein Λ-vacuum equations can be phrased in terms of the existence of an **I**-structure that holds globally for all the local twistor spaces $\mathbb{T}_\gamma$ for the ray space $\mathcal{N}$.

5. Twistor quantization and cohomological wavefunctions

Up to this point, I have been concerned only with classical twistor theory. For the *quantized* theory, we need to introduce the *commutation laws* **[22]** §6.10:

$$Z^\alpha \bar{Z}_\beta - \bar{Z}_\beta Z^\alpha = \hbar\delta^\alpha_\beta$$

and

$$Z^\alpha Z^\beta - Z^\beta Z^\alpha = 0, \qquad \bar{Z}_\alpha \bar{Z}_\beta - \bar{Z}_\beta \bar{Z}_\alpha = 0,$$

where now Z^α and $\bar{Z}$ are taken to be *linear operators* generating a non-commutative algebra $\mathbb{A}$, acting on some appropriate "ket space" **[24]**. We should think of that space as a quantum state space of some kind, that I shall sometimes refer to as "$|\ldots\rangle$", but it is best not to be too specific about this for the time being. (In the language of standard quantum mechanics, a ket space may be thought of as a complex linear space with a basis that is a "complete set of commuting variables".) In fact, it will be a key part of the arguments in §6 that for *conformally curved* space-times, the ket space will *not* be provided in a globally consistent way, though having a *local* (but non-unique) existence in appropriately defined "sufficiently small" regions of the ray spacer $\mathbb{P}\mathcal{N}$. The proposal, according to palatial twistor theory, is that in the case of a curved $\mathcal{M}$ we obtain an algebra $\mathcal{A}$, that generalizes the role that $\mathbb{A}$ plays for $\mathbb{M}$ (or for $\mathbb{D}$), where $\mathcal{A}$ is defined *completely globally* for $\mathcal{M}$, even though there would be *no* globally consistent ket space.

There are, however, issues concerning the nature of $\mathbb{A}$ (and certainly of $\mathcal{A}$) that are not completely resolved at the time of writing. We would certainly require that $\mathbb{A}$ contain polynomial expressions in Z and $\bar{Z}_\beta$, but, as we shall be seeing, expressions that are *analytic* in these quantities must also play a role. In a (conformally) *curved* space-time $\mathcal{M}$, we would have a *deformed* such algebra $\mathcal{A}$ that is, in some appropriate *local* sense, the same as $\mathbb{A}$, but whose global structure would encode the entire (conformal) geometry of a given curved space-time $\mathcal{M}$. The algebra $\mathbb{A}$ itself is to be thought of as, in some sense, the algebra of linear operators acting on (germs of?) holomorphic entities of some kind defined on $\mathbb{T}$, but the precise notion of what is required has not yet become completely clear. In basic terms, $\mathbb{A}$ is to be taken as the algebra generated by Z^α and $\partial/\partial Z^\beta$, but where infinite series in these (non-commuting) operators would also need to be considered as belonging to $\mathbb{A}$. This raises issues of *convergence* and *locality* that will need to be sorted out in due course, but for our present purposes I shall ignore these subtleties and merely explain the general idea of what is required.

In the case of $\mathbb{M}$, the above commutation laws are *almost* implied by the standard quantum commutators for position and momentum

$$p_a x^b - x^b p_a = \mathrm{i}\hbar\delta_a^b$$

but there appears to be an additional input related to the issue of helicity. By direct calculation, we may verify that the twistor commutation laws reproduce exactly the (considerably more complicated-looking) commutation laws for p_a and M^{ab} that arise from their roles as translation and Lorentz-rotation generators of the Poincaré group. In this calculation, we take note of the fact that there is no factor-ordering ambiguity in the expressions for p_a and M^{ab} in terms of the spinor parts of Z^α and $\bar{Z}_\alpha$ (because of the symmetry brackets in the spinor expression for M^{ab}). However, when we examine the calculation for obtaining the helicity s, we do not retrieve the classical expression $2s{=}Z^\alpha\bar{Z}_\alpha$ (or $2s{=}\bar{Z}_\alpha Z^\alpha$) but, specifically (writing the helicity *operator* as $\mathbf{s}$, in bold type):

$$\mathbf{s} = \tfrac{1}{4}\,(Z^\alpha\bar{Z}_\alpha + \bar{Z}_\alpha Z^\alpha).$$

In analogy with the standard quantum-mechanical procedures, if we wish to consider what the *wavefunction* for a massless particle should be in twistor terms, we need to think of functions of Z^α that are "independent of $\bar{Z}_\beta$". This means "annihilated by $\partial/\partial\bar{Z}_\beta$", in other words *holomorphic* in Z^α (by the Cauchy–Riemann equations). Thus, a twistor wavefunction (in the **Z**-description) must be holomorphic in **Z** and we have the operators representing Z^α and $\bar{Z}_\alpha$::

:

$$Z^\alpha{::}\rightsquigarrow Z^\alpha \times\,, \qquad \bar{Z}_\alpha \rightsquigarrow \hbar\frac{\partial}{\partial Z^\alpha}\,.$$

We can alternatively consider wavefunctions expressed in terms of the conjugate variables $\bar{Z}_\alpha$, which are *dual* twistors, and re-labelling $\bar{Z}_\alpha$ as W_α, we have commutation laws $W_\alpha\bar{W}^\beta{-}\bar{W}^\beta W_\alpha{=}{-}\hbar\delta_\alpha^\beta$, $W_\alpha W_\beta{-}W_\beta W_\alpha{=}0$, $\bar{W}^\alpha\bar{W}^\beta{-}\bar{W}^\beta\bar{W}^\alpha{=}0$, leading to a *dual*-twistor **W**-description of wavefunctions.

For purposes of being definite, I just adopt the **Z**-description here. It should be remarked that, in the **Z**-description, the above quantization procedure allows the removal of all operations that involve $\bar{\mathbf{Z}}$, replacing them completely by operations in **Z**, and thereby providing us with entirely *holomorphic* descriptions. This will prove to be of *central* importance to the theory.

If we are asking that a wavefunction describe a (massless) particle of *definite helicity*, then we need to put it into an eigenstate of $\mathbf{s}$, which, by the above, is

$$\mathbf{s} = -\tfrac{1}{2}\hbar\Big(Z^\alpha\frac{\partial}{\partial Z^\alpha} + 2\Big).$$

This is simply a displaced *Euler homogeneity operator* $Z^\alpha\dfrac{\partial}{\partial Z^\alpha}$ so that for a helicity eigenstate, with eigenvalue s, we need a twistor wavefunction $f(\mathbf{Z})$ that is not only holomorphic but also *homogeneous* of degree

$$n = -2s - 2$$

where, for convenience, I henceforth choose $\hbar$=1. Then $2s$ is an integer (odd for a fermion and even for a boson).

We need to see the relation between such a twistor wavefunction and the space-time description, in terms of the zero-mass field equations in flat space-time $\mathbb{M}$ —or in a conformally flat $\mathcal{M}$—for each helicity s. These field equations are expressed in the 2-spinor form

$$\nabla^{AA'}\psi_{AB\ldots E} = 0, \quad \text{or} \quad \Box\psi = 0, \quad \text{or} \quad \nabla^{AA'}\tilde{\psi}_{A'B'\ldots E'} = 0,$$

for the integer $2s$ satisfying s<0, s=0, or s>0, respectively, where we have *total symmetry* for each of the $|2s|$-index quantities

$$\psi_{AB\ldots E} = \psi_{(AB\ldots E)}, \quad \tilde{\psi}_{A'B'\ldots E'} = \tilde{\psi}_{(A'B'\ldots E')}.$$

These equations give the spinor form of (the anti-self-dual and self-dual parts of) the free Maxwell equations (if $|s|$=1) and of the source-free linearized free gravitational field (if $|s|$=2).

The required relation between $f(\mathbf{Z})$ and the appropriate ψ (or $\tilde{\psi}$) can be achieved by a simple contour integral expression (see **[25]** and **[22]** §6.10), where the contour lies within the Riemann sphere **R**, representing a point **r** of complex Minkowski space $\mathbb{CM}$, **R** being the locus of projective twistors $\mathbb{P}\mathbf{Z}$ that are incident ($\boldsymbol{\omega}$=i$\mathbf{r}$•$\boldsymbol{\pi}$) with **r**, where $\mathbf{Z}$=($\boldsymbol{\omega}$,$\boldsymbol{\pi}$). A free wavefunction $\psi(\mathbf{r})$ (or $\tilde{\psi}(\mathbf{r})$), should be of *positive frequency*, and this is achieved if $\psi(\mathbf{r})$ (or $\tilde{\psi}(\mathbf{r})$) remains holomorphic when we allow **r** to be any complex point lying in the *forward tube* $\mathbb{M}^+$. This is that part of $\mathbb{PM}$ consisting of points whose imaginary parts are timelike past-pointing), and is represented, in projective twistor space $\mathbb{PT}$, by the lines **R** lying entirely in $\mathbb{PT}^+$. See Fig.2 for a picture of this arrangement (the shaded region indicating where f is free of singularities).

The details are best not entered into here, but what we find is that the twistor wavefunction f is not really to be thought of as “just a function” in the ordinary sense, but as a representative of an element of *1st cohomology* (actually 1st *sheaf* cohomology) of the space $\mathbb{PT}^+$. It is at this stage that we begin to realize the deeper and more subtle aspects of the *non-locality* of the twistor picture of physical reality, this non-locality finding expression in the essential non-locality of cohomology **[26]**. A good intuitive way of appreciating the idea of 1st cohomology is to contemplate the “impossible tribar” depicted in Fig.3. Here we have a picture that for each local region, there is an interpretation provided, of a 3-dimensional structure that is unambiguous, except for an uncertainty as to its distance from the viewer’s eye. As we follow around the triangular shape, our interpretation remains consistent until we return to our starting point, only to find that it has actually become *in*consistent! The element of 1st cohomology that is expressed by the picture is the measure of this *global* inconsistency **[27]**, where *locally* there is no inconsistency, but merely a mild-seeming ambiguity of the distance from the viewer’s eye of the pictured object.

In the case of a twistor wavefunction there is an additional subtlety, in that the global inconsistency arises from the "rigidity" of *holomorphic functions* rather than that of the solid structures conjured up by the local parts of Fig.3. To be more specific, we may think of $\mathbb{PT}^+$ as being broken into pieces—and simply the *two* overlapping open regions $\mathcal{U}_1$ and $\mathcal{U}_2$ depicted in Fig.2 will do—which together cover the whole of $\mathbb{PT}^+$, and where there is a region $\mathcal{R}$ of intersection of $\mathcal{U}_1$ and $\mathcal{U}_2$ which is where our holomorphic function *f* is actually defined

$$\mathbb{PT}^+ = \mathcal{U}_1 \cup \mathcal{U}_2, \quad \mathcal{R} = \mathcal{U}_1 \cap \mathcal{U}_2;$$

(see Fig.2). We can think of this as being analogous to the tribar of Fig.3 by imagining a splitting of the tribar picture into two overlapping parts, a left-hand one ("$\mathcal{U}_1$") and a right-hand one ("$\mathcal{U}_2$") where the (disconnected) overlap region provides us with instructions as to how to glue the two parts together. These instructions are the analogue of the holomorphic function *f*, and it is the *rigidity* of holomorphic functions (as expressed in the local uniqueness of analytic continuation) that provides the analogy with that of the rigid-body structures depicted (locally) in Fig 3.

The non-locality of twistor 1st cohomology that is illustrated here reflects a *physical* non-locality that is exhibited in 1-particle wavefunctions that had worried Einstein, way back in 1927 (*Einstein's boxes*; see **[28]**), though much milder and elementary than the 2-particle non-locality that Bell established in 1964. We may imagine that a photon source, aimed at a photo-sensitive screen some distance away, emits a single photon towards the screen, the state of this photon being described by a wavefunction ψ. As soon as the screen registers reception of the photon at any one of its points—say **x**—this detection event instantly forbids every other point of the screen from detecting the photon, despite the fact that at another point **y** on the screen, the wavefunction itself may have had, a moment previously, a ψ-value comparable with that at **x**. The probability of detection at each point of the screen is determined in the same way (by some form of Born rule), but as the detection probability just refers to a *single* particle, it cannot be detected at more than one point. It is a global thing, quite unlike the situation with a water wave, say, whose effect on each point on a cliff face is *locally* determined, being independent of its effect at other points on the cliff.

Of course, taken on its own, such a situation can be explained in accordance with the point of view that the wavefunction is simply a kind of "probability wave" with no actual *reality* attached to it. This simple picture cannot be maintained, however, when interference effects are involved, and for such reasons I would myself insist on attributing some measure of physical reality to the wavefunction. Yet, this "reality" has to acquire some kind of non-locality, as this simple example demonstrates.

For an extreme illustration of this, we may imagine that an astronomer detects a photon from a galaxy some millions of light years away. Prior to detection, one would consider that the photon's wavefunction had spread, enormously diluted, over a region several millions of light years across. Yet, the astronomer's detection of the photon at once forbids its detection at any other place in that vast region (where I here ignore complicating matters of Bose statistics and quantum field theory). The twistor picture of this non-locality, is like that of a vast impossible tribar, whose impossibility is removed once it is broken at any *one* place, this "breaking" being the analogy of a quantum state reduction (**R**) occurring at that place.

Of course, as we know, a "local-realistic" model (i.e. a "Bertelmans's socks" type of explanation, see **[29]**) can easily be provided for this kind of single-particle non-locality, and we need to pass to multi-particle situations in order to provide instances of genuine Bell non-locality. The twistor description of an n-particle wavefunction would require n^{th} cohomology, so the twistor picture becomes more complicated (and not yet adequately discussed, as far as I am aware). I am more concerned, in this article, with how one might attempt to address, in twistor terms, the issue of the *state reduction* **R** that occurs when a *measurement* is applied to a (twistor) wavefunction. Since, according to §1, I adopt the view that this is an *objective* physical process (**OR**) which is gravitational in nature, we shall need to see how genuine curved-space geometry might be incorporated into the twistor formalism. The basic ideas for this are outlined in the next section, and we see a new kind of non-locality arising in the twistor picture.

6. Palatial twistor theory

Before turning to the proposal of "palatial twistor theory" for a general curved space-time $\mathcal{M}$, it will be helpful to give a brief outline of the earlier procedure referred to as the "non-linear graviton" construction **[30]** (and see also **[31]** for the case of non-zero Λ), whereby the general (complex) solution of Einstein's Λ-vacuum equations, in the anti-self-dual case, may be expressed in twistor terms. The gist of this construction can be gleaned from Fig.2, where we think now of the two overlapping open regions $\mathcal{U}_1$ and $\mathcal{U}_2$ as being separate portions of twistor space that are to be "glued together" over the shaded intersection region. Now, rather than thinking of f as just being "painted on" the overlap region $\mathcal{M}$ we can think of f as playing a more *active* role, whereby it effects a sliding of one patch over the other, so as to obtain some form of "curved" twistor space, achieved by a process of "coordinate patching". In the infinitesimal case, this can be expressed as a shift in $\mathcal{U}_1$ as matched to $\mathcal{U}_2$, along the vector field:

$$I^{\alpha\beta} \frac{\partial f}{\partial Z^{\alpha}} \frac{\partial}{\partial Z^{\beta}},$$

(defined on $\mathcal{R}$) where f is homogenous of degree 2 (corresponding to helicity s=–2). Exponentiating this, we get a genuinely curved twistor space of required type (with a globally defined **I**-structure). The *points* of the complex 4-manifold $\mathcal{R}$ arise as completed Riemann spheres, that deformed versions of the line **R**, in Fig.2. A remarkable theorem due to Kodaira **[32]** tells us that such deformed **R**s do, indeed form a complex 4-parameter family (at least for deformations that are not too large). It is a striking fact that the non-trivial local structure of the resulting $\mathcal{R}$, which involves a non-zero anti-self-dual Weyl curvature (Ψ_{ABCD}; see **[16]**, **[30]**), arises from *global* structure of the deformed twistor space $\mathcal{T}$, itself having merely a *local* **I**-structure, identical with that of $\mathbb{M}$, or of $\mathbb{D}$ (with the correponding Λ).

The idea behind palatial twistor theory **[18]** is that we try to mimic this "non-linear-graviton" procedure, but where instead of matching the complex-manifold structure of $\mathcal{U}_1$ to that of $\mathcal{U}_2$ in order to get a curved twistor space $\mathcal{T}$, we try to match their respective twistor *quantum algebras* $\mathbb{A}_1$ and $\mathbb{A}_2$ to give a "curved" quantum algebra $\mathcal{A}$, but without involving actual "spaces", like the regions of $\mathcal{U}_1$ and $\mathcal{U}_2$, or $\mathcal{T}$. Had we maintained a notion of a twistor *manifold*, for the regions $\mathcal{U}_1$, $\mathcal{U}_2$, this would force us into the *left*-handed space-time framework, rather than allowing both helicities. Matching the twistor *algebras* rather than the twistor *spaces* of the non-linear graviton construction enables us to resolve a long-standing (~40-year-old) conundrum known as the "googly problem" (see **[22]**, footnote on p.164). That problem demands finding a suitable procedure which would yield (complex) space-times possessing *self-dual* conformal curvature ($\tilde{\Psi}_{A'B'C'D'}$), i.e. *right*-handed (s=2), for which the linearized version would require a twistor function f of the awkward-looking homogeneity degree –6, which had proved very problematic. Of course, that could be addressed if the *dual* (**W**-description) were adopted, but then the same problem would arise for the *anti*-self dual part, which doesn't help, as a single formalism has to be found, able to cope with *both* parts at once. This is what palatial twistor theory is proposed to achieve.

Since these twistor quantum algebras are non-commutative, we are led into the kind of picture provided by the ideas of *non-commutative geometry* **[33]**, and ordinary "spaces" like $\mathcal{U}_1$ and $\mathcal{U}_2$ are not determined uniquely by such algebras. Nevertheless, we need some notion of locality (or "topology") in order to express the concept of building up the entire structure out of "flat" pieces. This is achieved by appealing to the light-ray spaces $\mathbb{P}\mathcal{N}$, $\mathbb{p}\mathcal{N}$, and $\mathcal{N}$, described in §2 and §3. We can imagine that $\mathbb{P}\mathcal{N}$ is divided into, say, n partially overlapping regions $\mathbb{P}\mathcal{N}_1$, $\mathbb{P}\mathcal{N}_2$, …, $\mathbb{P}\mathcal{N}_n$, providing an open covering ($\mathcal{N}_1$,$\mathcal{N}_2$,…,$\mathcal{N}_n$) of $\mathcal{N}$. The idea is that if the individual spaces $\mathcal{N}_k$, together with their intersections, are, in an appropriate sense, "simple", then they can be (non-canonically) assigned respective "flat" twistor quantum algebras $\mathbb{A}_1$, $\mathbb{A}_2$, .., $\mathbb{A}_n$. On the various overlaps $\mathcal{N}_j \cup \mathcal{N}_k$ the algebras needs to match appropriately, but the resulting "patched" algebra will *not* be "flat" in its total structure if $\mathcal{M}$ is conformally curved.

I need to explain some of these terms a little more fully. Basically, the algebra will be considered to be *flat* if it has a consistent "ket space" $|\ldots\rangle$, and the idea is to find such a flat algebra for any "simple" region $\mathcal{U}$ in $\mathcal{N}$. Here, an open subregion of $\mathcal{N}$ is called *simple* if it is topologically and holomorphically trivial—by which I mean that it has Euclidean topology and is in some appropriate sense *convex*. To construct a (flat) twistor quantum algebra suitably assigned to any such simple region, we first consider the twistor space $\mathbb{T}_\gamma$, for each ray γ in $\mathbb{P}\mathcal{U}$ (from local twistor transport §4), and then construct the twistor quantum algebra $\mathbb{A}_\gamma$ from each, in the standard way (as in §4). We have no reason to expect a canonical isomorphism between these algebras for different rays γ in $\mathbb{P}\mathcal{U}$ when $\mathcal{M}$ is conformally curved. However, it is to be expected that we *can*, for a simple region $\mathcal{U}$ in $\mathcal{N}$, *deform*, continuously and holomorphically, the various $\mathbb{A}_\gamma$s at the different points of $\mathcal{U}$, so as to obtain a holomorphic "trivialization" of the bundle of $\mathbb{A}_\gamma$s over $\mathcal{U}$, thereby obtaining (by no means uniquely) a single algebra $\mathbb{A}_{\mathcal{U}}$, continuously and holomorphically isomorphic to each $\mathbb{A}_\gamma$ for $\gamma \in \mathbb{P}\mathcal{U}$ (and therefore isomorphic to $\mathbb{A}$) consistently over the whole of

$\mathcal{U}$. This is to be expected because the relation between the various local twistor spaces $\mathbb{T}_\gamma$ and their immediate neighbours (i.e. $\mathbb{T}_{\gamma'}$, where γ' is neighbouring to γ) becomes a *holomorphic* one between the corresponding $\mathbb{A}_\gamma$s, by virtue of the holomorphic nature of the twisor quantization process (and by virtue of the pre-quantization connection of §3. This notion of "consistency" (though still not fully understood in mathematical details) demands that there be a consistent "ket-space" $|\ldots\rangle$ for $\mathbb{A}_{\mathcal{U}}$, isomorphic to $\mathbb{A}$, over each region $\mathcal{U}$.

Each such flat $\mathbb{A}_{\mathcal{U}}$ is to be thought of in the spirit of a "coordinate patch". Over the *intersection* $\mathcal{U}_i \cap \mathcal{U}_j$ of two simple regions $\mathcal{U}_i$ and $\mathcal{U}_j$ we require consistency of the algebras $\mathbb{A}_{\mathcal{U}_i}$ and $\mathbb{A}_{\mathcal{U}_j}$ in the sense of having a continuous and holomorphic deformation of one to the other, retaining a consistent ket space on the intersection, but we do not require a common ket-space to be present for the whole of their *union* $\mathcal{U}_i \cup \mathcal{U}_j$. Such consistency would not generally be possible globally. Instead, our fully "patched together" algebra $\mathcal{A}$ would *not* have a consistent ket space (unless $\mathcal{M}$ is conformally flat). The idea would be that a measure of the *departure* from global consistency of a ket space, over the whole of $\mathcal{N}$, would be something of the nature of a (non-linear) 1[st] cohomology element (as with the inconsistency expressed in Fig.3) and which, in space-time terms, would express the presence of a non-zero *Weyl conformal tensor*, i.e. conformal curvature for $\mathcal{M}$.

We need to be able to identify the *points* of $\mathcal{M}$ in terms of the algebra $\mathcal{A}$. These have to arise by *non-local* considerations (as was the case with the non-linear graviton construction). Corresponding to any particular point **r** of $\mathcal{M}$ there would be a locus **R** in $\mathbb{P}\mathcal{N}$ representing **r**, namely the family of all rays through **r**, which is topologically S^2. The idea is that the consistency (i.e. trivialization, in the above strong sense of having a consistent ket-space) of the $\mathbb{A}$-bundle over **R** is what determines such an S^2 locus as representing a point of $\mathcal{M}$. For this to work, as a sufficiently restrictive proposal for locating $\mathcal{M}$'s points in terms of such a twistorial construction, we need to establish the validity of various technical issues that have been skimmed over in the previous two paragraphs. Moreover, to ensure that the construction outlined above actually provides us with a 4-dimensional $\mathcal{M}$, we would certainly need some suitable generalization of the Kodaira theorem **[32]** that was central to the non-linear graviton construction.

None of this yet encodes the formulation of Einstein's equations. It is perhaps remarkable, therefore, to find that Einstein's Λ-vacuum equations are themselves very simply encoded into this structure. For these equations provide *precisely* the necessary and sufficient condition that the local twistor spaces $\mathbb{T}_\gamma$ possess an **I**-structure (for given Λ), so all we now require that is that the needed continuous and holomorphic deformations of the $\mathbb{A}_{\mathcal{U}}$ algebras preserve their nature as algebras on $\mathbb{T}_\gamma$ with this **I**-structure. If all these procedures (or something like them) indeed work as intended (with generalizations to the Yang–Mills equations and other aspects of physics), then there would appear to be significant openings for twistor theory in a non-local basic physics, not envisaged before.

Nevertheless, we have still not addressed the issue raised in §1 of the need for a (non-local) physics capable of describing the **R**-process as a realistic gravitational phenomenon. Indeed, the formalism as so far described cannot yet be taken as a "quantum-gravity" theory if only for the simple reason that no place has been found for the Planck length l_P or equivalently the Planck time t_P. A tempting way to incorporate such dimensional quantities might be to modify the commutators of §5 as follows:

$$Z^\alpha \bar{Z}_\beta - \bar{Z}_\beta Z^\alpha = \hbar\delta^\alpha_\beta$$

and

$$Z^\alpha Z^\beta - Z^\beta Z^\alpha = \varepsilon I^{\alpha\beta}, \qquad \bar{Z}_\alpha \bar{Z}_\beta - \bar{Z}_\beta \bar{Z}_\alpha = \bar{\varepsilon} I_{\alpha\beta}, \qquad ,$$

where ε is a very small (complex?) constant related to the Planck length. These commutator equations have not yet been significantly explored, and it cannot yet be said whether or not they supply anything of the kind of "quantum gravity" framework that might be needed.

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Figure captions

Fig.1 A positive-norm twistor **Z** lies in the portion $\mathbb{T}^+$ of non-projective twistor vector space $\mathbb{T}$, this being is the disjoint union of $\mathbb{T}^+$, $\mathbb{N}$, and $\mathbb{T}^-$. The projective versions of these three spaces are $\mathbb{PT}^+$, $\mathbb{PP}$, and $\mathbb{PT}^-$, respectively.

Fig.2 The contour integral arrangement for a twistor wavefunction. The two open sets $\mathcal{U}_1$ and $\mathcal{U}_2$ (regions of $\mathbb{PT}^+$ depicted above those letters) together cover the whole of $\mathbb{PT}^+$, the function f being defined on their intersection. The complex point **r** is represented by a Riemann sphere (complex projective line) **R** in $\mathbb{PT}^+$.

Fig.3 The "impossible tribar" illustrates 1st cohomology. This arises from a local ambiguity in the distance of the depicted object from the viewer.