Experimental tests of Bell inequalities

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1

Introduction

1.1 Annus mirabilis 1964

1964 was a wonderful year for physics, in this "annus mirabilis", summarising and simplifying a little, quark model was proposed by Gell-Mann and Zweig and charm and colour properties were introduced, Higgs published his work on the scalar boson, Ω boson and CP violation were discovered, cosmic background radiation was identified and, finally, Bell inequalities were suggested.

Most of these works were later subjects of a Nobel prize. Unluckily John Bell left us too early for this achievement, but, at least in my opinion, his work [1] is probably the most fruitful among all these huge progresses, since now we can recognise that it was the root of a fundamental change in our vision of the physical world, providing, on the one hand, the possibility of a test of quantum mechanics against a whole class of theories (substantially all the theories that we could define as "classical") and, on the other hand, introducing a clear notion of quantum non-locality, that represents not only a challenge in understanding quantum mechanics, but also one of the most important resources paving the way to quantum technologies that were developed in the last decades based on the quantum correlations discussed in Bell paper.

In the following we will present the experimental progresses in testing Bell inequalities. Rather amazingly even 50 years after Bell's paper, still a conclusive test of his proposal is missing [2, 3].

Nevertheless, huge progresses have been achieved respect to the first experiments. We will discuss in detail these developments and the remaining problems for a conclusive test of Bell inequalities.

1.2 The Bell inequalities, their hypotheses and the experimental loopholes.

In order to understand the experimental difficulties in achieving a final test of Bell inequalities, let us analyse a little in detail how these inequalities are derived and the explicit and implicit hypotheses they contain.

The proof of Bell inequalities usually starts by considering two separated subsystems of an entangled state, which are addressed to two independent measurement apparatuses measuring the expectation value of two dichotomic observables,A_a and B_b. a, b are two parameters describing the setting of the measuring apparatus A and B respectively.

The two measurements must be independent (which at the end means they should be space like separated events).

Bell demonstrated [1] that joint expectation values of the two observables satisfy some inequality for every Local realistic theory (LRT), the so called Bell inequalities (BI). On the other hand, this inequality can be violated in Standard Quantum Mechanics (SQM) for specific choices of parameters.

As discussed in the theoretical chapters, several different equivalent (at least in two dimensional case, that is the one of experimental interest) "Bell inequalities" exist. In the following we consider the one proposed by by Clauser-Horne-Shimony-Holt (CHSH), that found a wide experimental use.

The proof is relatively simple. By supposing that the results of measurements are determined by a (or a set) hidden variable x distributed with a certain probability distribution $\rho(x)$.

After having introduced the expected value for joint measurements $C(a, b)$, let us consider the inequality:

$$|C(a,b) - C(a,c)| \leq \int_X |A_a(x)B_b(x) - A_a(x)B_c(x)|\rho(x)dx =$$
$$\int_X |A_a(x)B_b(x)|[1 - B_b(x)B_c(x)]\rho(x)dx = \int_X [1 - B_b(x)B_c(x)]\rho(x)dx \tag{1.1}$$

Introducing $0 \leq \delta \leq 1$ such that for some values b, b' we have $C(b', b) = 1 - \delta$, by splitting the set X into two regions $X_{\pm} = \{x | A_{b'}(x) = \pm B_b(x)\}$ we have

$$\int_{X_-} dx\rho(x) = \delta/2 \tag{1.2}$$

being $C(b', b) = 1 - \delta = \int_X B_b^2(x)\rho(x)dx - 2\int_{X_-} B_b^2(x)\rho(x)dx$. Hence:

$$\int_X B_b(x)B_c(x)\rho(x)dx \geq \int_X A_{b'}(x)B_c(x)\rho(x)dx - 2\int_{X_-} |A_{b'}(x)B_c(x)|\rho(x)dx =$$
$$= C(b', c) - \delta \tag{1.3}$$

From this result and Eq. 1.1 follows CHSH inequality [4]

$$S = |C(a,b) - C(a,c)| + C(b',b) + C(b',c) \leq 2. \tag{1.4}$$

Let us now consider the hypotheses needed for deriving this result:

- it exists one (or more) "hidden variable" predetermining the values of observables (realism)

- there is no non-local effect, i.e. $C(a,b) = \int_X A_a(x) B_b(x) \rho(x) dx$ (Bell definition of locality)

- the measurements can be chosen independently and randomly by the two observers (freedom of choice)

These hypotheses condition how an eventual experiment must be realised in order to be able to perform a conclusive test of Bell inequality.

A first point is that the two measurements must be set independently. A part some "conspiracy" (absence of free will, absence of the possibility of having independent random numbers, ...) this requires that the settings of the two measuring apparatuses are space like separated events, i.e. no communication is possible, even in principle, that could influence one setting on the basis of the other. If this is not the case the experiment will suffer of the so called **space like loophole**.

A second relevant point is that one should be able to detect all the pairs involved in the experiment or, at least, a sufficiently large fraction of them. If only a subsample of the total number of produced entangled systems is really detected one needs to introduce an additional assumption, i.e. one must suppose that the measured sample is a faithful representation of the whole. Indeed, a subsample could contain a distribution in the hidden variables different from the total one, since the hidden variable values can also be related to the probability of the state to be observed. This means that when the observed sample is not a sufficiently large fraction of the total set of pairs, then the additional hypothesis of having an unbiased measured subsample is introduced [9, 10, 11, 12]. If this is the case the experiment will suffer of the so called **detection loophole**.

By considering the effect of losses (including all causes, from losses on the paths to detection efficiency), one can deduce that for maximally entangled states, a detection-loophole free test of LRT requires to observe at least a 82.84% of the total sample. This value can eventually be reduced to 66.7% for non-maximally entangled systems [13] [1].

Additional requirements are also:

- that the number of emitted particle is independent by measurement settings, "**production rate loophole**".

- that the presence of a coincidence window does not allow in a hidden variable scheme a situation where local setting may change the time at which the local event happens ("**coincidence loophole**") [6].

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[1] i.e. for a pure bipartite state when the two components do not have an equal weight, see subsection 2.2.3.

2

Testing Bell Inequalities

2.1 Starting to test BI

2.1.1 70's experiments

Immediately after the publication of Bell's work, the quest for a suitable system for an experimental test started.

Several different systems were analysed (as $K\bar{K}$, $\Lambda\bar{\Lambda}$, entangled pairs of ions, etc.), however entangled photons appeared since the beginning to be the most suitable candidate.

The first experiments were performed in 70s using polarisation entangled photon pairs as

$$|\Phi^{+}\rangle = \frac{|H\rangle|H\rangle + |V\rangle|V\rangle}{\sqrt{2}} \tag{2.1}$$

where H and V denote the horizontal and vertical polarisation, respectively.

These states were produced using either a cascade atom decay or positronium decay.

Nevertheless, all these experiments suffered of the space like loophole and presented other additional hypotheses.

Experiments with positronium produced in atomic decays were realised by [15, 16, 17, 18] with results in agreement with SQM, i.e. showing a BI violation (with the exception of Ref. [16]).

However, the difficulty of selecting polarisation of high-energy (gamma) photons produced in positronium decay substantially limited the performances of this source (the polarisation was estimated through the scattering distribution by means of the Klein-Nishina formula, introducing the hypothesis that this result can be correctly related by using Quantum Mechanics to the one that would have been obtained by using linear polarisers). For this reason this source was later abandoned.

An alternative was offered by entangled photons produced in atomic cascade decay, which are in the visible region of the spectrum so that polarisation can be easily selected. However, in this case, the atom carries away part of the momentum and thus photon directions are not well correlated. This led to a severe detection loophole being detection efficiencies typically under 1%.

The first BI test was indeed performed already in 1972 by Freedman and Clauser [19] using polarisation entangled photon pairs at 551 and 423 nm produced in $4p^2\,{}^1S_0 \to 4p4s\,{}^1P_1 \to 4s^2\,{}^1S_0$ cascade in calcium. Subsequent experiments were realised by Clauser [21] and by Fry and Thompson [22] (photons at 436 and 254 nm produced in $7^3S_1 \to 6^3P_1 \to 6^1S_0$ ^{200}Hg decay).

These pristine seminal experiments demonstrated a clear violation of Bell inequalities, with the only exception of [20] where a systematic error was later identified in the form of stresses in the walls of the bulb containing the electron gun and mercury vapour.

2.1.2 A first attempt to eliminate space like loophole

The experiment realised in 1982 in Orsay [23] was the culminating effort of this series of experiments, addressing for the first time the space-like separation between the two observers.

Here [23] entangled photons had wave-lengths of 422.7 and 551.3 nm respectively. They were generated by a cascade decay in calcium-40 $(J = 0) \to (J = 1) \to (J = 0)$ particularly suited for coincidence experiments since the life time of the intermediate level is rather short (about 5 ns).

The two entangled photons were then sent to two photomultipliers, preceded by a polarisation measurement set, 6 m apart from the source. The possibility of achieving a space-like separation between the two detections was due to the use of rapid acousto-optic switches operating at 50 MHz, addressing on two different paths photons with different selected polarisation.

It must be noticed that some concerns about the real elimination of space like loophole still remained due to the fact that the switch among polarisation measurement bases was not random, but periodic.

The observed violation of BI was of 5 standard deviations. However, only a very small fraction of generated pairs was really observed, leading to a very severe detection loophole

2.2 A new source: PDC experiments

2.2.1 Early experiments with PDC

After Orsay experiment for a few years no new experimental development in testing BI followed.

Essentially photon cascade experiments had reached their limit and new sources were necessary.

The breakthrough was represented by the development of sources of entangled photons based on parametric down conversion (PDC) at the end of 80s, together with the availability of Silicon Avalanche Photo Diode Detectors (SPAD), with a much higher detection efficiency that earlier phototubes.

PDC is an exclusively quantum effect arising from vacuum fluctuation that occurs in the interaction between a high energy electromagnetic field and the atoms of a nonlinear dielectric birefringent crystal, as lithium iodate ($LiIO_3$) or β-barium borate (BBO). This spontaneous emission consists in a very small probability ($\approx 10^{-9}$) decay of a photon with higher frequency in twin conjugated photons such that the sum of their frequencies and wave vectors correspond to the frequency and wave vector of the decaying photon (i.e. energy and momentum are conserved).

Two types of PDC exist. In type I PDC photons with the same polarisation are emitted in circumcentric cones. In Type II PDC photons with orthogonal polarisation are emitted in cones shifted due to birefringence.

The type I PDC two photons state presents a phase and momentum entanglement that can be eventually applied for a Bell inequality measurement by using interferometers.

On the other hand polarisation entangled states can be obtained by recombining on a beam splitter two correlated photons emitted in type I PDC after having rotated the polarisation of one of the two photons.

This scheme allowed a first application of PDC to test BI in 1988 [24, 25] . However, this kind of set ups required a post selection of the cases when both the photons had exited different ports of the beam splitter, limiting the usefulness to overcome the detection loophole.

Therefore, further developments of PDC BI tests required either the use of schemes based on interferometry or the developments of bright sources of polarisation entangled photons. These two lines of research will be the subject of the next two subsections.

2.2.2 Interferometric experiments

The first scheme for testing BI through an interferometer was proposed by Franson [26] and consisted in placing two Mach-Zehnder interferometers (MZI) on the path of the two correlated photons. If the long arms of the interferometers add a tunable phase ϕ_i ($i = 1, 2$ corresponding to interferometer 1,2 respectively) respect to the short ones, the final state is

$$\Psi_{fr} = \frac{1}{2}\left[|s_1\rangle|s_2\rangle + |l_1\rangle|l_2\rangle e^{i(\phi_1+\phi_2)} + e^{i(\phi_1)}|l_1\rangle|s_2\rangle + e^{i(\phi_2)}|s_1\rangle|l_2\rangle\right] \qquad (2.2)$$

where s, l denote short and long path, respectively.

If both photons follow either the short or the long path they are registered as a coincidence count from the detectors placed at the exits of the MZIs. If not they are not registered as coincidences (i.e. only 50% of the pairs is selected).

The coincidence rate of photons arriving simultaneously is (a_i being the destruction operator of a photon in mode i)

$$R_c \propto \eta_1\eta_2\langle\Psi_{fr}|a_1^\dagger a_2^\dagger a_1 a_2|\Psi_{fr}\rangle = \frac{1}{4}\eta_1\eta_2[1 + cos(\phi_1 + \phi_2)] \tag{2.3}$$

where $\eta_1\eta_2$ are quantum efficiencies of the detectors following the MZI on path 1 and 2 respectively. The striking fact about this equation for the coincidence rate is that it can be perfectly modulated by using either of the widely separated phase plates. This "non-local" effect can be used for testing Bell inequalities (e.g. as 1.4) using the phases ϕ_1 , ϕ_2.

Franson's scheme was realised the first time in [31], where a BBO crystal was pumped by an argon ion laser beam in collinear regime producing type I PDC photons pairs at 916 nm. A beam splitter separated the two photons that were then addressed to the two MZIs leading to a 7 standard deviations BI violation. Later a 16 standard deviations (only inferred from visibility) was achieved in [100, 28].

It is also worth mentioning that a modified scheme for exploiting momentum - phase entanglement was used in a previous experiment [30] where a 10 standard deviations violation was observed.

As mentioned, all these schemes suffered of 50% pairs postselection.

A set-up overcoming this problem, i.e. without the problem of eliminating the long-short terms, was then realised [34] by using type II PDC and polarising beam splitters in the interferometers, reaching (95.0 ± 1.4)% visibility (even if no real test of Bell inequalities was made). In this configuration either the horizontally polarised photon follows the long path, while its vertical twin brother travel through the long one, or both follow short paths.

An important advantage of interferometric schemes is that they can be "easily" implemented in fiber. This represents an important issue when one wants to propagate photons for long distances either for achieving clear space–like separation of measurements or for quantum communication application.

A first Franson's scheme experiment on long distance entanglement transmission was presented in [35], achieving a 86.9% visibility. Of the two entangled photons, the one at 820 nm was addressed to a single mode fiber interferometer, the other, with a 1.3 μm wave length, was propagated through a 4.3 km single-mode telecom communication fiber before reaching the interferometer.

In a following experiment a separation longer than 10 km [36, 37] allowed definitively closing locality loophole (if assuming that the passive coupler randomly selects which interferometer analyzes the photon), at the same time with a polarisation entanglement scheme [75] (where a real random selection was done). In this case CHSH inequality violation of $S = 2.92 \pm 0.18$ was achieved. Finally, a following upgrading [29] allowed realising measurements such that the temporal order of them was invertible by changing the reference frame.

2.2.3 Bright sources of polarisation entangled photons

The development of bright sources of polarisation entangled photons was the following significant step in the experimental path toward a final test of BI (see Fig.1).

In the middle of 90s two ideas circulate: one, proposed by Hardy [78] consisted in superimposing the emissions, with orthogonal polarisation, of two type I crystals, the other in selecting the intersections of two degenerate (i.e. with the same wavelength) emission cones (of orthogonal polarisation) in type II PDC [5].

For what concerns type II sources, the first example of source was realised in collinear regime. Here the two degenerate photons are emitted in two tangent cones, then by selecting their intersection point the two orthogonally polarized correlated photons exit in the same direction and can be separated by a beam splitter, generating an entangled state when one postselects events in which they have exited the beam splitter on different directions

This set up allowed a 10 standard deviations violation of BI [38].

However, in collinear regime, due to postselection after the beam splitter, only a 50% of original pairs is selected. Therefore, it is advantageous to use non collinear schemes [70], selecting the two intersections of orthogonally polarised cones. It must be noticed that the state deriving from this superposition is not yet entangled, since, due to birefringence in the non-linear crystal, ordinary and extraordinary photons propagate with different velocities and different directions inside this medium. Thus, longitudinal and transverse walk-offs (i.e. the optical beam displacement due to birefringence) must be compensated for restoring indistinguishability between the two polarizations and producing an entangled state. This can be achieved by inserting some birefringent medium along the optical path of photons.

Ref. [8] realised the first non-collinear type II PDC set up by pumping a 3mm long BBO crystal with a 351 argon laser beam. The longitudinal (the transverse one being negligible) walk off (385 fs) was compensated by an additional BBO crystal. All four Bell states were generated leading to a 102 standard deviations violation of CHSH inequality ($S = -2.6489 \pm 0.0064$).

In the following years several very bright sources have been realised [71, 72, 73, 74] up to dozens of coincidences counts per second for mW pump power ($77s^{-1}$ in [71]) in a traditional crystal and even up to a measured coincidence flux of hundreds per second for mW of the pump by using Periodically Poled crystals (i.e. crystals whose susceptibility is periodically modulated producing a constructive interference in the emission), e.g. $300s^{-1}$ in [72].

These bright sources found application both in closing without any doubt space like loophole [75] and in long distance open air transmission and test of BI (more than 600m with $S = 2.41 \pm 0.10$ in [74]; 13 km in [76] with $S = 2.45 \pm 0.09$, and finally 144 km in [77]).

As hinted, in developing bight sources of polarisation photons entangled states an alternative to type II PDC stems from superimposing the emissions, with orthogonal polarizations, of two type I PDC crystals whose optical axes are orthogonal [78] generating the state (eventually non maximally entangled when $f \neq 1$ [72]):

$$|\psi_{NME}\rangle = \frac{|H\rangle|H\rangle + f|V\rangle|V\rangle}{\sqrt{(1+|f|^2)}} \tag{2.4}$$

where the value of the parameter f can be tuned by tuning the pump beam properties.

In 1999 two different set–ups realising this scheme and demonstrating large BI violation were produced. In the first, the emission of two thin adjacent 0.59 mm long type I BBO crystals was superimposed [79] achieving $S = 2.7007 \pm 0.0029$. The scheme is very simple and then found found widespread use. Nevertheless, the short length of the crystals is dictated by the need of having a good superposition of the subsequent emissions.

The second was based on a more complicate set–up, where the emissions of two 1cm long $LiIO_3$ crystals were superimposed by an optical condenser [80].

These two setups were realized by using a continuous wave pump laser. However, for timing reasons, the pulsed regime is preferable. When the pump pulses are very short (femtoseconds regime), the photon pairs production amplitudes corresponding to different regions inside the crystal do not sum coherently [81]. This initially required either using thin ($\approx 100\mu m$) non-linear crystals [82] or narrow band spectral filters (for increasing the coherence length) [83, 84, 85]. However, these solutions significantly reduce the available flux of entangled photon pairs.

More recently, bright sources in pulsed regime were realised by inserting non-linear crystals in interferometers [39, 86, 87, 88] (as a Sagnac one).

Altogether, efficient bright sources of bipartite polarisation entangled photons became available in the second part of 90s. Very high visibility experiments were realised, measuring BI violation next to theoretical bound (Cirel'son bound), see Fig.2.

Space like loophole was clearly eliminated and other loopholes concerning the statistic of the samples [89] did not appear to be a major concern, since a clear elimination of detection loophole would largely solve these issues as well.

Thus, the last step for a conclusive test of BI remained the elimination of detection loophole (at the same time of space like one), having approached in these last experiment at most 30% total detection efficiency: thus some space was still left for specific LRT [2, 12, 90] exploiting this loophole (and some of them was falsified by specific experimental tests [2, 91]).

Two possible directions were considered (the possibility of using higher dimensional systems requiring smaller detection efficiencies being substantially limited by the difficulty of producing these states): either to abandon photons for other systems or to look for high detection efficiency photo-detectors.

2.3 Toward a conclusive test of Bell inequalities

2.3.1 Test of local realism with atoms

Even if most of tests of local realism performed up to now have been realised with photons, several other systems were considered. Some of them has mainly a historical interest, since the problems connected with their use stopped the research in this sense. Among them we can mention polarisation correlation of $^1S_0$ proton pairs produced in nuclear reactions [69] or K mesons [40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58] (where detection loophole reappears due to decay channel selection [63, 64, 65]). In other cases they are still very far from a conclusive test, as neutron interferometry [66], where large losses in interferometers make impossible to eliminate the detection loophole, or atoms entangled in a superposition involving two circular Rydberg states produced by single photon exchange in high Q cavity [67] where it is difficult to have high purity of states.

Also entangled superconducting systems, even if they allowed to close detection loophole [97], are probably not the best choice for eliminating space like loophole at the same time.

Larger interest has the use of entangled atoms or photon-atom entangled states [68] (that should nonetheless solve the detection efficiency problem for the photon).

In principle atomic states can be detected with very high efficiency an, indeed, in an experiment with Be^+ ions [94] a clear violation of BI was observed with a detection efficiency of $\approx 98\%$. However, in this case the measurement of the two ions was not disjoined. More recently, Yb^+ in two traps separated of 1 metre [95] and 20 metres [96] were used. Nevertheless, for eliminating the space like loophole, at least 300 m would be required.

In principle these schemes could lead to a conclusive test of BI, in particular in configurations where distant atoms are entangled by exchanging entangled photons.

2.4 The elimination of detection loophole in photon experiments

As mentioned the most important problem of BI tests with photons is the detection efficiency of SPADs, that, albeit by far larger than the photubes one, is substantially limited to be smaller than 70%, preventing, also in principle, a final test of BI without fair sampling assumption.

A possible way out to this problem is represented by cryogenic detectors, that in principle can have detection efficiency next to 1.

After a decennium without specific progresses in testing BI with photons, recently two experiments claimed to have overcome detection loophole by exploiting Transition-Edge Sensors (TESs), i.e. microcalorimeters based on a superconducting thin film operating next to transition and working as a very sensitive thermometer. TESs can effectively reach very high detection efficiency, have a discrimination in number of impinging photons and are substantially free from dark counts.

In the first experiment [98] degenerate pairs of photons were produced in a Sagnac type II PDC source where a periodically poled KTP was pumped by a laser beam at 405 nm. Non-maximally entangled states (with a weight $f \approx 0.3$) were used for reducing the detection efficiency needed for eliminating detection loophole. After a polarisation measurement, performed with a half-wave plate followed by a calcite polariser, both photons were addressed through fibres to two TESs (with a total arm efficiency of 73.8% and 78.6% respectively).

Possible lurches with coincidence and production rates loopholes, that were suggested for these experiment, were later shown not to be a real problem [99].

In the second one [100] non maximally entangled pairs of photons (with a weight $f \approx 0.26$) were produced by pumping two thin type I BBO crystals with a 355 nm laser beam pulsed (avoiding in this way problems with coincidence loophole) with a 25 kHz repetition rate through a Pockels cell. Again entangled photons were detected by TES detectors.

Of course the task of eliminating at the same time detection and space like loopholes is formidable, since propagating the photons at a sufficient distance for eliminating the space like loophole unavoidably compromises the overall channel efficiency.

Nevertheless, both groups seem to proceed in this direction. Thus, there are chances that a conclusive experiment on BI will arrive from photons experiments in the next years.

3

Conclusions

In this ten lustra since Bell's paper a continuous progress led from seminal experimental tests of Bell inequalities to several experiments with a high statistical violation of them.

This huge experimental work leaves very few doubts on the fact the the world described by quantum mechanics is really non-local and nowadays this "spooky action at distance" has become a common notion not only among physicists, but diffused in a larger scientific community being now a resource for quantum technologies.

Even if a very small space is left for a different result, a completely loophole free violation of Bell inequality is still missing. Nonetheless, as described in this contribution, a conclusive result could be realised within a few years.

In conclusion, Bell inequalities and their experimental tests have represented a fundamental contribution to our understanding of nature, confirming the most paradoxical aspect of quantum mechanics, i.e. non-locality and allowing to exclude any possible deterministic extension of quantum mechanics.

Furthermore, quantum correlations have not only played a fundamental role in the discussion on foundation of quantum mechanics [**?**], but also represent nowadays a fundamental tool for developing quantum technologies and in this field BI are become often a tool for testing quantum resources in ordinary laboratory work.

If Bell inequalities have played an exponential growing role in the last 50 years physics, all suggests that their role will keep on growing in the next 50 one both in application to quantum technologies and as instrument for understanding the secrets of Nature.

Felix qui potuit rerum cognoscere causas.

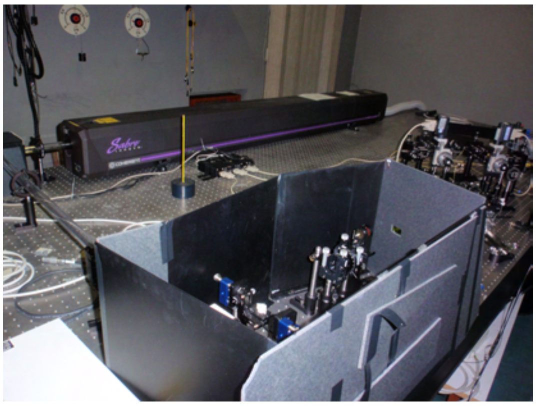

Figure 3.1 Typical set-up for Bell inequalities measurement with PDC light. In foreground one can observe the non-linear crystals where entangled photons are generated by the pump beam emitted by a laser (in background). On the right hand side the polarisation selection apparatuses and photo-detectors.

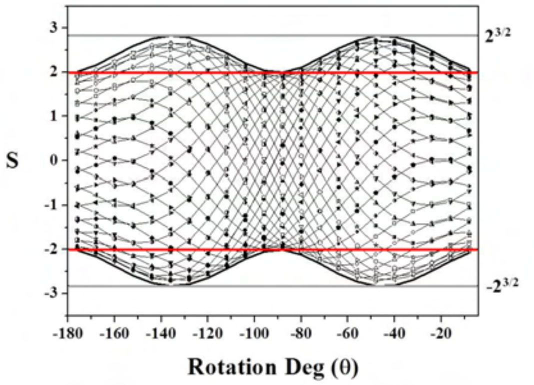

Figure 3.2 Measured values of the CHSH parameter S (F.Bovino,I.P. Degiovanni concession) are plotted versus θ and the various curves are associated to different values of the parameter ξ (used to parametrize the entangled states $|\varphi(\xi)\rangle = \cos(\xi)|\phi^+\rangle + \sin(\xi)|\psi^-\rangle$). The thicker curves correspond to the theoretically predicted S bounds [92]. Plot points are the measured values [93] of S for any pair $\{\xi, \theta\}$. A good qualitative and quantitative agreement between theoretical and experimental bounds, even if the experimental upper (lower) bounds stands slightly below (above) the theoretical predictions. These effects are, as usual, imputable to noise and imperfections associated to the polarization preservation and measurement of same setup components, namely HWPs, PBSs, and fibers.

References

[1] Bell,J. *Physics*, **1**, (1964) 195.
[2] M.Genovese, *Physics Reports*, **413**, (2005) 319.
[3] N.Brunner et al., *Rev. Mod. Phys.* **86**, (2014) 419.
[4] J. Clauser et al., *Phys. Rev. Lett.* ,**23**, (1969) 880.
[5] A. Garuccio, in "Fundamental Problems in Quantum Theory", Ed. D. Greenberger , (New York Academy of Sciences, 1995);
[6] J.Larson and R.Gill, *EPL* ,**67**, 707 (2004).
[7] T.E. Kiess et al., *Phys. Rev. Lett.* , **71** (1993) 3893.
[8] P.G. Kwiat et al., *Phys. Rev. Lett.* ,**75**, 4337 (1995).
[9] E. Santos, *Phys. Lett.* A ,**212**, (1996) 10.
[10] L. De Caro and A. Garuccio *Phys. Rev.* A, **54** (1996) 174.
[11] J.F. Clauser and M.A. Horne, *Phys. Rev.* D ,**10**, (1974) 526.
[12] P.M. Pearle, *Phys. Rev.* D ,**2**, (1970) 1418.
[13] P. H. Eberhard, *Phys. Rev.* A, **47**, (1993) R747. ,
[14] M. Fiorentino, *Phys. Rev.* A, **69**, 041801(R) (2004).
[15] L.R. Kasday, J.D. Ullman and C.S. Wu, Nuovo. Cim. B 25 (1975) 633.
[16] G. Faraci et al., *Lett. Nuov. Cim.* **9**, (1974) 607.
[17] A.R. Wilson et al., *J. Phys. G.*,**2**, (1976) 613.
[18] M. Bruno et al., *Nuovo Cim. B*, **40** (1977) 142.
[19] S.J. Freedman and J.F. Clauser, *Phys. Rev. Lett.* ,**28**, (1972) 938.
[20] R.A. Holt and F.M. Pipkin, unpublished (1973).
[21] J.F. Clauser, *Phys. Rev. Lett.* ,**36**, (1976) 1223.
[22] E.S. Fry and R.C. Thompson, *Phys. Rev. Lett.* ,**37**, (1976) 465.
[23] A. Aspect et al., *Phys. Rev. Lett.* ,**49**, (1982) 1804.
[24] Y.H.Shih et al., *Phys. Rev.* A, (1993) 1288.
[25] Z.J. Ou and L. Mandel, *Phys. Rev. Lett.* ,**61**, (1988) 50.
[26] J. P. Franson, *Phys. Rev. Lett.* ,**62**, (1989) 2205. ,
[27] P.G. Kwiat, A.M. Steinberg and R.Y. Chiao, *Phys. Rev.* A, **47**, (1993) R2472.
[28] R.Y. Chiao, P.G. Kwiat and A.M. Steinberg, Quant. Semicl. Opt. **7** (1995) 259.
[29] H. Zbinden et al., *J. Phys. A: Math. Gen.* **34** (2001) 7103.
[30] J. G. Rarity, and P. R. Tapster, *Phys. Rev. Lett.* ,**64**, (1990) 2495.
[31] J. Brendel et al. *Eur.Phys.Lett.* **20** (1992) 275.
[32] P. G. Kwiat el al, *Phys. Rev.* A, **41**, (1990) 2910.,
[33] W. Tittel et al, *Phys. Rev. Lett.* ,**81**, (1998) 3563.
[34] D.V. Strekalov et al., *Phys. Rev.* A, **54**, (1996) R1.

[35] P.R. Tapster, J.G. Rarity and P.C.M. Owens, ,*Phys. Rev. Lett.* ,73 (1994) 1923.

[36] W. Tittel et al., *Phys. Rev. Lett.* ,**81**, (1998) 3563.

[37] W. Tittel et al., *Phys. Rev.* A, **59**, (1999) 4150.

[38] T.E. Kiess et al., *Phys. Rev. Lett.* , **71**, (1993) 3893.

[39] Y. Kim et al., *Phys. Rev.* A, **63**, (2001) 060301(R); *Phys. Rev.* A, **63**, (2001) 062301.

[40] F. Uchiyama, *Phys. Lett.* A ,**231**, (1997) 295.

[41] R.A. Bertlmann, W. Grimus and B.C. Hiesmayr, *Phys. Lett.* A ,**289**, (2001) 21.

[42] F. Benatti and R. Floreani, *Phys. Rev.* D ,**57**, (1998) R1332.

[43] F. Benatti and R. Floreani, *Eur. Phys. Journ. C* **13** (2000) 267.

[44] R.A. Bertlmann and W. Grimus, *Phys. Lett.* B ,**392**, (1997) 426; *Phys. Rev.* D ,**58**, (1998) 034014;

[45] G.C. Ghirardi et al., The DAΦNE Physical handbook , edited by L. Maiani, G. Pancheri, and N. Paver (INFN, Frascati, 1992) Vol. I.

[46] J. Six, *Phys. Lett.* B ,**114**, (1982) 200.

[47] P.H. Eberhard, *Nucl. Phys. B* **398** (1993) 155.

[48] A. Di Domenico, *Nucl. Phys. B* **450** (1995) 293.

[49] B. Ancochea, *Phys. Rev.* D ,**60**, (1999) 094008.

[50] A. Bramon and M. Nowakowski, *Phys. Rev. Lett.* ,**83**, (1999) 1.

[51] N. Gisin and A. Go, Am.J.Phys. **69**, (2001) 264.

[52] B.C. Hiesmayr, Found.Phys.Lett. **14**, (2001) 231.

[53] R.A. Bertlman et al., *Phys. Lett.* A ,**332**, (2004) 355; A. Bramon et al., quant-ph0410122.

[54] R.H. Dalitz and G. Garbarino, quant-ph 0011108.

[55] P. Privitera and F. Selleri, *Phys. Lett.* B ,**296**, (1992) 261.

[56] F. Selleri, *Phys. Rev.* A, **56**, (1997) 3493.

[57] A. Pompili and F. Selleri, *Eur. Phys. J. C* ,**14**, (2000) 469;

[58] A.Bramon and G.Garbarino, *Phys. Rev. Lett.* ,**88**, (2002) 040403.

[59] E. Leader and E. Predazzi, An introduction to gauge theoris and modern particle physics, (Cambridge Univ. press, Cambridge, 1996).

[60] Particle Data Book, S. Eidelman et al., *Phys. Lett.* B ,592 (2004) 1; G.V. Dass and K.V.L. Sarma, *Eur. Phys. J. C* ,**5**, (1998) 283.

[61] W.H. Furry, **Phys. Rev.** 49 (1936) 393.

[62] H. Albrecht, *Phys. Lett.* B ,**192**, (1987) 245.

[63] M.Genovese et al., *Phys. Lett.* B ,**513**, (2001) 401.

[64] M. Genovese et al., *Found. of Phys* ,**32**, (2002) 589.

[65] M. Genovese, *Phys. Rev.* A, **69**, (2004) 022103.

[66] Y. Hasegawa et al., *Nature* ,**425**, (2003) 45.

[67] E. Hangley et al., *Phys. Rev. Lett.* ,**79**, (1997) 1.

[68] D.L. Moehring et al., *Phys. Rev. Lett.* ,**93**, (2004) 090410.

[69] M. Lamehi-Rachti and W. Mittig, Phys. Rev. **14**, (1976) 2543.

[70] P.G. Kwiat et al., *Phys. Rev.* A, **49**, 3209 (1994).

[71] C. Kurtsiefer, M. Oberparlaiter and H. Weinfurter, *Phys. Rev.* A, 64, 023802 (2001).

[72] C. E. Kuklewicz et al., *Phys. Rev.* A, **69**, 013807 (2004).

[73] G. Di Giuseppe et al., *Phys. Rev.* A, **66**, 013801 (2002).

[74] M. Aspelmeyer et al., *Science* ,**301**, 621 (2003).

[75] G. Weihs et al., *Phys. Rev. Lett.* ,**81**, (1998) 5039.

[76] C.-Z. Peng et al., *Phys. Rev. Lett.* ,**94**, 150501 (2005).

[77] R. Ursin et al., quant-ph 0607182; *Nature* ,Physics **3**, 481 (2007).

[78] L. Hardy, *Phys. Lett.* A ,**161**, 326 (1992)

[79] A. G. White et al., *Phys. Rev. Lett.* , **83**, 3103 (1999).

[80] G. Brida et al., *Phys. Lett.* A , **299**, 121 (2002).

[81] T.E. Keller and M.H. Rubin, *Phys. Rev.* A, **56**, 1534 (1997).

[82] A. V. Sergienko et al., *Phys. Rev.* A, **60**, R2622 (1999).

[83] W. P. Grice and I.A. Walmsley, *Phys. Rev.* A, 56, 1627 (1997).

[84] W. P. Grice et al., *Phys. Rev.* A, 57, **R2289**, (1998).

[85] G. Di Giuseppe et al., *Phys. Rev.* A, **56**, R21 (1997) .

[86] H. Poh et al., *Phys. Rev.* A, **80**, 043815 (2009).

[87] M. Barbieri et al., quant-ph 0303018.

[88] C. Cinelli et al., quant-ph 0406148.

[89] A. Khrennikov, Adv. Sci. Lett. 2 (2009) 488; Accardi, L. "Foundations of Probability ans Physics - 5", Ed.s L.Accardi et al., AIP conf. proc. 1101 (2009, New York) 3; C. Garola and S. Sozzo, *EPL* ,, **86** (2009) 20009;

[90] N.Gisin and B. Gisin, *Phys. Lett.* A ,**260**, (1999) 323; T.W. Marshal and E. Santos, *Found. of Phys* ,**18**, (1988) 185; A. Casado et al., *Eur. Phys. J. D* ,**11** (2000) 465; D **13** (2001) 109.

[91] G. Brida et al., *Phys. Lett.* A ,**328** (2004) 313; G. Brida, M. Genovese, F.Piacentini. *Eur. Phys. J. D* ,**44**, (2007) 577

[92] A. Cabello, *Phys. Rev. Lett.* ,**92** (2004) 060403.

[93] F. A. Bovino, G. Castagnoli, I. P. Degiovanni, and S. Castelletto, *Phys. Rev. Lett.* ,**92**, 060404 (2004).

[94] M.A. Rowe et al., *Nature* ,**409**, (2001) 791.

[95] Matsukevitch et al, *Phys. Rev. Lett.* ,**100**, 140404 (2008).

[96] Hofman, J. et al. *Science* ,337 (2012) 72.

[97] M.Ansmann et al, *Nature* **461** (2009) 504.

[98] M.Giustina et al., *Nature* **497** (2013) 227.

[99] J. Larsson, M. Giustina, J. Kofler, B. Wittmann, R. Ursin and S. Ramelow *Phys. Rev.* A, **90**, (2014) 032107.

[100] B.G. Christensen et al., *Phys. Rev. Lett.* ,**111** (2013) 130406.