Interactions and Inequality

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Abstract

John Bell was a staunch supporter of the dynamical wave function collapse approach to making a well-defined quantum theory. Through letters from him, I reminisce on my handful of interactions with him, all of which were memorable to me. Then, non-locality, violation of Bell's inequality and some further implications are discussed within the framework of the CSL (continuous spontaneous localization) model of dynamical collapse.

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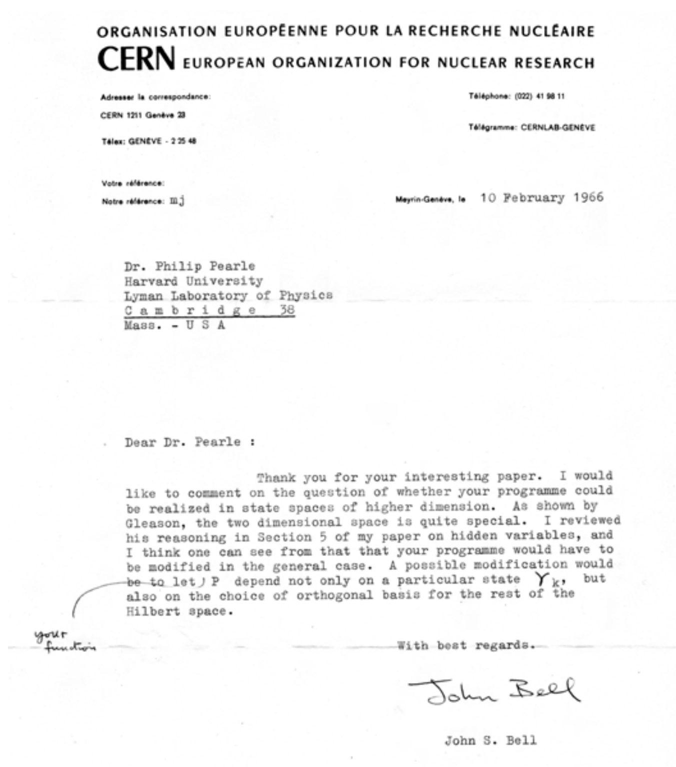
I. INTERACTIONS

I was an Instructor at Harvard with a newly minted PhD, earned with an indifferent thesis from MIT on particle theory. It took me two years to come to terms with, dare I say it, that I did *not care* about the S-matrix. I was going to be out of a job anyway in 1966 so, in the Fall of 1965, I put my full energy into writing my very first paper, about what I did care. It was, to my eyes, that there is something deeply wrong with the quantum theory I had been taught. It is that the rules of how to use it are inadequate. The title of the paper[1] was "Elimination of the Reduction Postulate from Quantum Theory and A Framework for Hidden Variable Theories." It was a long title, because it was really two rather separate papers.

The first was based upon the perceived inadequacy, that the collapse (reduction) postulate of the theory is ill-defined.

The second stated some postulates I thought a good hidden variable theory ought to obey, and gave a model for their satisfaction in a two-dimensional (i.e., spin 1/2) Hilbert



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Meyrin-Genève, le 10 February 1966

Dr. Philip Pearle
Harvard University
Lyman Laboratory of Physics
Cambridge 38
Mass. - U S A

Dear Dr. Pearle :

Thank you for your interesting paper. I would like to comment on the question of whether your programme could be realized in state spaces of higher dimension. As shown by Gleason, the two dimensional space is quite special. I reviewed his reasoning in Section 5 of my paper on hidden variables, and I think one can see from that that your programme would have to be modified in the general case. A possible modification would ~~be to let~~ P depend not only on a particular state Υ_k, but also on the choice of orthogonal basis for the rest of the Hilbert space.

your function

With best regards.

John Bell

John S. Bell

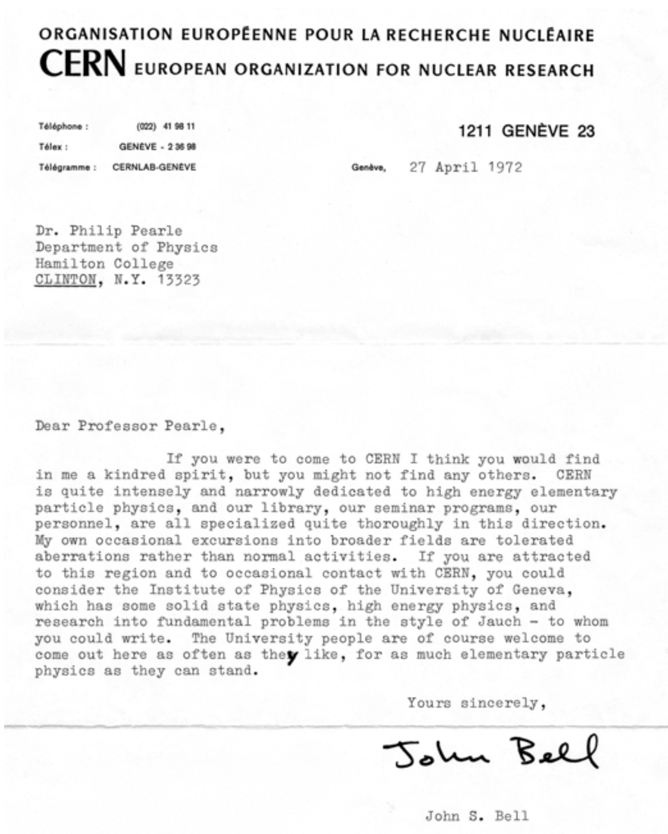
FIG. 1. Letter from John Bell, 2/1966

space. Try as I might, I was not able to generalize my model to a higher dimensional (i.e., spin 1) Hilbert space but I put it out there for someone more clever than I to achieve. I sent the paper to a very few people whom I thought might be interested. One was John Bell, and he replied(Fig.1)!

So, someone more clever than I pointed out that generalizing my model could not be achieved. I therefore excised the model and, after rewriting and retitling, published my very first paper[2]!

After a three year stint at Case Institute of Technology, which graduated to become Case Western Reserve University while I was there, in 1969 I became ensconced at Hamilton College. In 1973, through good behavior, it was time for my first sabbatical. In the Spring of 1972, I wrote to John Bell, asking him if I would be able to spend it at CERN. He wrote the note in Fig.2.

So, my family and I ended up spending a glorious year at the University of Geneva, under the benevolent eye of Josef Jauch (who, tragically, died at age 60 a couple of months after



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Genève, 27 April 1972

Dr. Philip Pearle
Department of Physics
Hamilton College
CLINTON, N.Y. 13323

Dear Professor Pearle,

If you were to come to CERN I think you would find in me a kindred spirit, but you might not find any others. CERN is quite intensely and narrowly dedicated to high energy elementary particle physics, and our library, our seminar programs, our personnel, are all specialized quite thoroughly in this direction. My own occasional excursions into broader fields are tolerated aberrations rather than normal activities. If you are attracted to this region and to occasional contact with CERN, you could consider the Institute of Physics of the University of Geneva, which has some solid state physics, high energy physics, and research into fundamental problems in the style of Jauch - to whom you could write. The University people are of course welcome to come out here as often as they like, for as much elementary particle physics as they can stand.

Yours sincerely,

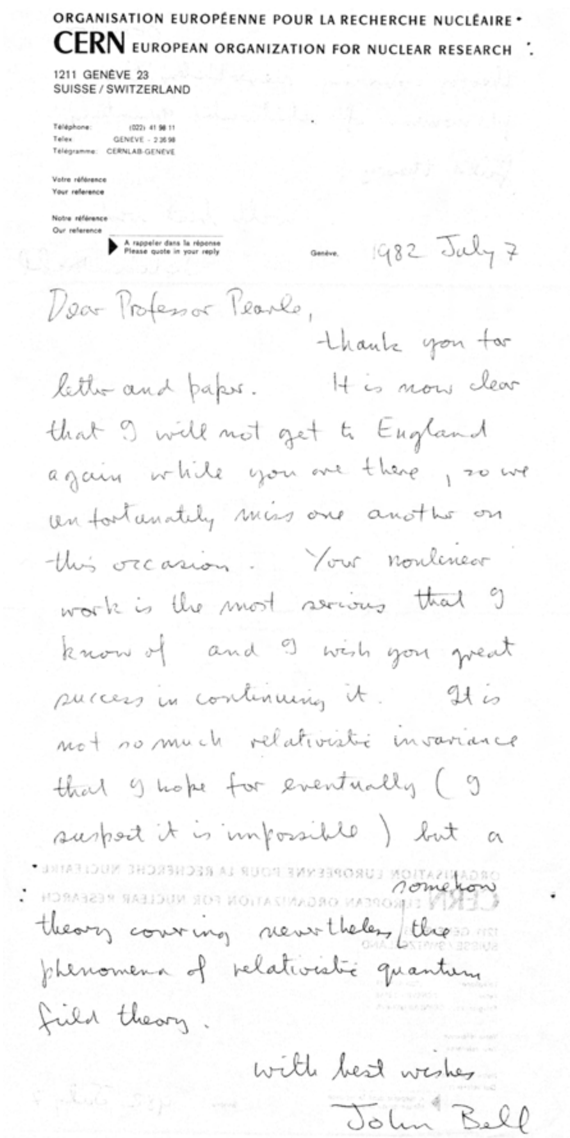
John Bell

John S. Bell

FIG. 2. Letter from John Bell, 4/1972

we had left Switzerland). While there, I visited with John Bell at CERN only once, and we discussed my beginning excursions into constructing a dynamical theory of wave function collapse, which emerged a few years later[3] . He was interested and supportive, but I felt diffident enough to not bother him again without something new to report.

While on sabbatical once more, this time in 1981-2 at Oxford with the group of Roger Penrose (whom gravitational considerations had led to argue for dynamical collapse[4]), it looked like John Bell might come by, but it fell through. I attach a note he sent(Fig.3), which contains an interesting concluding remark.



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A rappeler dans la réponse
Please quote in your reply

Genève, 1982 July 7

Dear Professor Pearle,

thank you for letter and paper. It is now clear that I will not get to England again while you are there, so we unfortunately miss one another on this occasion. Your nonlinear work is the most serious that I know of and I wish you great success in continuing it. It is not so much relativistic invariance that I hope for eventually (I suspect it is impossible) but a theory covering nevertheless somehow the phenomena of relativistic quantum field theory.

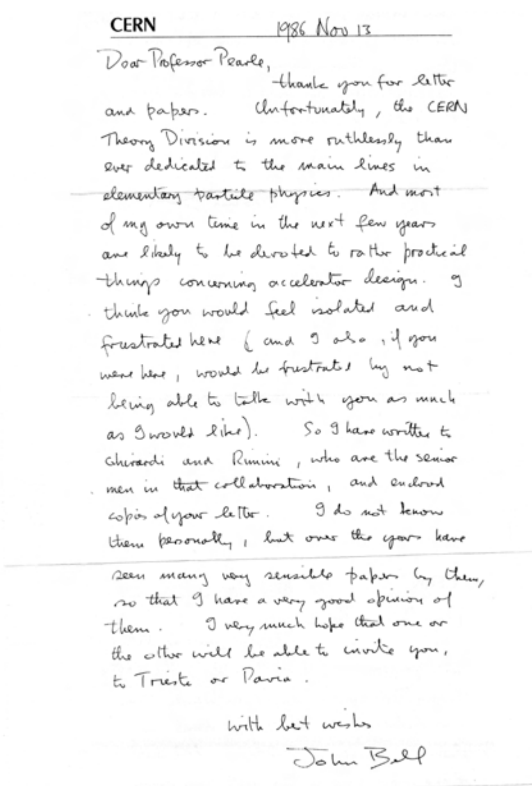
With best wishes
John Bell

FIG. 3. Letter from John Bell, 7/1982

Sabbatical time yet again, 1987-88, which proved to be the most productive and exciting time of my professional life: July-October at South Carolina working with with Yakir Aharonov and Lev Vaidman (which resulted in a joint paper on the Aharonov-Casher effect) and getting to know Shmuel Nussinov and Frank Avignone (with whom I subsequently wrote papers) and Jeeva Anandan, November to January at Cambridge with Michael Redhead, Jeremy Butterfield and Rob Clifton (who subsequently edited a book to which I contributed), February to April at Trieste with GianCarlo Ghirardi, and May-June at Pavia with Alberto Rimini.

The last two engagements, which led to my discovering CSL, I owed to John Bell. He had sent me a preprint of "Are there Quantum Jumps?"[5] from which I learned about Ghirardi, Rimini, Weber's Spontaneous Localization (SL) theory of wave function collapse. I wrote to him on Nov. 5, 1986, expressing my appreciation for their work, and I added that I had a sabbatical coming up, ending with:

"I teach at a small liberal arts college in the U. S. and thus have relatively little time for research and interaction with other physicists, so these sabbaticals are precious to me.


CERN 1986 Nov 13

Dear Professor Pearle,
thank you for letter and papers. Unfortunately, the CERN Theory Division is more ruthlessly than ever dedicated to the main lines in elementary particle physics. And most of my own time in the next few years are likely to be devoted to rather practical things concerning accelerator design. I think you would feel isolated and frustrated here (and I also, if you were here, would be frustrated by not being able to talk with you as much as I would like). So I have written to Ghirardi and Rimini, who are the senior men in that collaboration, and enclosed copies of your letter. I do not know them personally, but over the years have seen many very sensible papers by them, so that I have a very good opinion of them. I very much hope that one or the other will be able to invite you, to Trieste or Pavia.

with best wishes
John Bell

FIG. 4. Letter from John Bell, 11/1986

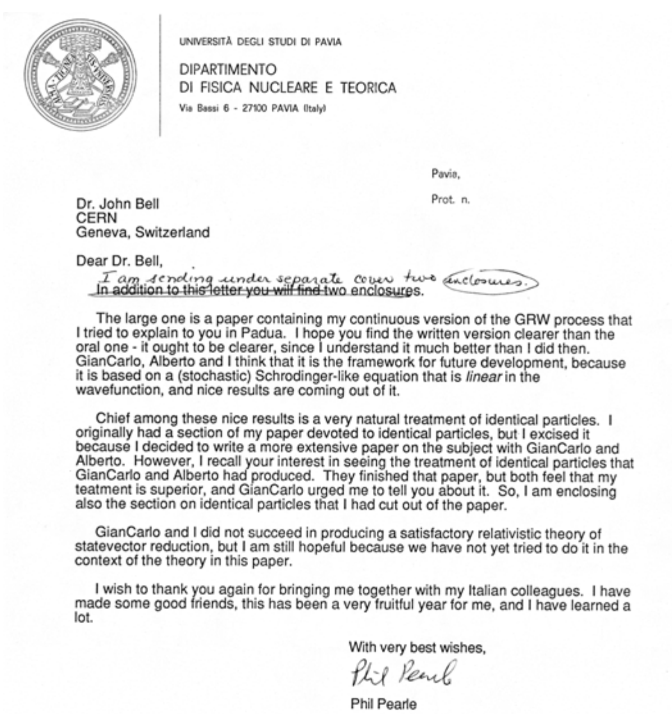
I recall the one time I visited you at CERN, in 1973, when I was on sabbatical at the University of Geneva with Prof. Jauch, you let me know that the Powers at CERN were not too encouraging of research in Foundational questions but, nonetheless, I will ask you if you think CERN might be possible or appropriate. Or, perhaps you might be able to recommend a suitable place, for example, do you think Professors Ghirardi, Rimini or Weber might be interested in inviting me for part of that time? I do not know to which one of them it would be appropriate for me to write. I would appreciate any advice or help you could offer."

I received an immediate response.(Fig.4)

GianCarlo and I worked hard at making a relativistic version of the SL theory, but did not succeed in finding one (subsequently found by Euan Squires and Chris Dove[6] and later by Rodi Tumulka[7]). But, during this time, I had an inkling of what turned out to be CSL.

A week before I was to leave Trieste for Pavia, GianCarlo and I drove two hours to the University of Padua, to hear John Bell giving a talk entitled "Six Possible Realities." Afterwards, we had a chance to talk with him. In trying to explain my new ideas, I realized that I didn't understand them very well.

So, arriving at Pavia, I asked Alberto if he would mind my trying to clarify my thoughts,



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DIPARTIMENTO
DI FISICA NUCLEARE E TEORICA

Via Bassi 6 - 27100 PAVIA (Italy)

Pavia,

Prot. n.

Dr. John Bell
CERN
Geneva, Switzerland

Dear Dr. Bell,

I am sending under separate cover two enclosures.
~~In addition to this letter you will find~~ two enclosures.

The large one is a paper containing my continuous version of the GRW process that I tried to explain to you in Padua. I hope you find the written version clearer than the oral one - it ought to be clearer, since I understand it much better than I did then. GianCarlo, Alberto and I think that it is the framework for future development, because it is based on a (stochastic) Schrodinger-like equation that is *linear* in the wavefunction, and nice results are coming out of it.

Chief among these nice results is a very natural treatment of identical particles. I originally had a section of my paper devoted to identical particles, but I excised it because I decided to write a more extensive paper on the subject with GianCarlo and Alberto. However, I recall your interest in seeing the treatment of identical particles that GianCarlo and Alberto had produced. They finished that paper, but both feel that my teatment is superior, and GianCarlo urged me to tell you about it. So, I am enclosing also the section on identical particles that I had cut out of the paper.

GianCarlo and I did not succeed in producing a satisfactory relativistic theory of statevector reduction, but I am still hopeful because we have not yet tried to do it in the context of the theory in this paper.

I wish to thank you again for bringing me together with my Italian colleagues. I have made some good friends, this has been a very fruitful year for me, and I have learned a lot.

With very best wishes,

Phil Pearle

Phil Pearle

FIG. 5. Letter to John Bell, 5/1988

instead of working on a project he had proposed, and he graciously acquiesced. In 17 days, I had finished my paper[8]. Alberto read it, and invited GianCarlo over the weekend without my knowledge, with the result that I was surprised on Monday to learn of their excitement about it, and also learn of some excellent ideas of their own, so we decided to write another paper together[9]. Here is the letter I wrote to John about this development.(Fig.5)

(Incidentally, a modification of SL that satisfactorily treats identical particles was found by Euan Squires and Chris Dove[10] and independently by Rodi Tumulka[7].)

I saw John Bell two more times.

At a conference in Erice in August 1989 entitled "Sixty-Two Years of Uncertainty," he gave a simply wonderful talk, "Toward an Exact Quantum Mechanics," but published under the bolder title "Against 'Measurement' "[11]. In it, he incisively critiqued the texts of L.D. Landau/ E.M. Lifshitz, K. Gottfried and a paper by N. G. van Kampen, with regard to their treatments of the mysterious transition of the state vector from AND to OR as he put it (i.e., collapse), condemning them for their imprecision by the acronym he introduced specially for the purpose, FAPP (for all practical purposes).

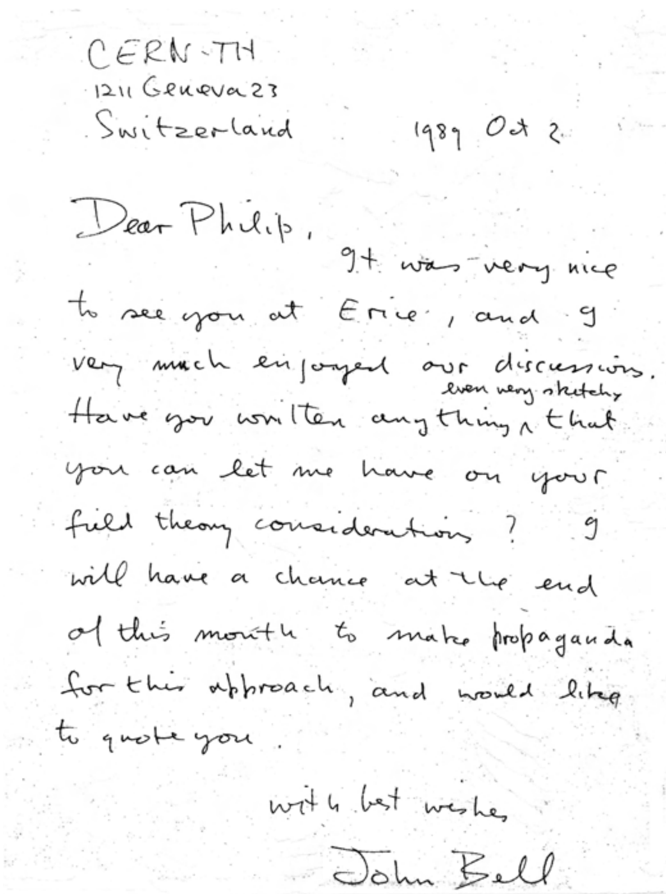
David Mermin was sitting with Vicky Weisskopf up front, and David rose to defend orthodoxy. After listening patiently to a few minutes of this remonstration, Bell interrupted with the single loud word "FAPPtrap" (for those unfamiliar with this idiom, "claptrap" is an 18th century word referring to theatrical techniques employed to garner applause, but which has come to mean "absurd or nonsensical talk or ideas"). This broke up the audience, and David sat down.

Afterwards, I spoke with John and expressed my appreciation, remarking that during his talk I had this vision of him as a knight, behind whose shield people like me who dared to challenge orthodoxy could safely work. While thanking me he commented, nonetheless, that he would be critical of me as well should he perceive the necessity. Apparently, the necessity arose a few days later.

Prior to the mandatory banquet, a number of us were standing around and talking about dynamical collapse. This included Abner Shimony: in my early attempts at constructing collapse dynamics, Abner had argued for the necessity of what is now called the "tails" criteria. This is that the state vector has to end up with 0 amplitude in a finite time for any macroscopic state but one. In my pre-CSL dynamical models, I had acquiesced and used this as a criteria to distinguish between some alternatives[12]. However, with the advent of CSL, for which other amplitudes become exponentially small but not 0, I have found a number of good reasons to dis-acquiesce.

Abner still maintained that position. So, I raised the tails issue as if it was a concern of mine, thinking it would be nice for Abner to hear John's opinion. In a disquisition which began with the booming phrase for all to hear in his inimitable charming brogue, "Phil, don't be a ninny (for those unfamiliar with this idiom, "ninny" is an 16th century word referring to "innocent," but which has come to mean "a foolish or simple-minded person") which made me feel rather warm, he stressed that one can unambiguously tell the difference between a really small amplitude term compared to very much larger amplitude term. I had already believed this and, though tarted up, it is the main argument today[13], but Abner sticks by his guns.

At Erice, I also gave a talk, on an attempt to make a relativistic version of CSL[14] which culminated in a paper by Ghirardi, Grassi and myself[15]. While talking, I would look up



CERN-TH
1211 Geneva 23
Switzerland

1989 Oct 2.

Dear Philip,

It was very nice to see you at Erice, and I very much enjoyed our discussions. Have you written anything even very sketchy that you can let me have on your field theory considerations? I will have a chance at the end of this month to make propaganda for this approach, and would like to quote you.

with best wishes

John Bell

FIG. 6. Letter from John Bell, 10/1989

every once in a while to see John sitting at the end of the row with his feet in the aisle and a grin on his face. Soon after the conference, I got a note from him(Fig.6), so I sent him as much of [14] as I had finished, and received this postcard(Fig.7) in an envelope in return, my last written communication from him.

However, on June 10-15 in 1990, at a Workshop on the Foundations of Quantum Mechanics convened at Amherst College, to all our delight, John showed up with his wife Mary largely, I believe, because Mary had an old friend who lived in the vicinity. The format was very informal and congenial to private conversations, and Kurt Gottfried and John conversed privately a good deal. I thought to myself, here is a modern Einstein-Bohr debate, I can't wait to hear the outcome. But, both men were too courteous for my expectations, and I guess they agreed to disagree, for nothing memorable that I recall emerged.

At the end of the conference we all sat around in a relaxed mood and were asked to say something we had learned. It came my turn, and I said that I had previously characterized the tails problem of CSL, rather poetically I had thought, that a little bit of everything that might have been coexists with what is (at which point I snuck a peek at a frowning John), but that I had learned from John that one should not express the ideas of a new theory in an old language (at which point John positively beamed!).

Three months later, we learned of our irreparable loss.

David Mermin who had been at Amherst too, Kurt Gottfried and I, perhaps feeling the need to do *something*, arranged to meet for private discussions concerning Foundational issues at Cornell a couple of times. However, it is a matter of core beliefs, of religion if you will: which is holier to you, quantum theory or reality? If the former, one may muck up reality. If the latter, one may muck up quantum theory. None of us was converted.[16]

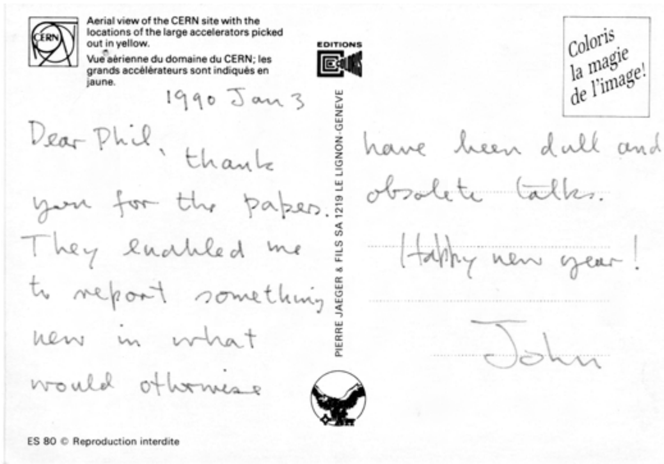
II. CSL

Here is a brief introduction to the muck-up of quantum theory that is CSL, without proofs.[17]

The state vector evolution in standard quantum theory is

$$|\psi, t\rangle = \mathcal{T}e^{-i\int_0^t dt' H(t')}|\psi, 0\rangle,$$

where the time-ordering operator $\mathcal{T}$ is necessitated if $[H(t), H(t')] \neq 0$ for some $t \neq t'$.



Aerial view of the CERN site with the locations of the large accelerators picked out in yellow.
Vue aérienne du domaine du CERN; les grands accélérateurs sont indiqués en jaune.

EDITIONS

Coloris la magie de l'image!

1990 Jan 3

Dear Phil, thank you for the papers. They enabled me to report something new in what would otherwise have been dull and obsolete talks.

Happy new year!

John

PIERRE JAEGER & FILS SA 1219 LE LIGNON-GENEVE

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FIG. 7. Letter from John Bell, 1/1990

The CSL evolution is

$$|\psi,t\rangle = \mathcal{T}e^{-i\int_0^t dt' H(t') - \frac{1}{4\lambda}\int_0^t dt'[\mathbf{w}(t')-2\lambda\mathbf{A}]^2}|\psi,0\rangle. \tag{1}$$

Here, λ is a collapse rate parameter. $\mathbf{w}(t') = (w_1(t'), w_2(t'), ...)$ is a vector whose components are independent functions of time of white noise character, i.e., they can take on any value between $-\infty$ and ∞ at any time. $\mathbf{A} = (A_1, A_2, ...)$ are a set of mutually commuting "collapse generating" operators, so they have a joint complete eigenbasis. If $H = 0$, one of these basis vectors is the end result of collapse, i.e., is the state vector at $t \to \infty$.

Nature is supposed to provide a particular $\mathbf{w}(t')$ under which the state vector evolves. The probability of any $\mathbf{w}(t')$ is postulated to depend only upon the state vector $|\psi,t\rangle$ which has evolved under it:

$$\text{Probability} \sim \langle\psi,t|\psi,t\rangle. \tag{2}$$

State vectors have different norms since their evolution is not unitary: this *probability rule* says that state vectors of largest norm are most probable. The proportionality constant in (2) can be chosen so that the integrated probability is 1 (the integral is the product of $\int_{-\infty}^{\infty} dw_j(t')$ for each fixed j and each fixed t', $0 \le t' \le t$.)

That's all there is to the formalism of CSL, Eqs.(1) and (2). It may be applied to many different situations. The most familiar is non-relativistic CSL, where the indices j refer to the points of space $\mathbf{x}$ and $A_{\mathbf{x}}$ is chosen to be essentially proportional to the mass density operator at $\mathbf{x}$[8],[9]. This makes a superposition of states such as a pointer in the state $\alpha|\text{here}\rangle + \beta|\text{there}\rangle$ collapse toward either $|\text{here}\rangle$ (which happens for a fraction $|\alpha|^2$ of the outcomes) or $|\text{there}\rangle$ (which happens for a fraction $|\beta|^2$ of the outcomes). The bigger the pointer mass, the faster proceeds the collapse. However, the CSL formalism may be applied otherwise, e.g. it has recently been applied to inflaton field fluctuation operators in the early universe[18] so that a particular universe (presumably ours) is chosen by collapse dynamics, instead of the superposition of universes given by the standard theory.

If one sets $H = 0$ to see the collapse mechanism operate without interference, $\mathcal{T}$ is no longer needed in Eq.(1) since the A_j commute. Upon expanding the exponent, one sees it may be written in terms of $B_j(t) \equiv \int_0^t dt' w_j(t')$:

$$-\frac{1}{4\lambda}\int_0^t dt'[\mathbf{w}(t') - 2\lambda\mathbf{A}]^2 = -\frac{1}{4\lambda}\int_0^t dt'\mathbf{w}^2(t') - \mathbf{B}(t)\cdot\mathbf{A} + \lambda\mathbf{A}^2$$
$$= \Big[-\frac{1}{4\lambda}\int_0^t dt'\mathbf{w}^2(t') + \frac{1}{4\lambda t}\mathbf{B}^2(t)\Big] - \frac{1}{4\lambda t}[\mathbf{B}(t) - 2\lambda t\mathbf{A}]^2. \quad (3)$$

It may be shown that the first bracket in the last line of Eq.(3) makes no dynamical or probabilistic contribution, so in this case the evolution (1) simplifies to

$$|\psi, t\rangle = e^{-\frac{1}{4\lambda t}[\mathbf{B}(t)-2\lambda t\mathbf{A}]^2}|\psi, 0\rangle, \quad (4)$$

where (2) says that the probability that $\mathbf{B}(t)$ lies in the range $d\mathbf{B}(t)$ is

$$\text{Probability} = \prod_j \frac{dB_j}{\sqrt{2\pi\lambda t}}\langle\psi, t|\psi, t\rangle. \quad (5)$$

We shall use Eqs.(4) and (5) in the rest of this paper.

III. NONLOCALITY IN CSL

The deBroglie-Bohm pilot wave theory and quantum theory yield the same predictions. In the former, spatially separated objects influence each other non-locally through the quantum potential. Bell suspected the nexus of these two features, identical predictions and non-locality, for *any* alternative to quantum theory, and was thus led to his inequality. Here we consider the nexus of these two features in a simplified version of non-relativistic CSL.[19]

Consider an EPR-Bohm state of two spin 1/2 particles, $\alpha|\Uparrow\rangle_L|\Downarrow\rangle_R + \beta|\Downarrow\rangle_L|\Uparrow\rangle_R$ ($|\alpha|^2 + |\beta|^2 = 1$), where L and R refer to two widely separated locations. At the left and right there are identical apparatuses, each attached to a pointer containing N nucleons. The usual Hamiltonian evolution is such that each apparatus locally, faithfully and very rapidly measures the spin in the vertical direction, resulting in the associated pointers either up (state $|\uparrow\rangle$) or down (state $|\downarrow\rangle$), correlated to the spin which is encountered at each location. The resulting quantum state, in usual quantum theory, evolves no further. However, this state,

$$|\psi, 0\rangle = \alpha|\uparrow\rangle_L|\downarrow\rangle_R + \beta|\downarrow\rangle_L|\uparrow\rangle_R. \tag{6}$$

is the initial quantum state for applying CSL dynamics. (Here we have neglected the spin particles as well as the rest of the apparatus, which are assumed to have negligibly different mass distributions in the different states, and therefore have negligible effect on the collapse dynamics.)

Simplifying non-relativistic CSL, we define four collapse-generating operators which represent the number of nucleons (i.e., $\approx$ mass of the pointer divided by the mass of a nucleon) at left or right in the relevant directions: $N_{L\uparrow}\otimes 1_R, N_{L\downarrow}\otimes 1_R, 1_L\otimes N_{R\uparrow}, 1_L\otimes N_{R\downarrow}$ They are defined by

$$N_{L\uparrow}|\uparrow\rangle_L = N|\uparrow\rangle_L, N_{L\uparrow}|\downarrow\rangle_L = 0; N_{L\downarrow}|\downarrow\rangle_L = N|\downarrow\rangle_L, N_{L\downarrow}|\uparrow\rangle_L = 0;$$
$$N_{R\uparrow}|\uparrow\rangle_R = N|\uparrow\rangle_R, N_{R\uparrow}|\downarrow\rangle_R = 0; N_{R\downarrow}|\downarrow\rangle_R = N|\downarrow\rangle_R, N_{R\downarrow}|\uparrow\rangle_R = 0.$$

Similarly, we introduce four random variables $B_{L\uparrow}, B_{L\downarrow}, B_{R\uparrow}, B_{R\downarrow}$ which fluctuate in each of the four regions Left-Right/Up-Down.

The apparatus on either side may be rotated so that spin in any direction may be measured, and so pointers may point in any direction. Therefore, we should have, in our expressions, collapse-generating operators and random variables associated to pointers pointing in any direction. But, all the associated collapse generating operators operating on the basis vectors in this example give 0, make no contribution to any physical result and may be ignored.

It only remains to apply Eqs.(4),(5) and draw the consequences.

The state vector at time t is

$$|\psi, t\rangle = \alpha|\uparrow\rangle_L|\downarrow\rangle_R e^{-\frac{1}{4\lambda t}\left[(B_{L\uparrow}-2\lambda Nt)^2+B_{L\downarrow}^2+B_{R\uparrow}^2+(B_{R\downarrow}-2\lambda Nt)^2\right]}$$
$$+\beta|\downarrow\rangle_L|\uparrow\rangle_R e^{-\frac{1}{4\lambda t}\left[B_{L\uparrow}^2+(B_{L\downarrow}-2\lambda Nt)^2+(B_{R\uparrow}-2\lambda Nt)^2+B_{R\downarrow}^2\right]} \tag{7}$$

and the probability density of the random variables is

$$P = |\alpha|^2 e^{-\frac{1}{2\lambda t}\left[(B_{L\uparrow}-2\lambda Nt)^2+B_{L\downarrow}^2+B_{R\uparrow}^2+(B_{R\downarrow}-2\lambda Nt)^2\right]} + |\beta|^2 e^{-\frac{1}{2\lambda t}\left[B_{L\uparrow}^2+(B_{L\downarrow}-2\lambda Nt)^2+(B_{R\uparrow}-2\lambda Nt)^2+B_{R\downarrow}^2\right]}. \tag{8}$$

Each term in (8) describes a gaussian in the four-dimensional $\mathbf{B}$-space: call them $G_\alpha(\mathbf{B}, t)$ and $G_\beta(\mathbf{B}, t)$. The peak of G_α, call it $\mathbf{B}_\alpha$ has components $B_{L\uparrow} = 2\lambda N t, B_{L\downarrow} = 0, B_{R\uparrow} = 0, B_{R\downarrow} = 2\lambda N t$. The peak of G_β, call it $\mathbf{B}_\beta$ has components $B_{L\uparrow} = 0, B_{L\downarrow} = 2\lambda N t, B_{R\uparrow} = 2\lambda N t, B_{R\downarrow} = 0$. Each gaussian has standard deviation $\sigma = \sqrt{\lambda t}$, so its width spreads much more slowly than $2\lambda N t$, the translation of the peaks. Thus, for $t >> 1/\lambda N^2$, $G_\alpha(\mathbf{B}, t)$ and $G_\beta(\mathbf{B}, t)$ only have a very small overlap of their tails.

So, the most probable $\mathbf{B}$-values either lie within a few standard deviations of $\mathbf{B}_\alpha$ or of $\mathbf{B}_\beta$. The other $\mathbf{B}$ values have negligible probability of occurring, so need not be considered. If $\mathbf{B}$ lies anywhere within a few standard deviations of $\mathbf{B}_\alpha$, the associated state vector given by (7) is $|\psi, t\rangle \sim |\uparrow\rangle_L |\downarrow\rangle_R$ to an excellent approximation, as the other term is exponentially smaller (a similar statement holds for $\mathbf{B}_\beta$). This is the collapse: CSL has described something that standard quantum theory never does, the occurrence of an event.

(The norm of a state in CSL is as unimportant as is the phase factor multiplying a state in standard quantum theory: one can normalize a state at any time. A state is defined by its direction in Hilbert space.)

If, using (5), one integrates the probability (8) over a few standard deviations of $\mathbf{B}_\alpha$, the result is $|\alpha|^2$ to an excellent approximation (similarly for $\mathbf{B}_\beta$). This is the identical prediction given by standard quantum theory for the outcome $|\psi, t\rangle \sim |\uparrow\rangle_L |\downarrow\rangle_R$.

Thus, in CSL, is achieved one half of the nexus described above, identical predictions. What about the second half, non-locality?

The state vector evolution is linear and local.

It is in the probability rule, non-linear in the state vector, where the non-locality resides. The two gaussians, G_α and G_β each depend on four variables: $B_{L\uparrow}, B_{L\downarrow}$ which exist at the left and $B_{R\uparrow}, B_{R\downarrow}$ which exist at the right. The probability rule therefore says the left variables must be *correlated* with the far-removed right variables, in order that a high probability $\mathbf{B}$ occur, resulting in a high probability outcome of the collapse.

In CSL, the probability rule, responsible for the non-local correlation, has the status of a law of nature, assumed but unexplained. As for any law of nature, it might some day be explained by something deeper. If the explanation involves non-local communication, this need not necessarily violate special relativity, since relativity allows tachyonic influences.

IV. BELL'S INEQUALITY VIOLATION

Bell's inequality[20] concerns a theory's expression, $P(\mathbf{a},\mathbf{b})$, for the ensemble average of the spin correlation $\sigma_1\cdot\mathbf{a}\sigma_2\cdot\mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are unit vectors making an angle θ with each other and the state is the angular momentum 0 (singlet) state, Eq.(6) with $\alpha=-\beta=1/\sqrt{2}$. Quantum theory gives $P(\mathbf{a},\mathbf{b})=-\cos\theta$, and this does not satisfy the inequality.

As we have seen, CSL's probability rule has a non-local influence, correlating the probabilities of detection at left and right. Certainly, at $t=0$, when the pointer state mimics the spin state, and at time $t=\infty$, when the outcomes of the collapses are precisely the same as predicted by quantum theory's "collapse postulate," $P(\mathbf{a},\mathbf{b};0)=P(\mathbf{a},\mathbf{b};\infty)=-\cos\theta$. But, what is $P(\mathbf{a},\mathbf{b};t)$ for $0<t<\infty$? While Bell showed that locality implies disagreement with quantum theory, non-locality doesn't necessarily imply agreement.

For example, one might envision an experiment where the apparatus discussed above involves pointers with a small value of N, so that the associated collapse proceeds "slowly." By this is meant that the ensemble of evolving state vectors is still in a superposition of pointer states with comparable amplitudes at some experimentally accessible time. At this time, suppose one utilizes two other apparatuses to make fast measurements of the orientation of the pointers. If the ensemble of state vectors at this time does not have the statistical distribution of pointer orientations initially predicted by quantum theory, the result will be different from that predicted by quantum theory. Such an experiment would be a test of CSL vs quantum theory.

So, we turn to analyze the predicted value of $P(\mathbf{a},\mathbf{b};t)$ according to CSL, where $\mathbf{a}=\hat{\mathbf{k}}$, $\mathbf{b}=\hat{\mathbf{k}}\cos\theta+\hat{\mathbf{i}}\sin\theta$. With the left apparatus measuring spin in the vertical direction, but with the right apparatus rotated by an angle θ in the $x-z$ plane, one readily finds the initial state vector (6) is to be replaced by

$$|\psi,0\rangle=\frac{1}{\sqrt{2}}\Big[\sin\frac{\theta}{2}|\uparrow\rangle_L|\nearrow\rangle_R+\cos\frac{\theta}{2}|\uparrow\rangle_L|\swarrow\rangle_R-\cos\frac{\theta}{2}|\downarrow\rangle_L|\nearrow\rangle_R+\sin\frac{\theta}{2}|\downarrow\rangle_L|\swarrow\rangle_R\Big], \quad (9)$$

where $|\nearrow\rangle_R$, $\swarrow\rangle_R$ are the states of the pointer at the right pointing parallel or anti-parallel to $\mathbf{b}$. $N_{R\uparrow}, N_{R\downarrow}$ are to be replaced by $N_{R\nearrow}, N_{R\swarrow}$ satisfying

$$N_{R\nearrow}|\nearrow\rangle_R=N|\nearrow\rangle_R, N_{R\nearrow}|\swarrow\rangle_R=0; N_{R\swarrow}|\nearrow\rangle_R=0, N_{R\swarrow}|\swarrow\rangle_R=N|\swarrow\rangle_R.$$

and $B_{R\uparrow}, B_{R\downarrow}$ are to be replaced by $B_{R\nearrow}, B_{R\swarrow}$. Then, the evolving state vector replacing

(7) is

$$
\begin{aligned}
|\psi,t\rangle = & \frac{\sin\frac{\theta}{2}}{\sqrt{2}}|\uparrow\rangle_L|\nearrow\rangle_R e^{-\frac{1}{4\lambda t}\left[(B_{L\uparrow}-2\lambda Nt)^2+B_{L\downarrow}^2+(B_{R\nearrow}-2\lambda Nt)^2+B_{R\downarrow}^2\right]} \\
& +\frac{\cos\frac{\theta}{2}}{\sqrt{2}}|\uparrow\rangle_L|\swarrow\rangle_R e^{-\frac{1}{4\lambda t}\left[(B_{L\uparrow}-2\lambda Nt)^2+B_{L\downarrow}^2+B_{R\nearrow}^2+(B_{R\swarrow}-2\lambda Nt)^2\right]} \\
& -\frac{\cos\frac{\theta}{2}}{\sqrt{2}}|\downarrow\rangle_L|\nearrow\rangle_R e^{-\frac{1}{4\lambda t}\left[B_{L\uparrow}^2+(B_{L\downarrow}-2\lambda Nt)^2+(B_{R\nearrow}-2\lambda Nt)^2+B_{R\swarrow}^2\right]} \\
& +\frac{\sin\frac{\theta}{2}}{\sqrt{2}}|\downarrow\rangle_L|\swarrow\rangle_R e^{-\frac{1}{4\lambda t}\left[B_{L\uparrow}^2+(B_{L\downarrow}-2\lambda Nt)^2+B_{R\nearrow}^2+(B_{R\swarrow}-2\lambda Nt)^2\right]}. \qquad (10)
\end{aligned}
$$

To express $P(\mathbf{a},\mathbf{b};t)$ in compact form, it is convenient to define Pauli operators $\mathbf{\Sigma}_L, \Sigma_R$, for the pointers, where $\mathbf{\Sigma}_L\cdot\mathbf{a}=\Sigma_3, \mathbf{\Sigma}_R\cdot\mathbf{b}=\Sigma_3\cos\theta+\Sigma_1\sin\theta$ and

$$
\mathbf{\Sigma}_L\cdot\mathbf{a}|\uparrow\rangle_L=|\uparrow\rangle_L, \mathbf{\Sigma}_L\cdot\mathbf{a}|\downarrow\rangle_L=-|\downarrow\rangle_L, \mathbf{\Sigma}_R\cdot\mathbf{b}|\nearrow\rangle_R=|\nearrow\rangle_R, \mathbf{\Sigma}_R\cdot\mathbf{b}|\swarrow\rangle_R=-|\nearrow\rangle_R.
$$

Then,

$$
\begin{aligned}
\langle\psi,t|\mathbf{\Sigma}_L\cdot\mathbf{a}\mathbf{\Sigma}_R\cdot\mathbf{b}|\psi,t\rangle = & \frac{\sin^2\frac{\theta}{2}}{2}|\uparrow\rangle_L|\nearrow\rangle_R e^{-\frac{1}{2\lambda t}\left[(B_{L\uparrow}-2\lambda Nt)^2+B_{L\downarrow}^2+(B_{R\nearrow}-2\lambda Nt)^2+B_{R\downarrow}^2\right]} \\
& -\frac{\cos^2\frac{\theta}{2}}{2}|\uparrow\rangle_L|\swarrow\rangle_R e^{-\frac{1}{2\lambda t}\left[(B_{L\uparrow}-2\lambda Nt)^2+B_{L\downarrow}^2+B_{R\nearrow}^2+(B_{R\swarrow}-2\lambda Nt)^2\right]} \\
& -\frac{\cos^2\frac{\theta}{2}}{2}|\downarrow\rangle_L|\nearrow\rangle_R e^{-\frac{1}{2\lambda t}\left[B_{L\uparrow}^2+(B_{L\downarrow}-2\lambda Nt)^2+(B_{R\nearrow}-2\lambda Nt)^2+B_{R\swarrow}^2\right]} \\
& +\frac{\sin^2\frac{\theta}{2}}{2}|\downarrow\rangle_L|\swarrow\rangle_R e^{-\frac{1}{2\lambda t}\left[B_{L\uparrow}^2+(B_{L\downarrow}-2\lambda Nt)^2+B_{R\nearrow}^2+(B_{R\swarrow}-2\lambda Nt)^2\right]}. \qquad (11)
\end{aligned}
$$

We must divide (11) by $\langle\psi,t|\psi,t\rangle$ to have the matrix element in (11) expressed in terms of the normalized state vector, and so obtain the correct expressions for the fractions of the various pointer directions associated with the state $|\psi,t\rangle$.

Then, for the complete ensemble of state vectors, we obtain

$$
\begin{aligned}
P(\mathbf{a},\mathbf{b};t) = & \int_{-\infty}^{\infty}\frac{dB_{L\uparrow}}{\sqrt{2\pi\lambda t}}\frac{dB_{L\downarrow}}{\sqrt{2\pi\lambda t}}\frac{dB_{R\nearrow}}{\sqrt{2\pi\lambda t}}\frac{dB_{R\swarrow}}{\sqrt{2\pi\lambda t}}\langle\psi,t|\psi,t\rangle\frac{\langle\psi,t|\mathbf{\Sigma}_L\cdot\mathbf{a}\mathbf{\Sigma}_R\cdot\mathbf{b}|\psi,t\rangle}{\langle\psi,t|\psi,t\rangle} \\
& =\sin^2\frac{\theta}{2}-\cos^2\frac{\theta}{2}=-\cos\theta. \qquad (12)
\end{aligned}
$$

Thus, it does not matter what time the ensemble of pointers is observed, the result will be the same as that predicted by quantum theory.

V. CONFLICT WITH QUANTUM THEORY

Although, for the situation described above, the considered measurement of the ensemble of state vectors at a time between the onset of CSL dynamics and the completion of collapse gives no conflict with the predictions of quantum theory, this need not be true for other situations. Here we shall consider the general case of $\langle\psi,t|Op|\psi,t\rangle$, where Op is an arbitrary operator. We shall see that there will be a conflict if Op does not commute with the collapse-generating operators. Indeed, in the example above, they do commute as $\mathbf{\Sigma}_L\cdot\mathbf{a}=N^{-1}[N_{L\uparrow}-N_{R\downarrow}], \mathbf{\Sigma}_R\cdot\mathbf{b}=N^{-1}[N_{R\nearrow}-N_{R\swarrow}]$, which is why there is no conflict.

In the general case, the initial state vector which CSL dynamics is to act upon (we are assuming the situation is such that the Hamiltonian is inconsequential) is

$$|\psi,0\rangle=\Sigma_n\alpha_n|\mathbf{a}_n\rangle, \tag{13}$$

where $|\mathbf{a}_n\rangle$ are eigenstates of the collapse generating operators, $A_j|\mathbf{a}_n\rangle=a_{jn}|\mathbf{a}_n\rangle$. It is assumed that there is no degeneracy, i.e., if $\mathbf{a}_n=\mathbf{a}_m$, then $m=n$. We are assuming that the $|\mathbf{a}_n\rangle$ are pointer states, so the prediction of quantum theory is that this will be the outcome state with probability $|\alpha_n|^2$.

Therefore, from (4), the matrix element to be considered is

$$\langle\psi,t|Op|\psi,t\rangle=\Sigma_{n,m}\alpha*_m\,\alpha_n|\langle\mathbf{a}_m|Op|\mathbf{a}_n\rangle e^{-\frac{1}{4\lambda t}\{[\mathbf{B}(t)-2\lambda t\mathbf{a}_n]^2+[\mathbf{B}(t)-2\lambda t\mathbf{a}_m]^2\}} \tag{14}$$

and, from (5), the ensemble average is

$$\overline{\langle\psi,t|Op|\psi,t\rangle}=\Sigma_{n,m}\alpha*_m\,\alpha_n\langle\mathbf{a}_m|Op|\mathbf{a}_n\rangle e^{-\frac{\lambda t}{2}[\mathbf{a}_n-\mathbf{a}_m]^2}. \tag{15}$$

If Op is a function of $\mathbf{A}$, then $\langle\mathbf{a}_m|Op|\mathbf{a}_n\rangle=\langle\mathbf{a}_n|Op|\mathbf{a}_n\rangle\delta_{nm}$, so (15) becomes time-independent:

$$\overline{\langle\psi,t|Op|\psi,t\rangle}=\Sigma_n|\alpha_n|^2\langle\mathbf{a}_n|Op|\mathbf{a}_n\rangle. \tag{16}$$

In any case, the right side of Eq.(16) also gives the ensemble average at $t\to\infty$, since $e^{-\frac{\lambda t}{2}[\mathbf{a}_n-\mathbf{a}_m]^2}\xrightarrow[t\to\infty]{}\delta_{nm}$. The right side of Eq.(16) is also the prediction of quantum theory for the value of this matrix element.

If Op does not commute with $\mathbf{A}$ then, according to (15), the ensemble average of the matrix element does not agree with the prediction (16) of quantum theory, and so its measurement at an intermediate time can serve as a test of CSL vs quantum theory. For example, in non-relativistic CSL, the collapse generating operators are mass density operators, which do not commute with momentum density operators. So, if one can contrive to measure the matrix element when Op is a function of momentum, for example in an interference experiment, one can test non-relativistic CSL[21].

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