Haag’s Theorem as a Reason to Reconsider Direct-Action Theories

R. E. Kastner

22 November 2014

ABSTRACT. It is argued that the severe consequences of Haag’s inconsistency theorem for relativistic quantum field theories can be successfully evaded in the direct-action approach. Some recent favorable comments of John Wheeler, often mistakenly presumed to have abandoned his own (and Feynman’s) direct-action theory, together with the remarkable immunity of direct-action quantum electrodynamics to Haag’s theorem, suggest that it may well be a good time to rehabilitate direct action theories. It is also noted that, as extra dividends, direct-action QED is immune to the self-energy problem of standard gauge field QED, and can also provide a solution to the problem of gauge arbitrariness.

1. Introduction.

Haag’s Theorem demonstrates that for interacting quantized fields, the field operators corresponding to the interacting fields do not belong to the same Fock space representation as the asymptotic free fields, despite the fact that all the operators obey the same commutation relations. (See [1], §4, especially equation (57) and ensuing discussion.) Haag showed that the interacting field demands an inequivalent representation from that of the free field; the vacuum states of the two fields cannot be defined in the same representation. This result presents a serious problem for the basic mathematical consistency of quantum field theories, and has led to much discussion [2-7].]

As noted by Haag, the source of the problem is the fact that the quantized field has, in principle, an infinite number of degrees of freedom (modes). This turns out to lead to inequivalent representations for field operators belonging to fields having the same commutations relations. For example, one can have two fields related by a simple continuous transformation, and having the same commutation relations, but the vacuum state will not exist for one of the fields in a representation for which the other field’s vacuum state is perfectly well-defined. As Haag notes, this problem could be surmounted if there were some principled way to fix the ‘correct’ representation, since strictly speaking, one does not need the infinite degrees of freedom. However, even if such a ‘correct’ representation could be chosen, the basic problem is not solved. A further manifestation of the inconsistency brought to light by Haag’s theorem is the lack of a unitary transformation connecting the operators for the free fields with their interacting counterparts. This point is discussed in [1], §4.

2. How direct-action theories can evade Haag's theorem

An immediate solution to the problem is to banish the notion of an independently existing field with its own degrees of freedom, and deal instead with a direct-action theory. While direct-action theories are widely considered to have significant drawbacks for pragmatic, computational purposes, they can be well-approximated by QFT considered as a computational tool only.[1] Indeed, Narlikar [8] showed that any interacting field theory of a field ϕ described by the usual invariant bilinear Lagrangian of the field and its derivatives, and an interaction term I of the form $I \sim g \langle \phi, j^{(i)} \rangle$, where the $j^{(i)}$ are the source currents and g a coupling constant, can be equivalently recast as a direct action theory. In such a theory, the field at a point x due to current $j^{(i)}(y)$ is given by

$$\phi(x) = g \int \bar{D}(x,y)\, j^{(i)}(y) dy \qquad (1)$$

where $\bar{D}(x,y)$ is the time-symmetric propagator for the field described by the given Lagrangian.

Taking Nature as described by the direct action theory rather than by the quantized field theory would resolve the problem. That is, if Nature is correctly described by the direct-action theory rather than by QFT, then Haag's theorem is of no consequence; one uses QFT merely as a computational tool rather than as a theory taken as describing the physical world.

This author recognizes that direct-action theories are not currently popular, but given the severity of the threat posed by Haag's theorem, it may well be time to reconsider them. Indeed, one of the founders of the Wheeler-Feynman direct action theory of electromagnetism [10], the late John Wheeler, was recently doing just that in connection with the search for a theory of quantum gravity [9]. Together with D. Wesley in 2003, he reviews the history of the development of the Wheeler-Feynman (WF) theory and comments:

---

[1] Actually, Wesley and Wheeler dissent from this common perception of direct-action theories as computationally cumbersome: "In addition to the conceptual simplicity of the theory, it is also more convenient mathematically. One need not calculate the dynamics of the field, a complex dynamical quantity with an infinite number of degrees of freedom; only the particles, with their finite number of degrees of freedom." [9], p. 428.

> [WF] swept the electromagnetic field from between the charged particles and replaced it with "half-retarded, half advanced direct interaction" between particle and particle. It was the high point of this work to show that the standard and well-tested force of reaction of radiation on an accelerated charge is accounted for as the sum of the direct actions on that charge by all the charges of any distant complete absorber. Such a formulation enforces global physical laws, and results in a quantitatively correct description of radiative phenomena, without assigning stress-energy to the electromagnetic field. ([9], p. 427)

Wesley and Wheeler note that one motivation for retaining the idea of a mediating field has been to enforce locality, and that some objections to the direct-action picture are based on an aversion to the idea of a 'nonlocal' interaction between particles; i.e., that the particles evidently interact instantaneously. They address this concern in a section entitled "Is Physics Entirely Local?," concluding that in fact it is not:

> One is reminded of an argument against quantum theory advanced by Einstein, Podolsky and Rosen in a well-known paper (1931) …The implicit nonlocality of [the EPR entanglement experiment], they argue, is at odds with the idea that physics should be fundamentally local…As has been evidenced by many experimental tests, the view of nature espoused by Einstein *et al* is not quite correct. Various experiments have shown that distant measurements can affect local phenomena. That is, *nature is not described by physical laws that are entirely local*. Effect from distant objects *can* influence local physics…this example from quantum theory serves to illustrate that it may be useful to expand our notions regarding what types of physical laws are 'allowed'. ([9], pp 426-7; emphasis in original text)

It should be clear from the above excerpts that the surviving original co-founder of the 'nonlocal' Wheeler-Feynman direct-action theory of electromagnetism views that formulation as perfectly viable. Moreover, he suggests that its nonlocal character should not be shunned but instead embraced, and that the same direct-action approach be applied towards longstanding stubborn challenges such as quantum gravity. In particular Wesley and Wheeler are questioning whether such challenges are fruitfully addressed by way of the usual conceptual tool of invoking a 'field' in order to try to account for the phenomena in a local manner. The present author would like to suggest that Haag's theorem is yet another challenge of this type, in which the 'local,' mediating field description has turned out to be fundamentally inadequate. In perhaps a crude analogy, the mediating field plays the part of a 'bucket brigade' that is invoked in order to restrict the influence of the field to a local, continuously conveyance from spacetime point to spacetime point. But, as Wesley and Wheeler note, this sort of 'bucket brigade' account of the influences of fields apparently is not a feature of quantum entities. And the infinite number of degrees of freedom implicit in that local, mechanistic account clearly leads to various problems, such as Haag's theorem and the problem of infinite self-energies of field sources. These issues, as well as the advantage of a direct-action theory for the problem of gauge arbitrariness, are discussed further in the next section.

3. Various approaches to direct-action theories

A quantum relativistic version of the classical Wheeler-Feynman theory was developed in the early 1970s by Davies [11-13]. Davies noted that his theory naturally invokes the Coulomb gauge, since the Coulomb interaction is characterized by the time-symmetric propagator and can be considered a 'virtual photon' interaction only. In contrast, radiative phenomena in his theory correspond to Fock space states which must be on-shell and transversely polarized (i.e.. 'real photons'). (See [12], p. 843 for a discussion of the Coulomb gauge as the natural choice for the Davies direct-action theory.) An advantage of the Coulomb gauge is that it is a 'complete' gauge, i.e., lacking any residual arbitrariness, unlike other gauges such as the manifestly covariant Lorenz gauge.

A similar point, albeit arrived at from different perspective of seeking to avoid both the divergent energy of self-interaction and the 'light tight box' complete absorber condition, is made by F. Rohrlich:

> The solution to these difficulties came to me in the early sixties from the realization…that one wants to avoid only the self-interaction related to the Coulomb field and not the one related to radiation reaction…thus one is led to a theory which is of the action-at-a-distance type only for the Coulomb field but which remains a field theory with respect to the radiation field…
>
> This realization agrees beautifully with the quantum mechanical understanding of electromagnetic field: only the radiation field is composed of photons (i.e., must be quantized) while the Coulomb field is not (i.e., should not be quantized). This, in turn, leads evidently to the Coulomb gauge which is, in this sense, the natural gauge. In any case, the elimination of the Coulomb field is physically easily justified, the elimination of the radiation field, however, is not, because it would mean that the photon is not as elementary a particle as the

> electron, a notion that I find difficult to maintain on this level of theory. ([14], p. 350)

It should be noted that Rohrlich 's approach is a 'hybrid' one—he wishes to retain the quantized field for radiated photons but abolish it for the Coulomb interaction. One of his motivations was to eliminate the 'light tight box' boundary condition, and this can be done by using a quantized field for radiative processes only.[2] However, the cost of this approach is arguably somewhat of a theoretical 'patchwork'.

Nevertheless, at this point we may note the connection between the Wheeler-Feynman, Davies, and Rohrlich pictures and the transactional picture of quantum processes first developed by Cramer [15] and elaborated by the present author in a 'possibilist' and relativistic form [17-19] . In the latter, I have argued that the Davies theory naturally lends itself to a transactional account, in which radiative phenomena correspond to actualized transactions. The first step in a radiative process is the emission of a photon offer wave and confirming response from *all* absorbers—even those that do not actually receive any real energy. This offer/confirmation exchange sets up a set of incipient transactions corresponding to all possible spatial directions, but (for a single photon) only one such direction can actually be chosen; that choice corresponds to the 'collapse' process.[3] This is the point at which one of the incipient transactions is actualized and a real photon is transferred (radiated) from an emitter to a particular 'winning' absorber. Since the precursor to any such radiative process involves responses from all absorbers, the complete absorber response cannot be neglected.[4] This picture provides a unified explanation of the quantized radiation field in terms of actualized photons, even though the underlying dynamics is all mediated by direct-action.

The transactional picture also explains the apparently mysterious pole remaining in the Feynman propagator when it is derived, as in the Davies theory, from the confirming response of absorbers. Davies tacitly assumed that the Feynman propagator can remain applicable, at least in

---

[2] Rohrlich notes that one does not need to abolish the self-interaction for the 'free field' responsible for the radiation reaction because it does not lead to divergences (see [10], p. 347). In the transactional picture, this is because no single charge can be both emitter and confirming absorber for its own photon offer wave.

[3] This author has argued that the collapse can be understood as a kind of spontaneous symmetry breaking ([17], Chapter 4).

[4] As Davies notes in [13], p. 1035 , when one does not include the full absorber response in the system under study, the direct-action theory involves a nonunitary scattering matrix. While Davies regards this as puzzling, in the transactional picture it is a natural reflection of the fact that full absorber response is a key part of any radiative process: radiated photons are always a product of absorber response, ultimately being emitted and absorbed by the 'winning' absorber, and are not simply emitted as free-standing entities.

principle, as a description of virtual particle processes, since his primary aim was to demonstrate equivalence between the direct-action picture and standard QED. But in fact, as he shows by Fourier decomposition of the Feynman propagator into bound (time-symmetric, off-shell) and free (on-shell) parts (see [13], eq (5)), in the direct-action picture the internal lines in scattering diagrams are not really described by the Feynman propagator but rather by the time-symmetric propagator. Thus, the Feynman propagator becomes a 'hybrid' and somewhat awkward entity in the Davies account, since it is presumed capable of describing both virtual and real processes while imposing retarded propagation ('causality') on both. This ambiguity arises because neither Davies nor Feynman make a fundamental distinction between real and virtual photons at the level of the basic field propagation. In particular, Feynman considered them a matter of context. [5]

In contrast, in the transactional picture a confirming response from absorbers unambiguously leads to real photons, as opposed to virtual photons, and calls for the pole in the Feynman propagator, which corresponds to a Fock state. The latter is an external line only in terms of a scattering diagram; it is not properly considered an internal line. True internal lines do not prompt an absorber response and that is why they can be accurately described by the time-symmetric component only [19]. In this regard, Rohrlich's picture, in which the Coulomb interaction is non-quantized and never transfers real energy, is harmonious with the present author's 'possibilist' transactional account (PTI) in that virtual processes (i.e. the Coulomb interactions) do not rise to the level of incipient transactions, and therefore are not eligible to transfer conserved quantities such as energy and momentum via a real photon.

The relevance of the treatment of virtual photons in the direct-action approach is that one of the assumptions required for Haag's theorem is that field states are defined for virtual processes.[6] However, this is not part of the direct action theory. In the latter, virtual processes are described by the time-symmetric propagator which does not correspond to a radiative process, and therefore does not correspond to a real photon or Fock space state. Thus, again Haag's theorem is blocked by the direct-action approach. An immediate additional side-benefit of the

---

[5] Feynman has remarked that there is no fundamental difference between real and virtual particles ([16], as quoted in Davies [13], pp. 1027-8). This is not the case in the transactional picture, as emphasized in Kastner [18], [19].

[6] Indeed one of the inelegant features of QFT is that the off-shell "states," formally subject to creation and destruction, must be eliminated by rampant use of Dirac delta functions as bookkeeping devices. See [1], pp 23-24, where Haag notes that the delta function enforcing on-shell behavior "must appear in all relations of physical significance." It would probably be more accurate to say 'empirical significance' in this connection. In any case, the existence in the theory of creation and destruction operators for unphysical states that must be eliminated in this *ad hoc* way points again to the fundamental problem: namely, the QFT model treats treats virtual propagation as physically equivalent to real propagation.

direct-action picture is gaining a physically natural basis for the choice of gauge, resolving another notorious problem of conceptual consistency and interpretation of relativistic field theories: apparent gauge arbitrariness.

4. Conclusion

Teller has correctly observed that

> Everyone must agree that as a piece of mathematics Haag's theorem is a valid result that at least appears to call into question the mathematical foundation of interacting quantum field theory, and agree that at the same time the theory has proved astonishingly successful in application to experimental results. [7, p. 115]

If Nature is in fact described by a direct-action theory, then this apparent paradox is resolved: QFT is an empirically equivalent calculational stand-in for the direct-action theory, so it can continue to be used for practical calculations. Meanwhile, its mathematical inconsistencies can be rendered inconsequential since they can be understood as arising from its 'makeshift,' nonfundamental character. The other significant dividends gained by adopting the direct-action picture are: (i) a solution to the gauge arbitrariness problem and (ii) a solution to the self-energy problem of standard QED.

References

[1] Haag, R. (1955). "*On quantum field theories*," Matematisk-fysiske Meddelelser, **29**, 12 .

[2] Earman, J. and Fraser, D. (2006). "*Haag's Theorem and Its Implications for the Foundations of Quantum Field Theory*," Erkenntnis **64**, 305.

[3] Ruelle, D. (1962). "On the asymptotic condition in quantum field theory," *Helvetica Physica Acta* **35**: 147–163.

[4] Sklar, Lawrence (2000). *Theory and Truth: Philosophical Critique within Foundational Science*. Oxford University Press.

[5] Lupher, T. (2005). "Who proved Haag's theorem?", *International Journal of Theoretical Physics* **44**: 1993–2003.

[6] Bain, J. (2000). "Against Particle/field duality: Asymptotic particle states and interpolating fields in interacting QFT (or: Who's afraid of Haag's theorem?)". *Erkenntnis* **53**: 375–406.

[7] Teller, Paul (1997). *An interpretive introduction to quantum field theory*. Princeton University Press, p. 115.

[8] Narlikar, J. V. (1968) "On the general correspondence between field theories and the theories of direct particle interaction," *Proc. Cam. Phil. Soc. 64*, 1071.

[9] Wesley, D. and Wheeler, J. A., "Towards an action-at-a-distance concept of spacetime," In A. Ashtekar et al, eds. (2003). *Revisiting the Foundations of Relativistic Physics*: *Festschrift in Honor of John Stachel,* Boston Studies in the Philosophy and History of Science (Book 234), pp. 421-436. Kluwer Academic Publishers.

[10] Wheeler, J.A. and R. P. Feynman, "Interaction with the Absorber as the Mechanism of Radiation," *Reviews of Modern Physics*, 17, 157–161 (1945); and "Classical Electrodynamics in Terms of Direct Interparticle Action," *Reviews of Modern Physics*, 21, 425–433 (1949).

[11] Davies, P. C. W. (1970). "A quantum theory of Wheeler-Feynman Electrodynamics," *Proc. Cam. Phil. Soc. 68*, 751.

[12] Davies, P. C. W. (1971)."Extension of Wheeler-Feynman Quantum Theory to the Relativistic Domain I. Scattering Processes," *J. Phys. A: Gen. Phys. 6*, 836.

[13] Davies, P. C. W. (1972)."Extension of Wheeler-Feynman Quantum Theory to the Relativistic Domain II. Emission Processes," *J. Phys. A: Gen. Phys. 5*, 1025-1036.

[14] Rohrlich, F. (1973). "The Electron: Development of the First Elementary Particle Theory," in Mehra, J. (ed.) *The Physicist's Conception of Nature*. pp. 331-369. D. Reidel Publishing Co.

[15] Cramer, J. G. (1986). ``The Transactional Interpretation of Quantum Mechanics." *Reviews of Modern Physics 58*, 647-688.

[16]Feynman, R. P. (1998). *Theory of Fundamental Processes*. Westview Press.

[17] Kastner, R. E. (2012). *The Transactional Interpretation of Quantum Mechanics: The Reality of Possibility.* Cambridge University Press.

[18] Kastner, R. E. (2012). "The Possibilist Transactional Interpretation and Relativity," *Foundations of Physics 42*, 1094-1113

[19] Kastner, R. E. (2014). "On Real and Virtual Photons in the Davies Theory of Time-Symmetric Quantum Electrodynamics," *Electronic Journal of Theoretical Physics 11*, No. 30. Preprint version: http://arxiv.org/abs/1312.4007