Protective measurements and the reality of the wave function

Shan Gao
Institute for the History of Natural Sciences,
Chinese Academy of Sciences, Beijing 100190, China.
E-mail: gaoshan@ihns.ac.cn.

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Abstract

The ontological status of the wave function in quantum mechanics is usually analyzed in the context of conventional impulsive measurements. These analyses are always based on some nontrivial assumptions, e.g. a preparation independence assumption is needed to prove the PBR theorem. In this paper, we point out that the reality of the wave function can be argued without resorting to nontrivial assumptions by analyzing protective measurements, by which one can measure the expectation values of observables on a single quantum system. The existing objections to this argument are answered. Moreover, we also give a PBR-like argument for the reality of the wave function in terms of protective measurements.

The physical meaning of the wave function has been a hot topic of debate in the foundations of quantum mechanics. A long-standing question is whether the wave function relates only to an ensemble of identically prepared systems or directly to the state of a single system. Recently, Pusey, Barrett and Rudolph demonstrated that under a preparation independence assumption, the wave function is a representation of the physical state of a single quantum system (Pusey, Barrett and Rudolph 2012)[1]. This poses a further interesting question, namely whether the reality of the wave function can be argued without resorting to nontrivial assumptions such as the preparation independence assumption (cf. Lewis et al 2012; Leifer and Maroney 2013; Patra, Pironio and Massar 2013). In this paper, we will argue that protective measurements, by which one can measure the expectation values of observables on a single quantum system (Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1993), already provide such an argument.

[1]For more discussions about the Pusey-Barrett-Rudolph theorem or PBR theorem, see Colbeck and Renner (2012); Schlosshauer and Fine (2012, 2013); Wallden (2013).

The ontological status of the wave function in quantum mechanics is usually analyzed in the context of conventional impulsive measurements. Although the wave function of a quantum system is in general extended over space, one can only detect the system in a random position in space by an (impulsive) position measurement, and the probability of detecting the system in the position is given by the modulus squared of the wave function there. Thus it seems reasonable for a realist to assume that the wave function does not refer directly to the physical state of the system but only relate to the state of an ensemble of identically prepared systems. Although there are several interesting theorems such as the PBR theorem which reject this epistemic view of the wave function, these theorems always depend on some nontrivial assumptions. By denying these nontrivial assumptions, one can still restore the epistemic view of the wave function. Moreover, it has been demonstrated that additional assumptions are always necessary to rule out the epistemic view of the wave function when considering only conventional impulsive measurements (Lewis et al 2012).

Thanks to the important discoveries of Yakir Aharonov and Lev Vaidman et al, it has been known that there exist other kinds of quantum measurements such as weak measurements and protective measurements (Aharonov, Albert and Vaidman 1988; Aharonov and Vaidman 1990; Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1993). In particular, by a series of protective measurements on a single quantum system, one can detect the system in all regions where its wave function extends and further measure the whole wave function of the system (Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1993). During a protective measurement, the measured state is protected by an appropriate procedure (e.g. via the quantum Zeno effect) so that it neither changes nor becomes entangled with the state of the measuring device appreciably. In this way, such protective measurements can measure the expectation values of observables on a single quantum system, even if the system is initially not in an eigenstate of the measured observable, and the whole wave function of the system can also be measured as expectation values of certain observables.

Since the wave function of a single quantum system can be measured by a series of protective measurements, it seems natural to assume that the wave function refers directly to the physical state of the system. Several authors, including the discoverers of protective measurements, have given similar arguments supporting this implication of protective measurements for the ontological status of the wave function (Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1993; Anandan 1993; Dickson 1995; Gao 2013). However, these analyses have been neglected by most researchers, and they are also subject to some objections (Unruh 1994; Dass and Qureshi 1999; Schlosshauer and Claringbold 2014). Here we will first present a clearer argument for the reality of the wave function in terms of protective measurements, and then answer these objections.

According to quantum mechanics, we can prepare a single measured system whose associated wave function is $\psi(t)$ at a given instant t. The question is whether the wave function refers directly to the physical state of the system or merely to the state of an ensemble of identically prepared systems. As noted above, this question can hardly be answered by analyzing non-protective impulsive measurements of the system, by each of which one obtains one of the eigenvalues of the measured observable, and the expectation value of the observable as well as the value of $\psi(t)$ can only be obtained by calculating the statistical average of the eigenvalues for an ensemble of identically prepared systems. Now, by a protective measurement on the measured system, we can directly obtain the expectation value of the measured observable. Moreover, by a series of protective measurements of certain observables on *this* system, we can further obtain the value of $\psi(t)$. Since we can measure the wave function *only* from a single prepared system by protective measurements, the wave function represents the state of a single system. Similarly, the expectation values of observables are also properties of a single system.

That the wave function of a single prepared system can be measured by protective measurements can be illustrated with a specific example (Aharonov and Vaidman 1993). Consider a quantum system in a discrete nondegenerate energy eigenstate $\psi(x)$. In this case, the measured system itself supplies the protection of the state due to energy conservation and no artificial protection is needed. We take the measured observable A_n to be (normalized) projection operators on small spatial regions V_n having volume v_n:

$$A_n = \begin{cases} \frac{1}{v_n}, & \text{if } x \in V_n, \\ 0, & \text{if } x \notin V_n. \end{cases} \tag{1}$$

An adiabatic measurement of A_n then yields

$$\langle A_n \rangle = \frac{1}{v_n} \int_{V_n} |\psi(x)|^2 dv, \tag{2}$$

which is the average of the density $\rho(x) = |\psi(x)|^2$ over the small region V_n. Similarly, we can adiabatically measure another observable $B_n = \frac{\hbar}{2mi}(A_n \nabla + \nabla A_n)$. The measurement yields

$$\langle B_n \rangle = \frac{1}{v_n} \int_{V_n} \frac{\hbar}{2mi} (\Psi^* \nabla \Psi - \Psi \nabla \Psi^*) dv = \frac{1}{v_n} \int_{V_n} j(x) dv. \tag{3}$$

This is the average value of the flux density $j(x)$ in the region V_n. Then when $v_n \to 0$ and after performing measurements in sufficiently many regions V_n we can measure $\rho(x)$ and $j(x)$ everywhere in space. Since the wave function $\psi(x,t)$ can be uniquely expressed by $\rho(x,t)$ and $j(x,t)$ (except for an overall phase factor), the above protective measurements can obtain the wave function of the measured system.

There are two possible objections to the above conclusion that protective measurements support the reality of the wave function. The first is based on the requirement that the unknown state of a single system is measurable. It claims that since the unknown state of a single quantum system cannot be protectively measured, protective measurements do not have implications for the ontological status of the wave function (see, e.g. Unruh 1994). However, this requirement is too stringent to be true (see also Hetzroni and Rohrlich 2014). If it were true, then no argument for the reality of the wave function including the PBR theorem could exist, because it is a well-known result of quantum mechanics that an unknown quantum state cannot be measured. Moreover, it is worth noting that knowing the wave function does not mean knowing the physical state of the measured system in our argument. The wave function is only a mathematical object associated with the prepared physical system, and we need to determine whether it refers to the physical state of the system or to the state of an ensemble of identically prepared systems. In this sense, although the wave function is known, the physical state of the system is still unknown. Thus, precisely speaking, what the above protective measurements measure is not the known wave function, but the unknown physical state, which, as shown above, turns out to be represented by the wave function.

The second objection concerns realistic protective measurements (Dass and Qureshi 1999; Schlosshauer and Claringbold 2014). A realistic protective measurement can never be performed on a single quantum system with absolute certainty. For example, for a realistic protective measurement of an observable A on a non-degenerate energy eigenstate whose measurement interval T is finite, there is always a tiny probability proportional to $1/T^2$ to obtain a different result $\langle A \rangle_\perp$, where $\perp$ refers to a normalized state in the subspace normal to the measured state as picked out by the first order perturbation theory, and correspondingly, the measured state collapses into this state. It thus claims that this precludes an ontological status for the wave function. If in the argument one directly resorts to the Einstein-Podolsky-Rosen criterion of reality (see, e.g. Hetzroni and Rohrlich 2014), according to which "*If, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.*" (italics in the original) (Einstein, Podolsky and Rosen 1935), then this objection may be valid. However, one may avoid this objection by resorting to a somewhat different criterion of reality, which is similarly reasonable and more appropriate for realistic protective measurements.

The new criterion of reality is that if, with an arbitrarily small disturbance on a system, we can predict with probability arbitrarily close to unity the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity. Although a realistic protective measurement with finite measurement time T can never be performed on a single quantum system with absolute certainty, the uncertainty and the disturbance on the measured system can be made arbitrarily small when the measurement time T approaches infinity. Thus according to this criterion of reality, realistic protective measurements also support the reality of the wave function. Note that in order to argue for the reality of the wave function in terms of protective measurements, it is not necessary to directly measure the wave function of a single quantum system, and measuring the expectation value of an arbitrary observable on a single quantum system is enough. If the expectation values of observables are physical properties of a single quantum system, then the wave function, which can be reconstructed from the expectation values of a sufficient number of observables, will also represent the physical property or physical state of a single quantum system. This will avoid the scaling problem (see Schlosshauer and Claringbold 2014).

Interestingly, we can also give another argument for ψ-ontology in terms of protective measurements, which is similar to the argument used by the PBR theorem (Pusey, Barrett and Rudolph 2012). For two arbitrary (protected) nonorthogonal states of a quantum system, select an observable whose expectation values in these two states are different. Then the overlap of the probability distributions of the results of protective measurements of the observable on these two states can be arbitrarily close to zero (e.g. when the measurement interval T approaches infinity). If there exists a non-zero probability p that these two nonorthogonal states correspond to the same physical state λ, then when assuming the same λ yields the same probability distribution of measurement results as the PBR theorem assumes, the overlap of the probability distributions of the results of protective measurements of the above observable on these two states will be not smaller than p. Since p is a determinate number, this leads to a contradiction. This argument, like the previous one, only considers a single quantum system, and thus avoids the preparation independence assumption used by the PBR theorem[2]. Note that the above protective measurements on the two *protected* nonorthogonal states are the same.

Finally, we note that there might also exist other components of the underlying physical state, which are not measureable by protective measurements and not described by the wave function, e.g. the positions of the particles in the de Broglie-Bohm theory. In this case, according to our argument, the wave function still represents the underlying physical state, though it is not a complete representation. Certainly, the wave function also plays an epistemic role by giving the probability distribution of measurement results according to the Born rule. However, this role will be secondary and determined by the complete quantum dynamics that describes the measure-

[2]Different from the present argument, the PBR argument does not rely on knowing the state being prepared.

ment process, e.g. the collapse dynamics in dynamical collapse theories.

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