

QBism and the character of the world

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Quantum probabilities as Bayesian probabilities

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In the Bayesian approach to probability theory, probability quantifies a degree of belief for a single trial, without any *a priori* connection to limiting frequencies. In this paper, we show that, despite being prescribed by a fundamental law, probabilities for individual quantum systems can be understood within the Bayesian approach. We argue that the distinction between classical and quantum probabilities lies not in their definition, but in the nature of the information they encode. In the classical world, *maximal* information about a physical system is *complete* in the sense of providing definite answers for all possible questions that can be asked of the system. In the quantum world, *maximal information is not complete and cannot be completed*. Using this distinction, we show that any Bayesian probability assignment in quantum mechanics must have the form of the quantum probability rule, that maximal information about a quantum system leads to a unique quantum-

QBism, the Perimeter of Quantum Bayesianism

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[quant-ph] 26 Mar 2010

This article summarizes the Quantum Bayesian [1–7] point of view of quantum mechanics, with special emphasis on the view's outer edges—dubbed QBism.¹ QBism has its roots in personalist Bayesian probability theory, is crucially dependent upon the tools of quantum information theory, and most recently, has set out to investigate whether the physical world might be of a type sketched by some false-started philosophies of 100 years ago (pragmatism, pluralism, nonreductionism, and meliorism). Beyond conceptual issues, work at Perimeter Institute is focussed on the hard technical problem of finding a *good* representation of quantum mechanics purely in terms of probabilities, without amplitudes or Hilbert-space operators. The best candidate representation involves a mysterious entity called a symmetric informationally complete quantum measurement. Contemplation of it gives a way of thinking of the Born Rule as an *addition* to the *rules* of probability theory, applicable when an agent considers gambling on the consequences of his interactions with a newly recognized universal capacity: dimension (formerly Hilbert-space dimension). (The word “capacity” should conjure up an image of something like gravitational mass—a body’s mass measures

to outward appearances is nearly identical to a common yearly annoyance. There are lessons here for quantum mechanics. In the history of physics, there has never been a healthier body than quantum theory; no theory has ever been more all-encompassing or more powerful. Its calculations are relevant at every scale of physical experience, from subnuclear particles, to table-top lasers, to the cores of neutron stars and even the first three minutes of the universe. Yet since its founding days, many physicists have feared that quantum theory’s common annoyance—the continuing feeling that something at the bottom of it does not make sense—may one day turn out to be the symptom of something fatal.

There is something about quantum theory that is different in character from any physical theory posed before. To put a finger on it, the issue is this: The basic statement of the theory—the one we have all learned from our textbooks—seems to rely on terms our intuitions balk at as having any place in a fundamental description of reality. The notions of “observer” and “measurement” are



Pablo Picasso, *Le Vieux Marc* (oil on canvas), 1912.

QBism puts the scientist back into science

An interpretation of quantum mechanics

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- HENCE: A quantum state is a personal judgement of the agent who assigns it.

Heads or Tails

Tossing a “fair” coin, following *The Logic of Science* by E. T. Jaynes:

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Observation 3

Any probability assignment starts from a *prior probability*.

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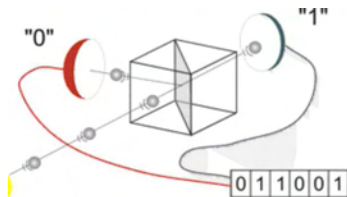
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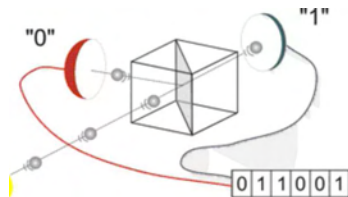
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- Probabilities can be assigned to single events as well as repeated trials.
- Different agents with different beliefs will in general assign different probabilities.

What about a quantum random number generator?



What about a quantum random number generator?



The probabilities for 0 and 1 are not a property of the device. They depend on the the prior quantum state for the device, i.e., on an agent's prior beliefs.

Epistemic quantum states

Einstein 1935 (not EPR)

Assuming λ (elements of physical reality) and locality (no spooky action at a distance) implies ψ is epistemic.

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Spekkens

Assuming ψ is epistemic explains a long list of otherwise puzzling quantum phenomena, from no cloning to teleportation and steering.

Schrödinger to Sommerfeld (1931):

One can only help oneself through something like the following emergency decree:

Quantum mechanics forbids statements about what really exists — statements about the object. It deals only with the object-subject relation. Even though this holds, after all, for any description of nature, it evidently holds in quantum mechanics in a much more radical sense.

Probability of what?

QBism: Quantum mechanics is a tool

that anyone can use to evaluate, on the basis of one's past [experience](#), one's probabilistic expectations for one's subsequent [experience](#).

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A quantum measurement finds nothing . . .

. . . but makes something:

A measurement is an action on the world by an agent that results in the *creation* of an outcome — a new experience for that agent.

An outcome does not preexist the measurement

Asher Peres:

Unperformed experiments have no results.

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Agent undertaking an experiment:

This experiment has no outcome until I experience one.

An outcome does not preexist the measurement

Asher Peres:

Unperformed experiments have no results.

Agent undertaking an experiment:

This experiment has no outcome until I experience one.

The experience *is* the outcome.

Experiences do not exist prior to being experienced.

- A quantum state is my personal judgement.
- Quantum mechanics is a tool to organize my experience.
- A measurement is an action on the world I take to elicit a new experience.

Nature cover, 27 March 2014

Quantum cryptography, randomness and cunning can outfox the snoopers
PAGE 443

QUANTUM PHYSICS
WHY IT'S ALL ABOUT ME
On the physical nature of the Now
PAGE 421

BIOMEDICINE
MAKE THE MOST OF MICE
Better use of disease models can save human lives
PAGE 423

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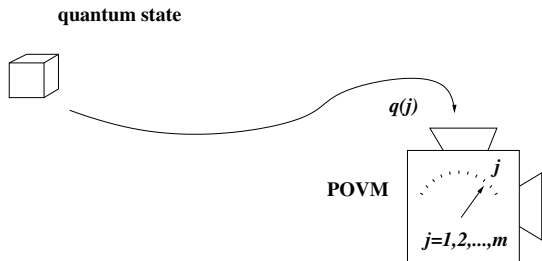
Informationally complete rank-1 POVMs

For any dimension d , there exists a (not necessarily symmetric) POVM $\{F_i\}$ with d^2 outcomes such that any density operator ρ is fully determined by the probabilities $p(i) = \text{tr}(F_i\rho)$ and any POVM $\{E_j\}$ is fully determined by the matrix of conditional probabilities $r(j|i) = \text{tr}(E_jF_i)/\text{tr}(F_i)$.

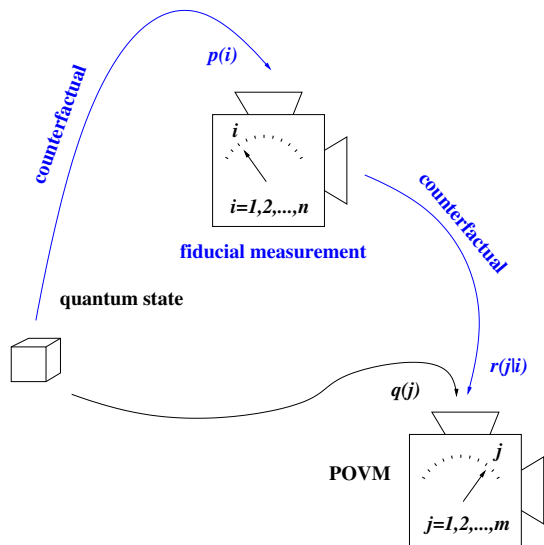
The Born rule as an object-subject relation

Born rule

$$q(j) = \text{tr}(\rho E_j)$$



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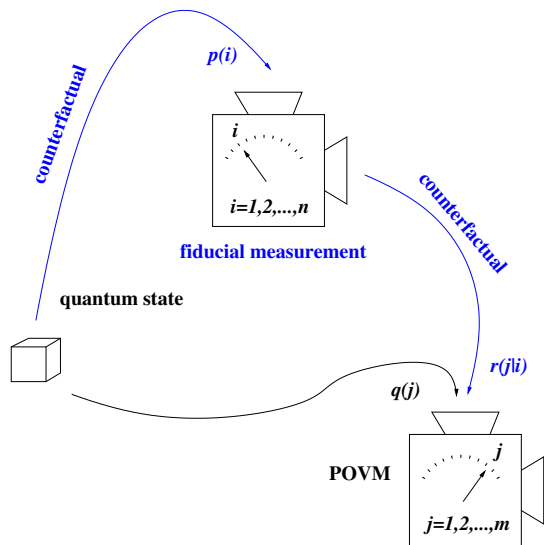
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$$\rho \longleftrightarrow p(i)$$

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Born rule, rewritten

$$q(j) = f(p(i), r(j|i))$$

The Born rule as an object-subject relation

- The Born rule connects an agent's outcome probabilities with his probabilities for a counterfactual fiducial measurement.

Dutch book (adapted from Wikipedia)

horse	odds offered				
1	even				
2	1:2				
3	1:3				

Dutch book (adapted from Wikipedia)

horse	odds offered		amount bet		
1	even		\$120		
2	1:2		\$80		
3	1:3		\$60		
	total		\$260		

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Definition

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Dutch book coherence

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Theorem

An agent's probability assignments are Dutch book coherent if and only if they obey the standard probability rules.

Probability theory provides explanations



Probability theory provides explanations



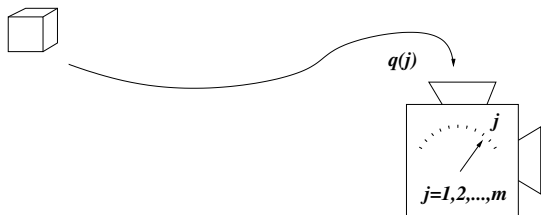
Quantum mechanics
is like probability
theory:

It gives no mechanism
for any particular
outcome, but a
formalism connecting
probabilities.

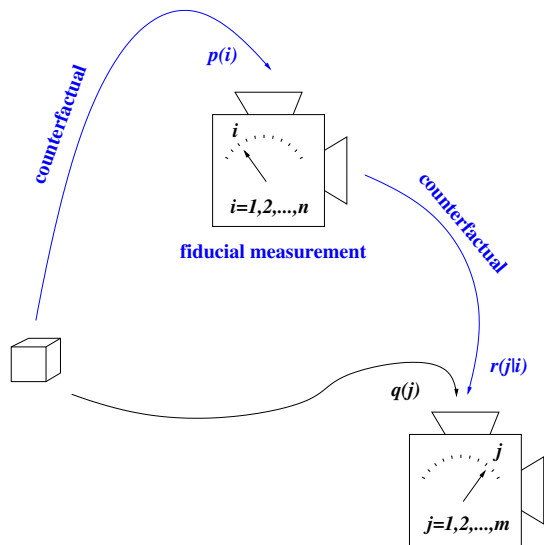
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probabilities

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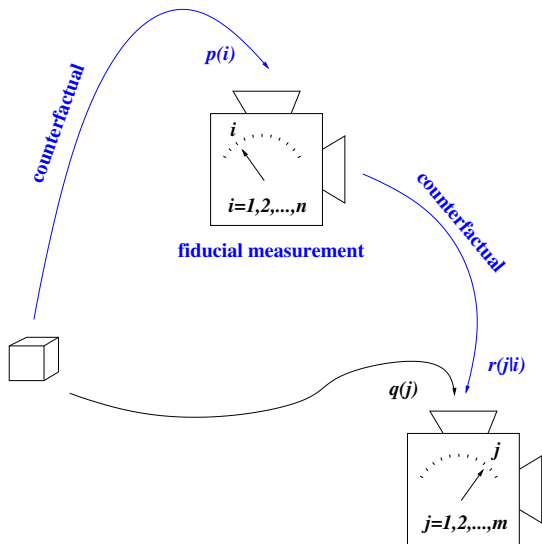
classical state

$$p(i)$$

conditional probabilities

$$r(j|i)$$

Classical mechanics as an object-subject relation



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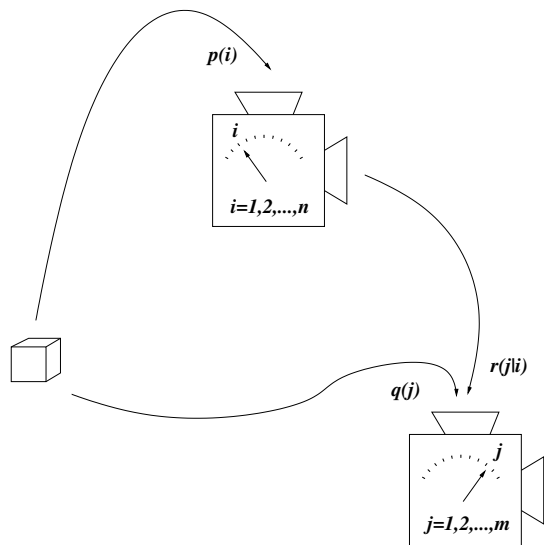
conditional probabilities

$$r(j|i)$$

classical probabilistic law

$$q(j) = \sum_i r(j|i)p(i)$$

Treating outcomes i as elements of reality



probabilities

$$q(j) = ?$$

classical state

$$p(i)$$

conditional probabilities

$$r(j|i)$$

law of total probability

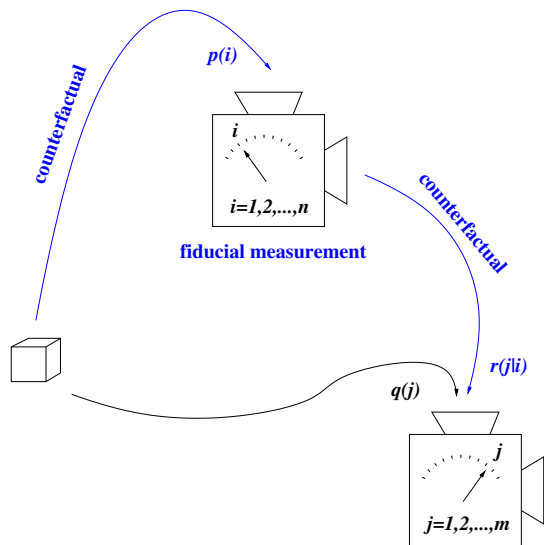
$$q(j) = \sum_i r(j|i)p(i)$$

A 2000-year old Greek maneuver

Schrödinger in *Nature and the Greeks*:

[...] the scientist subconsciously, almost inadvertently, simplifies his problem of understanding Nature by disregarding or cutting out of the picture to be constructed himself, his own personality, the subject of cognizance. [...]

Quantum theory is not classical



In quantum theory,
there is no fiducial
measurement such
that, in general

$$q(j) = \sum_i r(j|i) p(i)$$

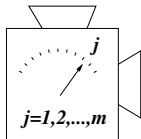
$p(i)$

$i=1,2,\dots,n$



$r(j|i)$

$q(j)$



In ontological models the i (from now on λ) are taken as elements of reality, implying that the law of total probability holds:

$$q(j) = \sum r(j|\lambda) p(\lambda)$$

What's wrong with λ -realism

It creates pseudo problems,

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If the world is fully described by a mathematical model, or a computer program, what is the difference between the world and the mathematical description?

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Hence it cannot be distinguished from solipsism:

If the world is equivalent to a computer program, then how can I tell that it is not my brain that is running it?

There is no experimental evidence for λ -realism

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Examples of pseudo-problems created by λ -realism:

- The problem of the collapse of the wavefunction.
- The measurement problem.
- The problem of quantum nonlocality.

Wigner's friend

Wigner's friend makes a measurement

in a closed lab and experiences an outcome. Wigner, outside the lab, doesn't experience an outcome and writes down an entangled state.

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The friend's measurement outcome is personal to the friend.

A paradox is created

by assuming the measurement outcome is an objective feature of the world. Then either the friend is hallucinating, or Wigner is inconsistent.

If Wigner were Alice, Bob would be her friend

Quantum correlations for a bipartite system:

they refer to Alice's experiences resulting from her (necessarily local) actions on A and B , respectively.

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Quantum correlations for a bipartite system:

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("spooky action at a distance") by assuming that Bob's outcome is an objective feature of the world.

QBism's realism

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When I act on the world, it changes,

as witnessed by my changed expectations for my future experiences.

A world under construction

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- In every quantum measurement something new is created.
- Every experience is an addition to the world.
- Quantum mechanics allows me to organize my experience with spectacular success.

I am not special

A Copernican principle:

By one category of thought we are agents, but by another category of thought we are physical systems. And when we take actions upon each other, the category distinctions are symmetrical.

I am not special

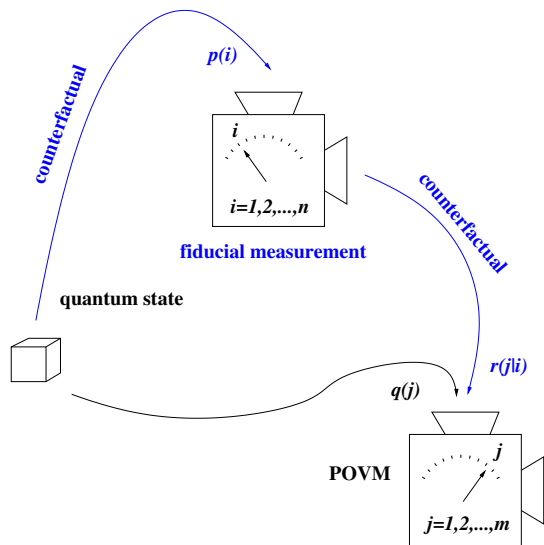
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This leads to the idea of a “pluriverse”

in which each agent's private experience is as real as mine.

The glue that holds the pluriverse together



Assuming that SICs exist in all d , there exists a fiducial measurement such that, in general,

$$q(j) = \sum_i r(j|i) \left((d+1)p(i) - \frac{1}{d} \right)$$

The “urgleichung”

A quantum postulate:

For every quantum system, there exists a fiducial measurement such that, for all measurements, $q(j) = \sum_i r(j|i) (\alpha p(i) - \beta)$

See C. A. Fuchs and RS, Found. Phys. (2011) and Rev. Mod. Phys. (2013).

Summary

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