Bell non-locality, Hardy's paradox and hyperplane dependence

by

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Abstract: I argue, in section **4**, that the 'elements of reality' of Hardy's famous gedanken experiment can retain their Lorentz invariance, i.e., their frame independence, if one recognizes the hyperplane dependence of their localization. This requires avoiding the conflation of hyperplane dependence with frame dependence, which occurs occasionally, and I argue against such conflation in **3**. Preparatory remarks on my general perspective concerning the interpretation of quantum mechanics are presented in **2** and I begin with some reminiscences of my delayed appreciation of the significance of John Bell's work. Finally, in **5**, I criticize a view of the nature of Lorentz transformations presented by Asher Peres and co-workers which conflicts with the view employed here.

1. Initial reactions to Bell's work: Like many others in the physics community, I was late to come to an appreciation of the significance of John Bell's papers on the foundations of quantum mechanics (QM). But unlike many of those, my indifference to Bell was not due to a dismissive attitude to foundational studies per se or to an intransigent commitment to some version of Copenhagenism. In 1971, for example, I was very much involved in the international conference on QM foundations at my home institution[1] where Bell presented the paper, "On the hypothesis that the Schroedinger equation is exact", published later as (Bell 1978), but I paid minimal attention to his presentation. The cause of this poor judgement (as I eventually came to realize it was) was that I was already convinced that the Schroedinger equation was not exact in the sense Bell meant, but must be augmented with primordial state reductions. I was among those who, in the words of Bob Wald, as reported by Roger Penrose (1997), were inclined to "take it seriously", the QM state that is, and, consequently, couldn't "really believe in it", i.e., really believe that purely unitary QM is a complete theory. I was delighted when the experimental tests of Bell's inequalities upheld the QM predictions. I was, furthermore, uninterested in hidden variable *reconstructions* of QM, such as Bohmian mechanics (Bohm 1952) and Many Worlds interpretations (Everett 1957) (DeWitt & Graham 1973), that were, in principle, not susceptible to empirical tests of their novel details. These attitudes, which I still, largely, subscribe to, were not, of course, due to any deep insights on my part into the workings of nature. They were due, rather, I suspect, to my personal allotment of psychological quirks and philosophical preferences

1. "International Colloquium on Issues in Contemporary Physics and Philosophy of Science, and their Relevance for our Society", Penn State University, September 6-18, 1971. A colloquium memorable for passionate presentations at which I first met Abner Shimony, received the withering declaration from Imre Lakatos that "Elementary particle physics is not a science!" and had the pleasure of participating in a dramatic reading of Joseph Jauch's then as yet unpublished mss. *"Are Quanta Real?"*. I read Sagredo to C.F. von Weisacker's Salviati and, if memory serves, a young Cliff Hooker's Simplicio.

(not to say prejudices) and they predetermined my favorable attitude to the later GRW (Ghirardi et al 1986) and other schemes for augmenting QM with *primordial spontaneous state reductions* (PSR).

My later recognition of the importance of Bell's work emerged only with the slow awareness that the violation of Bell's inequalities entailed a version of non-locality different from, but not wholly unrelated to the hyperplane dependence (HD) of many dynamical variables that I had been argueing for (Fleming 1965, 66) on the grounds of minimal compatibility with Lorentz covariance. Continued ruminations along these lines eventually led to my arguing for HD state reductions (Fleming 1985) as well. Only once, however, did I engage in a direct examination of Bell-related inequalities (Fleming 1995), those of the GHZ type (Greenberger et al 1989).

Quite apart from Bell's role in leading us to the recognition of the non-locality of *this* world, the incisive style of his writing and his arguments are invaluable! However much the choices we make among the many competing interpretations and approaches to QM may depend on the above mentioned personal allotments, Bell's work helps to make clear just where we stand (Maudlin 2014).

2. The ontology of quantum phenomena and the quantum state: Regarding the status of the quantum state, I belong in the ψ-ontic camp (Leifer 2014). But this camp is widely dispersed, with many conflicting subdivisions. In this section I try to pin down and clarify my place among the subdivisions. First, I do not hanker after determinism (Vaidman 2014). I regard the encounter with quantum phenomena, particularly value acquisition (or property actualization) implemented via state reduction, as the discovery of genuinely *uncaused events*! They are 'governed', in their occurrences, by the basic PSRs among them, by the unitarily evolving quantum probabilities, and, in the special case of measurements, by our experimental preparations, but by nothing else. Their individual occurrences are *instances of the suspension of the principle of sufficient reason* (Melamin & Lin 2010)! And in their turn, they cause abrupt changes in those quantum probabilities.

Quite generally, quantum *systems* can be characterized by the various possible complete sets of compatible properties (comsets), the members of which are, in principle, capable of being jointly actualized and the PSRs may be just such uncaused actualizations. The quantum state of a system, mathematically represented (relative to an inertial frame) by a state vector in the comparatively rare case of a pure state or by a density operator in the much more common case of an 'improper' mixed state, is our current best effort at conceptually grasping the ontological ground of objective physical reality. The evolving probabilities are determined by the relation of the dynamical variable operators and their eigenvectors, evolving via the Heisenberg equations of motion,[1] to the state *operators* from which the density *matrices*, of various representations, are formed. A generalized

1. My preoccupation with Lorentz covariant dynamics dictates the use of the Heisenberg picture. The Schroedinger picture is awkward, at best, in the presence of frame dependent time coordinates and becomes even more so when time dependence is generalized to HD. Finally, in the presence of PSR, the *generalized* Heisenberg picture admits, for closed systems, a clean separation between unitary and stochastic evolution.

Heisenberg picture (Weinberg 2012), which I am clearly employing here, restricts the dynamical variable operators to unitary evolution and the state operator, for otherwise closed systems, to stochastic evolution via PSRs and their exploitation for measurements. For open sub-systems the evolution of the sub-system state operator receives a unitary contribution as well as a stochastic contribution (Fleming 1996).

As yet we do not have a satisfactory theory of PSR, notwithstanding heroic and encouraging efforts. I have no unequivocal favorites but am impressed with the current relativistic, mass density versions (Bassi et al 2013)(Bedingham et al 2014) and the efforts of (Penrose 1996, 2009), (Diosi 1987, 1989, 2007) and others, to ground state reduction in quantum gravity and their search for empirical support in biological processes (Hameroff & Penrose 2013). Regarding the contribution of environmental decoherence (Schlosshauer 2008), it seems to me that while it can greatly enhance the effectiveness of PSR in avoiding / suppressing superpositions of macro-distinct states, it can not, itself, supplant PSR.

I do not share the widespread inclination (Albert & Ney 2014) to attribute fundamental status to the *wave function*, i.e., the position representation of a pure quantum state of a system of a definite number of quantons[1] or the local field theoretic functional representation with diagonalized fields. In the formalism of QM, all possible comset representations of a quantum state, whether pure or mixed, are equivalent in the sense that from any one such representation on a given hyperplane, any other one on the same hyperplane is, in principle, kinematically determined. I take this as indicative of a symmetry in nature that extends very deep. Nothing remotely like it holds in classical physics. The classical mathematical equivalence of all possible choices of canonical variables or all possible choices of generalized coordinates and velocities in Hamiltonian or Lagrangian formulations, respectively, is a poor analogue.[2]

It is the quantum system and its state, mathematically represented, in relation to an inertial frame, by the abstract state vector or the abstract trace class operator, in which I tend to see ontological status or real existence. But not exclusively so; the possible / potential properties of the system, mathematically represented by the dynamically evolving Heisenberg picture operators really exist as well, or so I think. It is in the uncaused events of PSR that these two modalities of existence, the system states and their potential properties, come together in the collapse to eigenvectors of an actualized property. But which property? My hesitancy to grant primordial status to any particular representation or, equivalently, to any particular comset would seem to require a random distribution of PSR to any and all possible discrete spectrum eigenvectors. But perhaps not. In the case of systems of definite numbers of quantons, collapse to coherent states which do not precisely actualize any commonly recognized dynamical variable may be

1. quanton := boson, fermion or anyon.
2. For a single classical particle, neither momentary position nor momentary momentum determines the other. The whole history of momentum fails to determine position at any time. For a single, spinless quanton the momentary position or momentum state representation determines the other and all possible others!

the way. Of course in the context of quantum field theory the number of quantons of any given type is almost always indefinite and then what does PSR lead to. Could coherent states again, but now of the field theory variety, be candidates? Alternatively, might the PSR of quantized field states aim directly at rendering quanton numbers momentarily definite? Finally what becomes of the whole PSR scheme in the context of a future quantum gravity theory where the ‘system’ subject to PSR is space-time itself.

But where do the abstract system states live, it is often asked. In the generalized Heisenberg picture I have in mind they live on space-like hyperplanes in space-time ‘between’ the state reductions. Non-relativistically, the state represents the history of the system literally between consecutive (collections of simultaneous) state reductions. The state lives at those time intervals. The unitarily evolving dynamical variables live, more particularly, at the individual times and, for mass conserving fields, at points of space at those times as well.

Lorentz covariantly, the states of closed systems live on the *collections* of space-like hyperplanes that each divide all the state reductions into those lying in the past and those lying in the future of the collection in question. In section **4** below, I will present yet another[1] account of the need for this HD of quantum states, this time by examining the gedanken experiment of (Hardy 1992) which leads to the so-called Hardy paradox of elements of reality that must violate Lorentz invariance. If they partake of the non-locality of HD, as I claim they must, then they regain Lorentz invariant status. This approach neither supports, necessarily, nor conflicts with other analyses such as (Aharonov et al 2001) but, rather, indicates that the appearance of paradox in the first place is due to neglect of HD in quanton localization and state reduction.

The dynamical variables, if local fields, live at points of space-time; but if global quantities, such as total 4-momenta or angular momenta or various charges, they live on *individual* space-like hyperplanes . And then there may be non-local fields that live at *points-on-hyperplanes,* (Ardalan & Fleming 1975), (Boyer & Fleming 1974), examples of which appear to be gaining use for certain purposes (Piazza & Costa 2007), (Cacciatori et al 2009), (Schuster & Toro 2013). Somewhat more conjecturally, in curved space-times, the non-local dynamical variables will live on space-like hypersurfaces of zero extrinsic curvature while the states live on collections of such which separate the instances of PSR and so on.

I want to stress that I do not regard this association of states and dynamical variables with (sets of) space-like hyperplanes as simply a possible approach to achieving manifest covariance of description under the IHLG. Rather, I claim that the association is *forced* upon us by the presumed equivalence of all inertial observers and the success such observers have in describing dynamical evolution in terms of physical states of affairs (PSA) at instants of time. The argument proceeds as follows:

1. For earlier arguments see (Fleming 1985, 1989, 1992, 2003).

(1) Any PSA that one inertial observer can describe and/or measure can, as a matter of principle, also be described and/or measured by any other inertial observer.

(2) Instants of time and the PSAs that hold at them for one inertial observer are, for any other inertial observer, space-like hyperplanes with those PSAs holding on them.

(3) Any space-like hyperplane and the PSA that holds on it is an instant of time, with the PSA holding at it, in some inertial frame.

(4) Consequently, all inertial observers can describe and/or measure the PSAs on any and all space-like hyperplanes.

While some would and have (Schwinger 1948)(Tomonaga 1946) extended these allowed descriptions to PSAs on arbitrary curvilinear space-like hypersurfaces, such an extension in Minkowski space-time can not be grounded in the tradition of description at instants of time and is not needed for the manifest covariance of descriptions. The extension is, therefore, artificial and I avoid it. I also note that we can not extend this argument, *without modification*, to non-inertial observers in Minkowski space-time or even freely falling observers in generally curved space-times because of the occurrence of horizons.

I might mention, in passing, that while, as claimed above, I do not favor special ontological status for position representation state functions for N quantons, I also do not sympathize with the argument that they can not be real because they live in 3N dimensional configuration space. They need live there only if one insists on making the state function a *single point field*, as is traditional. They would live just as easily in physical 3-space as an N point field, i.e., a field assigned to each (appropriately identified) *set* of N, 3-space points. Be that as it may, it is the abstract state vector or state operator that I see as the mathematical representation of the objective quantum reality rather than any particular, basis defined, number valued, representation thereof.

Finally, the results of delayed choice experiments on entangled states by Aspect's (Jaques et al 2007) and Zeilinger's (Ma et al 2012) groups corroborating Bell inequality violating probabilities reveal that the pairs of measurements can not be objectively separated into triggering and testing measurements of a correlation. Being space-like separated, neither measurement is invariantly earliest or latest and the correlation is simply a single property of the state actualized by the *pair* of measurements. To paraphrase (Gisin 2005) "a quantum [space-like] correlation is not a correlation between 2 events, but a single event that manifests itself at 2 locations."

3: Hyperplane dependence vs frame dependence: I find that some writers, including some in the minority whom I regard as sympathetic to my orientation towards HD, conflate HD with frame dependence (FD). This seeming conflation is further confused by the variety of concepts of just what an inertial frame of reference is. An older concept identifies inertial reference frames with Minkowski coordinate systems. I will call these M-frames. A more contemporary concept identifies inertial reference frames with hyperplane foliations of space-time. I will call these F-frames. The F-frames are associated with equivalence classes of M-frames, each class consisting of M-frames that are at rest relative to one another. For either kind of frame, HD is not the same as FD. I will spell this out here (in possibly unnecessary detail) with the elementary example provided by the total 4-momentum of a time-like persistent and space-like extended system which may be open or closed. Once established here, the presence of HD and the independence of HD and FD will be exploited in **4** to defuse the Hardy paradox.

Beyond these considerations it turns out that some workers object to the very concept of the nature of a Lorentz transformation that I will employ here. I did not appreciate this fact until rather recently. The concept I use is effectively defined (at least for 4-vectors) by equations (3.1, 2) and (3.12) below and I will not defend it just yet as I believe most will not object to it. But in **5** I will defend it against the opposing view which has rather drastic consequences.

For a quantum state, ρ, and a closed system (having conserved 4-momentum) the expectation values of the 4-momentum components will be denoted by, $< P^\mu >_\rho$, and satisfy the transformation rule,

$$< P^\mu >'_\rho = \Lambda^\mu_\nu < P^\nu >_\rho \tag{3.1}$$

for two M-frames, M and M', with coordinates related by the IHLG transformation,

$$x^{\mu\,\prime} = \Lambda^\mu_\nu x^\nu + a^\mu . \tag{3.2}$$

Such expectation values are calculated using a quantum state operator, $\hat{\rho}$, and a total 4-momentum operator, $\hat{P}^\mu$. Thus,

$$< P^\mu >_\rho = Tr(\hat{\rho}\,\hat{P}^\mu) . \tag{3.3}$$

While the quantum state, ρ, is objective and M-frame independent, the quantum state *operator*, $\hat{\rho}$, represents the *relationship* of the state to the frame and thus is, itself, frame dependent. In particular we have,

$$\hat{\rho}' = \hat{U}(\Lambda,a)\hat{\rho}\,\hat{U}(\Lambda,a)^\dagger , \tag{3.4}$$

where $\hat{U}(\Lambda,a)$ is the unitary representation of the IHLG transformation. On the other hand, the operators, $\hat{P}^\mu$, are not frame dependent in the sense that the same set of operators are used to evaluate the expectation values in any frame. Thus,

$$< P^\mu >'_\rho = Tr(\hat{\rho}'\hat{P}^\mu) . \tag{3.5}$$

The preceding transformation rule, (3.1), for the expectation values emerges from the property of the frame independent momentum operators,

$$\hat{U}(\Lambda,a)^{\dagger}\hat{P}^{\mu}\,\hat{U}(\Lambda,a)=\Lambda^{\mu}_{\nu}\hat{P}^{\nu}\,, \qquad (3.6)$$

but it would be double counting to identify the left hand side as the 4-momentum operator for the frame, M'.

So far no hint of hyperplane dependence (HD) has appeared, notwithstanding the fact that the definition of $\hat{P}^{\mu}$ in quantum field theoretic terms can employ any hyperplane, whatsoever. Thus, if $\hat{\theta}^{\mu\nu}(x)$ is the symmetric, stress-energy-momentum (SEM) field where,

$$<\hat{\theta}^{\mu\nu}(x')>'=\Lambda^{\mu}_{\lambda}\Lambda^{\nu}_{\sigma}<\hat{\theta}^{\lambda\sigma}(x)>, \qquad (3.7a)$$

then,

$$\hat{P}^{\mu}=\int d^4x\,\delta(\eta x-\tau)\hat{\theta}^{\mu\nu}(x)\eta_{\nu}\,, \qquad (3.7b)$$

for any τ and any future pointing time-like unit vector, η^{μ}. The integral remains hyperplane independent due to the SEM field integrand being (for a closed system) locally conserved, i.e.,

$$\partial_{\mu}\hat{\theta}^{\mu\nu}(x)=0\,. \qquad (3.8)$$

We now consider an open subsystem of our closed system for which the local conservation of the subsystem SEM field, $\hat{\theta}^{\mu\nu}_{S}(x)$, does not hold. The field theoretic definition of the open subsystem total 4-momentum operator is now HD,

$$\hat{P}^{\mu}_{S}(\eta,\tau)=\int d^4x\,\delta(\eta x-\tau)\hat{\theta}^{\mu\nu}_{S}(x)\eta_{\nu}\,, \qquad (3.9)$$

and the behaviour of $\hat{P}^{\mu}_{S}(\eta,\tau)$ under unitary IHLG transformations is now,

$$\hat{U}(\Lambda,a)^{\dagger}\hat{P}^{\mu}_{S}(\eta',\tau')\,\hat{U}(\Lambda,a)=\Lambda^{\mu}_{\nu}\hat{P}^{\nu}_{S}(\eta,\tau)\,, \qquad (3.10a)$$

where,

$$\eta'=\Lambda\eta \qquad and \qquad \tau'=\tau+a\eta'\,. \qquad (3.10b)$$

Again, the left hand side of (3.10a) is not the 4-momentum operator for the frame, M', since the frame dependence is, as before, carried by the quantum state operator. The expectation values now (using the open subsystem analogues of (3.3 – 6)) are both FD and HD according to,

$$<P^{\mu}_{S}(\eta',\tau')>'_{\rho}=Tr(\hat{\rho}'\hat{P}^{\mu}_{S}(\eta',\tau'))=Tr(\hat{\rho}\,\Lambda^{\mu}_{\nu}\hat{P}^{\nu}_{S}(\eta,\tau))=\Lambda^{\mu}_{\nu}<P^{\nu}_{S}(\eta,\tau)>_{\rho}\,. \qquad (3.11)$$

We have seen FD quantities that were not HD quantities, (3.1), and now quantities that are both FD and HD, (3.11). Finally we consider the scalar quantities,

$$< P_S^\mu(\eta,\tau) >_\rho < P_{S,\mu}(\eta,\tau) >_\rho \quad and \quad < P_S(\eta,\tau)^2 >_\rho := Tr(\hat{\rho}\hat{P}_S^\mu(\eta,\tau)\hat{P}_{S,\mu}(\eta,\tau)). \qquad (3.12)$$

Being scalars, these are not FD, but they are HD for open systems and not HD for closed systems. Thus all possible combinations of FD and not FD, HD and not HD have been displayed. HD and FD are not the same.

Finally, in the presence of PSR the state operator, $\hat{\rho}$, for an otherwise closed system, will acquire a stochastic HD which, strictly speaking, would eliminate the possibility of any exactly non-HD expectation values! This does not undermine the sharp distinction between HD and FD!

4: The Lorentz covariance of elements of reality: I have previously presented the following kind of argument for a system of two entangled quantons subjected to space-like separated state reductions (Fleming 1989) and for a system of three entangled quantons similarly subjected (Fleming 1992). In each case the purpose of the argument is to demonstrate the manner in which considerations of HD can defuse (eliminate) apparent paradoxes that emerge from entanglement and EPR correlations that violate Bell inequalities in a relativistic context. This time I will again discuss the unitary evolution of a two quanton system preceding space-like separated state reduction. The quantons will

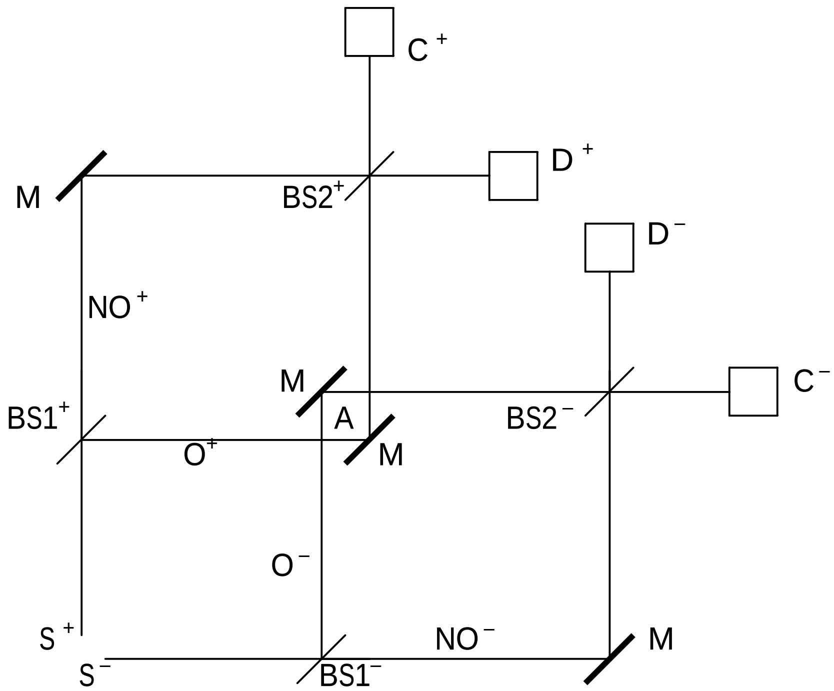

Fig. 1: Hardy's arrangement of Mach-Zender interferometers for an electron-positron pair with sources, $S^{\pm}$, overlapping arms, $O^{\pm}$, non-overlapping arms, $NO^{\pm}$, beam splitters, $BS1^{\pm}$ and $BS2^{\pm}$, mirrors, M, annihilation region, A, and detectors, $C^{\pm}$ and $D^{\pm}$.

be an electron-positron pair (e^-, e^+) and the experimental arrangement will be that of the well known gedanken experiment due to Lucien Hardy involving two Mach-Zender interferometers (Fig. 1). In that case each quanton interacts unitarily with macroscopic apparatus in space-like separated regions and then (i.e., time-like later), potentially, with each other and then again with a second set of macroscopic apparatus in space-like separated regions before the final state reductions.

The macroscopic apparatus that the system interacts with unitarily are four beam splitters, $BS1^\pm$ and $BS2^\pm$ and mirrors, M. The interactions of e^- with $BS1^-$ and e^+ with $BS1^+$ occur in space-like separated regions as do the interactions of e^- with $BS2^-$ and e^+ with $BS2^+$. But time-like later than the first set of interactions and time-like earlier than the second set of interactions is an annihilation region, A, where the electron-positron pair may annihilate into two photons, 2γ. The beam splitters and initial states of the quantons are arranged so that if either of the quantons traversed the arrangement alone, the quantum state evolution would follow the sequence,

$$|S^\pm> \underset{BS1^\pm}{\rightarrow} (|O^\pm> + |NO^\pm>)/\sqrt{2} \underset{BS2^\pm}{\rightarrow} |C^\pm> \tag{4.1}$$

where, $|S^\pm>$ are the initial states of the quantons, $|O^\pm> and\, |NO^\pm>$ are the partial states for the separate arms produced by interaction with $BS1^\pm$ and,

$$|O^\pm> \underset{BS2^\pm}{\rightarrow} (|C^\pm> + |D^\pm>)/\sqrt{2}\,, \tag{4.2}$$

and

$$|NO^\pm> \underset{BS2^\pm}{\rightarrow} (|C^\pm> - |D^\pm>)/\sqrt{2}\,, \tag{4.3}$$

where $|C^\pm> and \pm|D^\pm>$ are the partial states (and relative phases) produced for the separate detectors by interaction of $|O^\pm> or\, |NO^\pm>$ with $BS2^\pm$. Finally, the O arms can be arranged to have an overlap region, A, where the electron and positron could encounter one another and annihilate into two photons,

$$|O^+O^-> \underset{A}{\rightarrow} |2\gamma> \tag{4.4}$$

and in each interferometer, the final beam splitter encounter is space-like separated from the final detection event of the other interferometer – as indicated by the hyperplane lines, $h_R(O^+D^-)$ and $h_L(D^+O^-)$ in the somewhat busy Fig. 2

Let us now assume an inertial reference frame, F_0, in which the interactions of the quantons with their respective beam splitters, BS1, are simultaneous as are the interactions with the beam splitters, BS2 and the final detectors, CD. If the annihilation region were not in place, then the two quanton system would evolve according to the scheme,

$$|S^+S^-> \underset{BS1^{\pm}}{\rightarrow} (|O^+O^-> + |O^+NO^-> + |NO^+O^-> + |NO^+NO^->)/2 \underset{BS2^{\pm}}{\rightarrow} |C^+C^-> \tag{4.5}$$

If, instead, the overlap region, A, is in place (and we follow Hardy and consider only those instances in which the electron-positron pair is finally detected in $D^+ D^-$) , then the time evolution scheme in F_0 is,

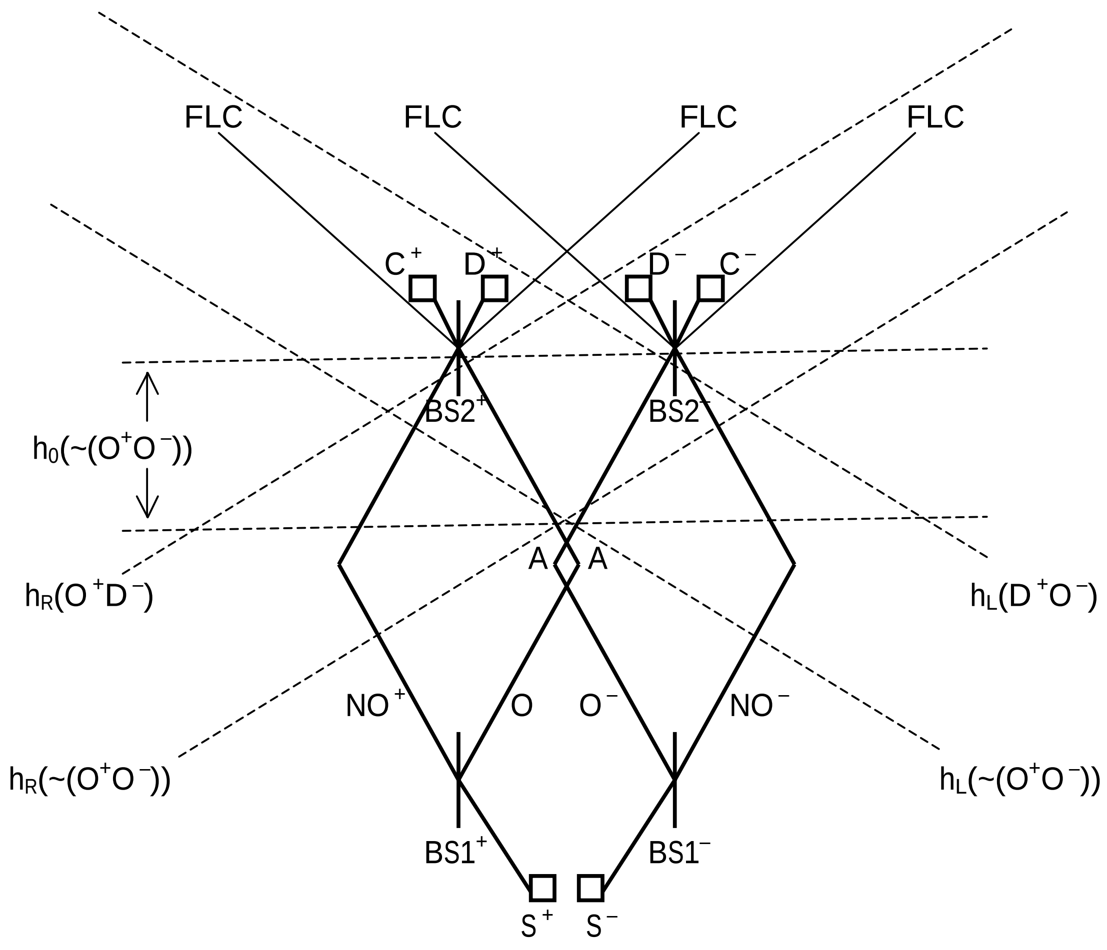

Fig. 2: Space-time relations among various stages of Hardy's gedanken experiment. FLC denotes future light cone lines. $h_{0,L,R}$ $(\sim(O^+O^-))$, $h_L(D^+O^-)$ and $h_R(O^+D^-)$ denote space-like hyperplane lines which are instantaneous in the $F_{0,L,R}$ frames, respectively.

$$|S^+S^- > \underset{BS1^{\pm}}{\rightarrow} (|O^+O^- > + |O^+NO^- > + |NO^+O^- > + |NO^+NO^- >)/2 \underset{A}{\rightarrow}$$

$$(|2\gamma > + |O^+, NO^- > + |NO^+, O^- > + |NO^+NO^- >)/2 \underset{BS2^{\pm}}{\rightarrow}$$

$$(1/4)(2|2\gamma > + 3|C^+C^- > + |C^+D^- > + |D^+C^- > - |D^+D^- >) \underset{D^+D^-}{\rightarrow} |D^+D^- > . \qquad (4.6)$$

Note that in F_0, after the possibility of annihilation and before the BS2 interactions, the probability, at any F_0 time, of finding the electron in the O^- state and the positron in the O^+ state is zero. That is the meaning of the symbol, $\sim(O^+O^-)$. Those instants of time are invariantly described as the hyperplanes of type $h_0(\sim(O^+O^-))$, indicated in Fig. 2.

In a reference frame, F_L, moving to the left relative to F_0, in which the $BS2^+$ interaction and the D^+ state reduction are earlier than the $BS2^-$ interaction, the F_L time sequence between the A event region and the $BS2^-$ interaction is,

$$(|2\gamma > + |O^+NO^- > + |NO^+O^- > + |NO^+NO^- >)/2 \underset{BS2^+}{\rightarrow}$$

$$(\sqrt{2}|2\gamma > + 2|C^+NO^- > + |C^+O^- > - |D^+O^- >)/2\sqrt{2} \underset{D^+}{\rightarrow} -(1/2\sqrt{2})|D^+O^- > . \qquad (4.7)$$

In such an F_L frame with D^+ detection there are instants of time, after A and before $BS2^-$, when, just as in F_0, there is no possibility of finding O^+O^-. See $h_L(\sim(O^+O^-))$.The probability of finding O^- at these times is 1/4. This probability remains 1/4 until the $BS2^+$ interaction when the O^- probability changes to 5/8 followed by the D^+ collapse when the O^- probability jumps to unity and remains there until the $BS2^-$ interaction. Those latter instants of F_L time are invariantly described as hyperplanes of the type, $h_L(D^+O^-)$ in Fig. 2.

Similarly, in a frame, F_R, moving to the right relative to F_0, in which the $BS2^-$ interaction and the D^- state reduction are earlier than the $BS2^+$ interaction, the F_R time sequence between the A event region and the $BS2^+$ interaction is,

$$(|2\gamma > + |O^+NO^- > + |NO^+O^- > + |NO^+NO^- >)/2 \underset{BS2^-}{\rightarrow}$$

$$(\sqrt{2}|2\gamma > + 2|NO^+C^- > + |O^+C^- > - |O^+D^- >)/2\sqrt{2} \underset{D^-}{\rightarrow} -(1/2\sqrt{2})|O^+D^- > . \qquad (4.8)$$

In such an F_R frame with D^- detection there are instants of time, after A and before $BS2^+$, when, just as in F_0, there is no possibility of finding O^+O^-. See $h_R(\sim(O^+O^-))$. The probability of finding O^+ at these times is 1/4. This probability remains 1/4 until the $BS2^-$ interaction when the O^+probability changes to 5/8 followed by the D^- collapse when the O^+ probability jumps to unity and remains there until the $BS2^+$ interaction. Those latter instants of F_R time are invariantly described as hyperplanes of the type, $h_R(O^+D^-)$, in Fig. 2.

The existence (in our subensemble of D^+ D^- instances) of F_L times after A at which the electron is certain to be in the O^- arm and F_R times after A at which the positron is certain to be in the O^+ arm while there are no F_0 times after A in which both localizations in O^- and O^+ is possible constitutes the apparent paradox, or, more precisely the frame dependence of the elements of reality, as Hardy called them.

But the two sets of hyperplanes of types, $h_L(D^+O^-)$ and $h_R(O^+D^-)$, are mutually disjoint and each set is disjoint with the three sets, $h_{0,L,R}$ $(\sim(O^+O^-))$. Consequently, there are no hyperplanes, 'later' than A, on which the electron–positron pair are in the $|O^+O^->$ state. The paradox, in which $\sim(O^+O^-)$ appeared to conflict with D^+O^- and O^+D^- dissolves by virtue of the absence of a common space-time substrate for the three assertions or any two of them. The elements of reality, as Hardy referred to them, retain their Lorentz invariance because all inertial observers agree as to the physical state of affairs on any given hyperplane, i.e., $F_{0,L,R}$ all agree that D^+O^- occurs on $h_L(D^+O^-)$, that O^+D^- occurs on $h_R(O^+D^-)$ and that O^+O^- does not occur on any of $h_{0,L,R}$ $(\sim(O^+O^-))$. Lorentz invariance is preserved via the non-locality of HD and the non-local, HD analysis is forced upon us by the 4 step argument at the end of 2. Referring back to the discussion in 3 against the conflation of HD with FD, we might emhasize here that while localization at instants of time is FD, localization, per se, is not FD. Rather, localization, per se, is HD and only which hyperplanes are instants of time is FD.

But to feel comfortable with this dissolution one has to take seriously the HD of quanton localization, which many do not. So let us test the HD probabilities by placing a detector in the O^+ arm, say, just before $BS2^+$.

First we notice that if we do not find the positron in the O^+arm (state) then the modified evolution on all three sets of hyperplanes, h_0, h_L and h_R yield zero probability for detecting the electron in D^-. So restricting to the final outcome of D^- guarantees finding the positron in O^+ as predicted on h_R. But then the state evolution on the h_L hyperplanes is modified early to ensure the electron is in NO^-. Similar considerations apply if we place the detector in the O^- arm, or a pair of detectors in two arms. One can never find the O^+O^- combination on any hyperplane 'later' than A. Nevertheless, this does not invalidate the predictions of certainty for finding O^+ on h_R 'after' D^- and 'before' $BS2^+$ or O^- on h_L 'after' D^+ and 'before' $BS2^-$, respectively. And, of course, 'after' both D^+ and D^-, no such predictions hold.

5. Kinematics, dynamics and the nature of Lorentz transformations: The view of QM presented in broad strokes in section 2 is very different from the view embraced and forcefully articulated over many years by the late Asher Peres (1995). A view that I would characterize as instrumentalist, epistemic and information theoretic. But instead of that broad and deep disagreement, what I will discuss here is the more narrowly focused disagreement over the meaning and nature of Lorentz transformations. Throughout this section we will not be considering any kind of state reductions, neither PSR or measurement induced. Thus state vectors here are always FD but not HD while the Heisenberg picture operators of interest may be HD, but are not FD (Their expectation values may be FD. See the discussion containing (3.4 - 6)).

It is, perhaps, surprising that physicists can still disagree about the nature of something as basic, in the sense of being elementary, as Lorentz transformations! But it is so. The simplest applications of such transformations are not at issue. Everyone agrees that if M and M' are two Minkowski coordinate systems for inertial frames, then the assignment of coordinate values in M and M', to any given space-time point, are related by equations of the form,

$$x'^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu} + a^{\mu} , \tag{5.1}$$

where the homogeneous coefficients, Λ^{μ}_{ν}, satisfy,

$$\Lambda^{\mu}_{\nu} \eta^{\nu\lambda} \Lambda^{\rho}_{\lambda} = \eta^{\mu\rho} , \tag{5.2}$$

with a diagonal metric tensor that can be chosen as, $\eta^{\mu\nu} = (1,-1,-1,-1)$. Agreement continues concerning the transformation rules for so-called local distribution-operator-valued fields. Denoting the general tensor-spinor components of any such field assigned to the space-time point with M coordinates, x, by $\hat{\phi}^{\alpha}(x)$, the general matrix elements must transform as,

$$< \Phi' | \hat{\phi}^{\alpha}(x') | \Psi' > = S^{\alpha}_{\beta}(\Lambda) < \Phi | \hat{\phi}^{\beta}(x) | \Psi > , \tag{5.3a}$$

where x' is related to x by (5.1) and $S(\Lambda)$ is a matrix representation of the Lorentz group up to a sign, i.e.,

$$S^{\alpha}_{\beta}(\Lambda_2) S^{\beta}_{\gamma}(\Lambda_1) = \pm S^{\alpha}_{\gamma}(\Lambda_2 \Lambda_1) . \tag{5.3b}$$

The state vectors ($| \mathrm{X} >$ for M and $| \mathrm{X}' >$ for M'), each representing the relationship between an objective state of affairs and the Minkowski frame in question, are, themselves, related by operator valued unitary representations, up to a sign, of the Poincare' group,

$$| \mathrm{X}' > = \hat{U}(\Lambda, a) | \mathrm{X} > , \tag{5.4a}$$

where,

$$\hat{U}(\Lambda_2, a_2) \hat{U}(\Lambda_1, a_1) = \pm \hat{U}(\Lambda_2 \Lambda_1, a_2 + \Lambda_2 a_1) . \tag{5.4b}$$

On all this agreement reigns! What unites the preceding examples is the assignment of quantities, coordinates or fields, to individual space-time points.

Disagreement emerges when one considers the assignment of quantities to individual times , but without the quantities in question also referring to specified individual space-time points. The quantities may or may not actually vary with time, but their assignment is to a definite time without an accompanying assignment to a definite position in space. A simple example of such a quantity would be the total 4-momentum of an open subsystem, S, $\hat{P}_S^{\mu}(t)$. The question then arises of the status of the transform of the quantity and what part of space-time it is assigned to?

Examples of such quantities are (1) the 4-momenta of open subsystems, which are likely to be time dependent but can not be associated with any definite space-time point at a given time by virtue of spacelike extension of the subsystem in question or by virtue of the Heisenberg uncertainty principle, (2) position operators of a system which 'locate', at any time, some localizable property of an otherwise spacelike extended system but which can not be associated with any a priori specified space-time point, and (3) marginal probability distributions at specified times where the quantities for which the probabilities are given are marginal in the sense of not being adequate to determine their own dynamical evolution with time.

The view I wish to criticize, while not new, has recently been very well represented in an important paper (Peres & Terno 2005). While they do not explicitly define the view, I will characterize it here as holding: (1) that some quantities do not have Lorentz transforms at all, (2) that when it exists, the Lorentz transform of any quantity assigned to instants of time in the original inertial frame, M, will, itself, be assigned to instants of time in the transformed inertial frame, M', and (3) that to achieve (2) considerations of dynamical evolution as well as kinematical relations may be admitted to determine the Lorentz transformed quantity. Peres and Terno adhere to this view in the process of employing a time dependent canonical formalism rather than a manifestly covariant formalism for discussing Lorentz transformations.

By contrast, the opposing view which I argue for asserts: (1) that any quantity or any physical state of affairs whatsoever, described from the perspective of any Minkowski inertial frame, has Lorentz transforms to the descriptions of that quantity or that same physical state of affairs from the perspectives of all such frames, (2) that assignment of a quantity to an instant of time in a Minkowski inertial frame is just a special case, of no special status in principle, of the assignment of quantities to arbitrary space-like hyperplanes, which hyperplanes do not change under Lorentz transformations but have their parameterization change since the parameterization of a fixed hyperplane is frame dependent, and (3) that, as a consequence of (2), the latter, generalized mode of assignment, being invariant in kind under all Lorentz transformations, will never require access to considerations of dynamical evolution for the evaluation of the transformed quantities; Lorentz transformations are purely kinematical.

5.1. Existence of Lorentz transforms: For our first example of the clash between these views, consider the claim of PT that, already within classical theory, a partial or marginal Liouville phase space density function in M can not determine a Lorentz transform for M' because the integrated variables, which in general will interact with the remaining variables, are required to determine a transformed density function in M'. This conclusion follows from defining the transformed function as associated with a definite time in M' if the original function was so in M. But a purely kinematical transform exists in M' requiring no access to the integrated variables or any details about their interactions. The kinematical transform is associated with the same space-like hyperplane that was an instant of time in M but is not so in M'. If the remaining variables are the positions and momenta of N point particles (the q'_n and p'_n are 4-vectors orthogonal to the time-like unit 4-vector, η', while the $\mathbf{q}_n$ and $\mathbf{p}_n$ are spatial 3-vectors), and using the abbreviations $\mathbf{q} := (\mathbf{q}_1,--,\mathbf{q}_N)$, $\mathbf{p} := (\mathbf{p}_1,--,\mathbf{p}_N)$, $q' := (q'_1,--,q'_N)$, $p' := (p'_1,--,p'_N)$, the original partial Liouville density, $L(\mathbf{q},\mathbf{p};t)$, in M has the kinematical Lorentz transform, $L'(q',p';\eta',\tau')$, in M', which satisfies,

$$L'(q',p';\eta',\tau')\Pi\delta(\eta' q')d^4q'\Pi\delta(\eta' p')d^4p' = L(\mathbf{q},\mathbf{p};t)\Pi d^3\mathbf{q}\Pi d^3\mathbf{p}\,, \qquad (5.5a)$$

where (in, hopefully, self explanatory notation), $\eta' = \Lambda[1,\mathbf{0}]$, $\tau' = ct + \eta' a$, $q' = \Lambda[0,\mathbf{q}] + a - \eta'(\eta' a)$ and $p' = \Lambda[0,\mathbf{p}]$. The hyperplane parameterized by (η',τ') in M' is the same hyperplane parameterized by $([1,\mathbf{0}],ct)$ in M, i.e., it is the instant of time, t, in M.

More generally, if we begin with the partial Liouville density on the hyperplane, (η,τ), in M, $L(q,p;\eta,\tau)$, the kinematical transform in M' (on the same hyperplane) is $L'(q',p';\eta',\tau')$, where now, $\eta' = \Lambda\eta$, $\tau' = \tau + \eta' a$, $q' = \Lambda q + a - \eta'(\eta' a)$, $p' = \Lambda p$ and,

$$L'(q',p';\eta',\tau')\Pi\delta(\eta' q')d^4q'\Pi\delta(\eta' p')d^4p' =$$

$$L(q,p;\eta,\tau)\Pi\delta(\eta q)d^4q\Pi\delta(\eta p)d^4p\,. \qquad (5.5b)$$

The change, in M, from any $L(q,p;\eta_1,\tau_1)$ to any other $L(q,p;\eta_2,\tau_2)$ is one of dynamical evolution and, in general, requires access to all the interacting variables for its calculation. Similarly for the change, in M', from any $L'(q',p';\eta'_1,\tau'_1)$ to any other $L'(q',p';\eta'_2,\tau'_2)$. But the change from $L(q,p;\eta,\tau)$ in M to the corresponding $L'(q',p';\eta',\tau')$ in M', given by (5.5b), is purely kinematical and is the proper content of the IHLG transform of L from M to M'.

5.2. Spin and spin entropy under kinematical Lorentz transformatons: We turn now from a consideration of a denial of the existence of a Lorentz transform (and a second such denial will be encountered here as well) to a consideration of a claim of very different behaviour under Lorentz transformations for two quantities which have the same tensorial rank, i.e., two 4-vectors. Our system of interest will be closed (originally, for PT, a free single quanton, but all the points to be made here apply to arbitrary closed systems) with self adjoint generators of the IHLG, $\hat{M}^{\mu\nu}$ and $\hat{P}^{\mu}$. The latter is the total 4-momentum of the system and the former contains the total angular momentum and the total internal angular momentum (spin) of the system, which will be our main concern.

By definition, a global 4-vector observable will be represented by a self-adjoint operator, $\hat{V}^{\mu}(\eta,\tau)$, that is assigned to entire individual space-like hyperplanes (parameterized by (η,τ)) rather than to limited or bounded portions of such hyperplanes. Under arbitrary transformations of the IHLG, the expectation values of such an observable in arbitrary pure states will transform as classical global 4-vectors do, i.e.,

$$<\Psi'|\hat{V}^{\mu}(\eta',\tau')|\Psi'>=\Lambda^{\mu}_{\nu}<\Psi|\hat{V}^{\nu}(\eta,\tau)|\Psi>, \tag{5.6a}$$

where, as above, $\eta'=\Lambda\eta$, $\tau'=\tau+\eta' a$, and $|\Psi'>=\hat{U}(\Lambda,a)|\Psi>$, where,

$$\hat{U}(\Lambda,a)=\exp\Big((i/\hbar)\hat{P}a\Big)\exp\Big(-(i/2\hbar)\hat{M}\omega\Big), \tag{5.6b}$$

and, $\Lambda=e^{\omega}$. The transformation, (Λ,a), between the Minkowski frames, M and M' say, relates the expectation values evaluated in the two frames but referring to the same hyperplane differently parameterized by (η,τ) and (η',τ'), respectively. Because of the generality of the state vector, $|\Psi>$, the expectation value equation implies the operator valued equation,

$$\hat{U}^{\dagger}(\Lambda,a)\hat{V}^{\mu}(\eta',\tau')\hat{U}(\Lambda,a)=\Lambda^{\mu}_{\nu}\hat{V}^{\nu}(\eta,\tau). \tag{5.6c}$$

Our interest here is the applicability of this transformation equation to the system 4-momentum, $\hat{P}^{\mu}$, the system Pauli-Lubyanski (PL) operator,

$$\hat{W}^{\mu}=-(1/2)\varepsilon^{\mu\alpha\beta\gamma}\hat{M}_{\alpha\beta}\hat{P}_{\gamma}, \tag{5.7}$$

(where $\varepsilon^{0123}=-\varepsilon_{0123}=+1$) and the system spin operator, $\hat{S}^{\mu}(\eta)$, which will be identified shortly. All three of these operators satisfy (5.6a, c) when substituted for $\hat{V}^{\mu}(\eta,\tau)$. They all transform under the IHLG in exactly the same way. To maximize the distinction between the spin operator on one hand and the 4-momentum and PL operator on the other hand, we will restrict the discussion to the case of a closed system. Extension of the results to the open system case in which $\hat{P}^{\mu}$ and $\hat{W}^{\mu}$ are HD is straightforward.

But PT identify $\hat{P}^{\mu}$ as a 'primary' operator and the system spin as a 'secondary' operator which is 'entangled' with the 4-momentum and transforms differently (PT do not mention $\hat{W}^{\mu}$ and I am hard put to assess how they would classify it). Besides the profound difference between $\hat{P}^{\mu}$ and $\hat{S}^{\mu}(\eta)$ that one is a linear momentum and the other an internal angular momentum, they also differ in that, for a closed system, $\hat{P}^{\mu}$, is completely independent of hyperplane assignment while $\hat{S}^{\mu}(\eta)$ retains, as we will see, a dependence on the orientation of the hyperplane. But, remembering again the discussion in 3, this is a difference in how they change (or not) from hyperplane to hyperplane from the perspective of a given frame. It does not undermine the identity of the way they transform from frame to frame on a given hyperplane! The difference in HD is significant and may well justify the 'primary / secondary' classification, but it has nothing to do with Lorentz transformation.

Now just what is the spin operator, $\hat{S}^{\mu}(\eta)$? It is given, in terms of $\hat{P}^{\mu}$ and $\hat{W}^{\mu}$, by,

$$\hat{S}^{\mu}(\eta) := \frac{\hat{W}^{\mu}}{\hat{M}} - \frac{\hat{P}^{\mu} + \eta^{\mu}\hat{M}}{\eta\hat{P} + \hat{M}} \frac{\eta\hat{W}}{\hat{M}}, \tag{5.8}$$

where $\hat{M} = \left|\sqrt{\hat{P}^2}\right|$. The justification of this identification is provided by the following considerations (in order of increasing importance):

(1) The constraint, $\eta\hat{S}(\eta) = 0$, which follows from (5.8), allows only three independent, space-like components which, for an instantaneous hyperplane, $\eta^{\mu} = [1,\mathbf{0}]$ [1], yields the familiar form, $\hat{S}^{\mu}([1,\mathbf{0}]) = [0,\hat{\mathbf{S}}]$, for a spin operator.

(2) From the commutation relations for the PL operator, $[\hat{W}^{\mu}, \hat{P}^{\nu}] = 0$ and,

$$[\hat{W}^{\mu}, \hat{W}^{\nu}] = i\hbar\,\varepsilon^{\mu\nu\alpha\beta}\hat{W}_{\alpha}\hat{P}_{\beta}, \tag{5.9a}$$

we obtain, $[\hat{S}^{\mu}(\eta), \hat{P}^{\nu}] = 0$ and,

$$[\hat{S}^{\mu}(\eta), \hat{S}^{\nu}(\eta)] = i\hbar\,\varepsilon^{\mu\nu\alpha\beta}\hat{S}_{\alpha}(\eta)\eta_{\beta}. \tag{5.9b}$$

If we now choose ξ_1^{μ}, ξ_2^{μ}, and $\xi_3^{\mu} = \varepsilon^{\mu\alpha\beta\gamma}\xi_{1,\alpha}\xi_{2,\beta}\eta_{\gamma}$, so that, $\xi_1^2 = \xi_2^2 = -1$, $\xi_1\xi_2 = \xi_1\eta = \xi_2\eta = 0$, then we find, $\xi_3^2 = -1$, $\xi_1\xi_3 = \xi_3\xi_2 = \xi_3\eta = 0$, and

$$[\xi_1\hat{S}(\eta), \xi_2\hat{S}(\eta)] = i\hbar\,\xi_3\hat{S}(\eta), \tag{5.10}$$

along with the cyclic permutations thereof. For instantaneous hyperplanes these are just the familiar spin commutation relations for spatial, Cartesian components.

(3) The square of the spin operator is equal to the ratio of the square of the PL operator and the square of the 4-momentum. For closed systems this is the ratio of the Casimir invariants for the IHLG and is not HD, i.e.,

1. For any 4-vector, V^{μ}, $V^{\mu} = [V^0, \boldsymbol{V}]$.

$$\hat{S}(\eta)^2 = \hat{S}^2 = \hat{W}^2 / \hat{M}^2 \to -\hbar^2 s(s+1), \qquad (5.11)$$

the last arrow indicating the value for an irreducible representation of the IHLG.

If a Lorentz transformation were to relate the spin on an instantaneous hyperplane in Bob's frame to the spin on an instantaneous hyperplane in Alice's frame (to use the popular Alice-Bob language employed by PT), then the combined momentum dependence and HD of the spin operator would require access to the momentum structure of the quantum state in order to calculate the spin transform expectation value. For example, consider Bob's reduced spin density operator,

$$\hat{\rho}_B := Tr_{P/S}(|\Psi_B><\Psi_B|), \qquad (5.12)$$

where the subscript, P/S, denotes that the trace is taken over all those dynamical variables, including the total 4-momentum, required to define a system comset, but excluding only the spin, S, and with the spin evaluated at $\eta_B^\mu = [1,\mathbf{0}]$. This quantity would have no Lorentz transform, as PT conceive it, that could be evaluated in Alice's frame, based solely on its value in Bob's frame and the relation between the frames. In particular, PT point out, the spin entropy, $Tr_S(\hat{\rho}_B \ln \hat{\rho}_B)$, could have value zero for Bob, due to $|\Psi_B>$ being a spin eigenstate for Bob, while $|\Psi_A> = \hat{U}(\Lambda_{AB}, a_{AB})|\Psi_B>$ would not be a spin eigenstate at $\eta_A^\mu = [1,\mathbf{0}]$ for Alice and $Tr_S(\hat{\rho}_A \ln \hat{\rho}_A)$ could have various values depending on details of the state vectors washed out by the partial tracing.

But the state vector does not lose spin eigenstate character due to passing from Bob's frame to Alice's. It loses spin eigenstate character due to dynamical evolution from hyperplanes that are instantaneous in Bob's frame to those that are instantaneous in Alice's frame. This dynamical evolution occurs within Bob's frame alone in going from $\eta_B^\mu = [1,\mathbf{0}]$ to $\eta_B^\mu = (\Lambda_{AB}^{-1})[1,\mathbf{0}]$ and in Alice's frame alone in going from $\eta_A^\mu = \Lambda_{AB}[1,\mathbf{0}]$ to $\eta_A^\mu = [1,\mathbf{0}]$. On any hyperplane for which the state is a spin eigenstate for Bob, it will also be a spin eigenstate for Alice with the same eigenvalue and for the 'same' component of spin. To see this suppose that,

$$\xi_B \hat{S}(\eta_B)|\Psi_B> = |\Psi_B> \hbar\mu, \qquad (5.13a)$$

where ξ_B is a space-like unit vector orthogonal to η_B. Then with, $\eta_A = \Lambda_{AB}\eta_B$ and $\xi_A = \Lambda_{AB}\xi_B$, we have,

$$\xi_A \hat{S}(\eta_A)|\Psi_A> = (\Lambda_{AB}\xi_B)\hat{S}(\Lambda_{AB}\eta_B)\hat{U}(\Lambda_{AB}, a_{AB})|\Psi_B> =$$

$$\hat{U}(\Lambda_{AB}, a_{AB})(\Lambda_{AB}\xi_B)\hat{U}^\dagger(\Lambda_{AB}, a_{AB})\hat{S}(\Lambda_{AB}\eta_B)\hat{U}(\Lambda_{AB}, a_{AB})|\Psi_B> =$$

$$\hat{U}(\Lambda_{AB}, a_{AB})(\Lambda_{AB}\xi_B)(\Lambda_{AB}\hat{S}(\eta_B))|\Psi_B> = U(\Lambda_{AB}, a_{AB})\xi_B \hat{S}(\eta_B)|\Psi_B> =$$

$$\hat{U}(\Lambda_{AB}, a_{AB}) | \Psi_B > \hbar\mu = | \Psi_A > \hbar\mu \;, \qquad (5.13b)$$

as claimed.

Dynamical evolution from one hyperplane to another changes the spin entropy in both frames. Lorentz transformation between the frames on a given hyperplane does not. Entropy is a Lorentz scalar. Failure to employ the HD language can obscure these matters!

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