IS ANY THEORY COMPATIBLE WITH THE QUANTUM PREDICTIONS NECESSARILY NON LOCAL ?

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Introduction

Important advancements in scientific knowledge may, not infrequently, be looked at from several angles. Here, for investigating the question the article title defines, part of our attention shall have to be turned on the relationships of Bell's theorem with the old, but still not fully cleared up, question of the nature of causality.

The word "nonlocality" means the violation of local causality, the initial meaning of which was that no influence, that is, no causal *action*, can be propagated faster than light. For a long time it was a received view that actions somewhat akin to the -basically time-directed- ones that living beings perform also take place within inanimate nature. This conception of causality may be called the *causality-action theory.* But later, having in view Hume's serious reservations concerning it, renowned philosophers of science claimed it might and should be dropped since the effects it was meant to account for could be explained just as well by considering that what is at work is simply a conjunction of physical laws expressed by means of differential equations. A view that later Hempel and Oppenheim generalized and called the D-N (deductive-nomological) model of explanation (reprinted in (Hempel, 1965). Here the (widely accepted as it seems) corresponding conception of causality will, for simplicity, be called the *law-centered theory of causality.*

But does the latter theory subsume, at least within physics, all the acceptations of the word 'cause' that true understanding calls for ? Among philosophers the matter is not settled. Hempel himself granted that in several respects the D-N model is somehow inadequate for analyzing the cause-effect relationship (Hempel, 1988). And it seems indeed that a significant argument favoring a 'no' answer to the question might have been put forward as early as in year 1907 (although, to our knowledge, it was not), when Einstein pointed out that according to his, then two years old, Special Relativity

(SR) theory, if we assumed an effect could follow its cause sooner than light velocity permits then in some other frames of reference it would occur before its cause. For, concerning this consequence of the assumption he wrote: "... in my opinion, regarded as pure logic... it contains no contradiction; however it absolutely clashes with the character of our total experience, and in this way is proved the impossibility of the hypothesis" (Einstein, 1907). Now, through the words "it contains no contradiction" Einstein granted that the premiss he invoked for inferring the limitation (local causality) inherent in the said impossibility could logically follow neither from the basic laws of pre-relativistic classical physics (be it only because it is time-directed while the said laws are not) nor, clearly, from the two basic postulates than induce the whole SR (relativity and independency of the speed of light from the speed of its source). In other words he then supplemented SR with a new element and the type of causality this element calls for is not reducible to the law-centered theory of causality. It seems therefore that it is only by referring to something akin to the older causality-action theory of causality, interpreting causality as a time-directed action of the cause on the effect, that the meaning of a local causality notion can be made fully explicit. To discriminate the resulting conception of local causality from other definitions of it that have been or could be proposed it is called here 'local causality in the first sense'.

Now, while the fact that Einstein thus indirectly invoked the causality-action theory is not of course a sufficient reason for taking it to be scientifically valid, still it should prevent us from branding it as unscientific just on the basis of controversial philosophical arguments. And anyhow for disproving local causality Bell clearly had to take the said theory into consideration, which in, for example, his last paper on the subject (Bell, 1990) he did through the words "The direct causes (and effects) are near by", which set in relief the notion of *direct* causes operating between purely physical events.

Two problems however arise. One of them bears on the exact nature of the premisses Bell had to assume. The other one, expressed by the very title of this article, is to infer from this study whether or not some remarkable general premiss such as realism must be assumed for a watertight proof of nonlocality. They are investigated in Sections 2 and 3 respectively. Section 4 draws conclusions.

Introduction

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Bell's premisses

In the course of years Bell developed several versions of his proof, described in the articles (Bell, 1964), (Bell, 1976), (Bell, 1981) and (Bell, 1990) respectively. The premisses of the first one still have close links with the EPR paper. In it the EPR criterion of reality is implicitly made use of, leading, through the consideration of a special case (an EPRB type experiment with parallel orientations of instruments) to a proof of determinism. Now, the said criterion and its implications –this one in particular– are somewhat generally held in suspicion within the physicists community. Moreover the article leads to an inequality that, presently at least, is not experimentally testable. For these reasons and shortness sake here its premisses are not to be analyzed. We shall focus on those of the three other papers, the general purpose of which is (and merely is) to look for an answer to a question that, in Section 3 of (Bell, 1976), Bell, in substance, stated as follows: could it not be that quantum theory is a fragment of a, more complete, hidden variable theory that, contrary to standard quantum theory, *has* local causality ?

2.1 The 1976 premisses

In non-deterministic theories the fact that 'local causality in the first sense' is grounded on the causality-action notion raises special difficulties. Which presumably is why in the 1976 paper Bell propounded an acceptation of the words "local causality" making the notion compatible with the law-centered theory of causality and apt therefore to be subjected to mathematical analysis, while maximally preserving, of course, the main features of our intuitive notion of what a cause is. It is in this spirit that, in that article, he defined locally causal theories as being those in which, R and R' being two finite, spatially separated space-time regions, the probability for an event A to happen in R cannot depend from any event B happening in R' when all the beables (roughly: 'elements of reality') within the overlap N of the backward light-cones of R and R' are specified. This condition of completeness is indeed necessary for taking into account the fact that events in N may well constitute causes common to both A and B and for preventing an information about B from affecting thereby the probability $P(A)$ of A without B being in any way a direct cause of A in the causality-action sense. With the consequence that it is only when all the beables in N are specified, hence assumed fixed, that absence of any direct causal action of B on A implies that the probability of A is independent from B. In other terms, it is only when the just stated "condition of completeness" is fulfilled that from 'local causality in the first sense' it is possible to infer consequences by applying standard probability rules, and that in particular it is possible to make use of the standard rule

$$P(A,B) = P(A|B)P(B)\,, \tag{2.1}$$

(where $P(A,B)$ is the "probability of both A and B" and $P(A|B)$ the "probability of A if B") and infer from it and the just inferred independence of $P(A|B)$ from B the factorization

$$P(A,B) = P(A)P(B) \tag{2.2}$$

that is, the mutual independence of the probabilities of A and B.

The rest of Bell's reasoning is well known. From the said independence and with the help of straightforward mathematical calculations Bell inferred an inequality (called 'Bell's inequality') and pointed out that, at least with some choices of the orientations of the instruments, it is at variance with both the quantum mechanical predictions and the experimental outcomes. From which he could conclude that 'local causality in the first sense' is disproved for all theories in which all the beables in the overlap N of the two backward light cones are specified. And it is only to such theories that, in this article, Bell gave the name "locally causal theories". Only they, therefore, are disproved by the article in question. Not all of those, if there are any more, merely satisfying 'local causality in the first sense'.

In the paper Bell stated his hope that gain in precision might be possible by concentrating on the notion the word 'beable' expresses rather than on the wooly one of "observables". Because by definition beables, he wrote, "are there". These two last terms, "are" and "there" neatly summarize in fact Bell's thinking as to what may be called realism and its necessity in physics. The need for the first one, a mood of "to be", springs, in his view, from the fact that a theory can truly be precise only if it describes "a bit of what exists [is], thought of independently of its modification through observation", as Einstein wrote (Einstein, 1949); a conception that may be called "Einsteinian realism". And as for the second, the word "there", apparently it is just a shortening for "embedded in space (or spacetime)". In that sense however the notion is still quite extensive (total energy, for example, is a beable). Most of the observables we deal with (instruments settings in particular) can be assigned to some bounded space-time region, and the corresponding beables Bell calls local.

2.2 The 1981 premisses

So, in the 1976 paper realism is unquestionably a premiss, explicitly stated and considered necessary for the reached conclusion to be valid. And, moreover, in it the class of the locally causal (and finally refuted) theories is, as shown above, subject to a definite limitation due to recourse to the condition of completeness mentioned above. Most remarkably in the 1981 paper, which aimed at maximal generality, Bell got rid of both conditions. There, neither the word "beable" nor any reference to the standard rule (2.1) (that caused the said limitation) appears. And still, just as the foregoing one, this article concluded in favor of local causality violation. A question thus arises: were these limitations truly necessary for proving the latter ?

The matter is worth a detailed examination. Since, in this domain, only correlations are experimentally reachable, in (Bell, 1981) Bell first considered a simple example of correlations and proposed next to transpose it, mutatis mutandis, to the experiment under study. To begin with, he noted that although heart attacks are typically stochastic, still, between the ones that take place in two distant cities not interacting in this field -Antwerp and Brussels say- statistics show significant correlations. But they are easily explainable, he pointed out, by attributing them to the fact that the probability of an attack depends on several causal factors, such as especially hot days or eating too much on Sundays, that are the same in the two cities and that, taken together, constitute the state λ of the outside world at a given time.

To investigate the pertinence of the planned transposition let us take a close look at this example. We observe first that when the said causal factors do not change -let it be granted that in one minute time this is the case- the probabilities $P(A)$ of one attack in Antwerp and $P(B)$ of one attack in Brussels are independent. So that, between them and the probability $P(A, B)$ that one attack in Antwerp and one in Brussels take place simultaneously to within one minute time, equality (2.2) above holds good. And clearly this is true during ordinary days (world state λ_1) as well as during hot days (world state λ_2), even though the involved probability values are not the same in the two cases. On the other hand, an elementary calculation shows that when they are not the overall, mean probability, calculated, say, within a one year period, that within any given minute an attack takes place in Antwerp and another one in Brussels is not a product of the similarly evaluated mean probabilities concerning Antwerp and Brussels separately. A correlation then exists, obviously due to the fact that within one year time the involved causal factor -here temperature- varied, so that we had to consider a combination of the two states λ_1 and λ_2 we assumed it could be in. To recover factorization we have to separate, by thought, the set of the minutes composing a whole year into two subsets E_1 and E_2 composed respectively of all the minutes of ordinary days ($\lambda = \lambda_1$) and all the minutes of hot days ($\lambda = \lambda_2$). At all the minutes composing one of them we assume the considered causal factor (temperature) is the same, so that in each of them $P(A, B)$ factorizes, which may be written:

$$P\left(A, B|a, b, \lambda_i\right) = P\left(A|a, \lambda_i\right) P\left(B|b, \lambda_i\right) \quad ; i = 1, 2 \tag{2.3}$$

where a and b are stable data specific to Antwerp and Brussels respectively, such as, say, the average sanitary level in the two cities. In each one of the two sub-ensembles independence of the probabilities is thus recovered (note in passing that in (Bell, 1981) the sentence introducing (2.3) (eq. (10) of the paper) is not entirely clear: Since this formula is an equality it can be true only if *all* causal factors are kept fixed: And then what Bell calls 'the residual fluctuations' are just the probabilities of attacks, assumed intrinsic and independent as a premiss).

This example, of course, is highly schematic. In reality many causal factors such as temperature intervene simultaneously. In the general case symbol λ_i stands for many numbers each one of which is the value one of these causal factors takes in sub-ensemble E_i. We assume here for simplicity that the number of causal factors is finite and that each one can take but a finite number of values; and we say that λ_i is complete if it specifies the value that every one of the existing causal factors has within E_i. Factorization (2.3) then applies for the said λ_i. The number of such λ_i may of course be very large and is unknown in general even though the above assumptions make it finite.

The idea that feasible measurements bearing exclusively on one particular E_i could actually be performed is unbelievable. It may in fact be considered that actual measurements can only bear on the above mentioned mean overall probability, which is the mean value $P(A, B|a, b)$, over all λ_i, of $P(A, B|a, b, \lambda_i)$. If $\rho(\lambda_i)$ is the (unknown) probability of λ_i this, because of (2.3), yields:

$$P(A, B|a, b) = \sum_i \rho(\lambda_i) P(A|a, \lambda_i) P(B|b, \lambda_i) \tag{2.4}$$

and, as we know, deriving from (2.4) the Bell or CHSH inequalities is just a matter of straightforward mathematical calculations. Were we to fancy (just for the sake of the argument since within this purely classical problem there is no reason to expect they should) that statistical results are at variance with the said inequalities, we would have to grant that our premisses were incomplete. One possibility is that parents of attack victims in Antwerp phoned the information to relatives in Brussels who were so moved that they themselves suffered attacks: for it is easily seen that under such an assumption the inequalities in question cannot be derived from (2.4). But note that this is assuming a typical direct action phenomenon of a cause on its effect.

Bell's transposition of this example to the study of experiments of the EPRB type simply consisted in the conjecture –hereafter called "the Conjecture"– that in these experiments observed correlations are explainable in the same way as those of the said example. More precisely, what is assumed in this Conjecture is that, when, on a given pair, Alice's instrument, oriented along a, registers outcome A and, on the same pair, Bob's one, oriented along b, registers outcome B the joint probability $P(A, B|a, b)$ must be given by formula (2.4) [(12) of (Bell, 1981)], from which the Bell and CHSH inequalities follow. Their experimental violation in the EPRB experiments therefore reveals, just as above, incompleteness of the premisses. And for adequately supplementing it, since the Conjecture was endorsed, no other way appears than to assume, as in the attack example above, that some direct causal influence of Bob's registered outcome B on the one, A, Alice registered indeed took place or vice versa. Which, when the two measurements are spacelike separated, implies a violation of 'local causality in the first sense'.

Compared to the way of justifying non-locality Bell had put forward in (Bell, 1976) this one is advantageous in several respects. First, it is more general: as we saw it has no need of the "beable" notion (in it Bell's mention of causal factors being "kept fixed" concerns but an intermediate stage of the argument), nor, as Bell himself pointed out, of any localization of the λ_i, the causal factors inside which might even involve wave functions. On these grounds it may seem that it does not call forth realism as a necessary premiss (a question to be investigated further in Section 3). Moreover, it would seem that it justifies rejecting *any* theory postulating 'local causality in the first sense' whereas, as we also saw, the theories the 1976 approach disproved were those called there “locally causal” in a somewhat restricted acceptation of the phrase.

On the other hand however, this way of proceeding also has the serious inconvenience that it implies endorsing the Conjecture and that the arguments that might justify this move are not crystal-clear. For indeed the attack model is grounded on the ideas, taken as premisses, that heart attacks are basically random, that in the absence of variable causal factors (such as temperature) the probabilities of an attack occurring in Antwerp and of one occurring in Brussels are independent, and that the need for considering subsets E_i and the corresponding labels λ_i essentially comes from the one of, by thought, grouping into sets and subsets the events in which every causal factor keeps the same value. Now, while in the model all these various ingredients (essential for the reasoning) are quite clear, not every one of them has a clear parallel in an EPRB experiment made on quantum systems. True there are similarities: In the model causal factors really exist, and in a quantum theory admitting of hidden variables it is assumed that the latter also exist. But on the other hand in the model eq. (2.4) could be derived from eq. (2.3) because (a) both involve probabilities and (b) for any λ_i these probabilities are mutually independent by assumption. In the case of an EPRB quantum experiment admitting of hidden variables the theory also involves probabilities because a definite wave-function is part of the premisses. But then independence, for any given λ_i, of the probabilities appearing in (2.3) is, to say the least, problematic. So it seems that in such an EPRB experiment eq. (2.4) cannot be properly derived. Or, more precisely, it appears that the mode of derivation of (2.4) made use of in (Bell, 1981) cannot be meaningfully carried over to such experiments. Which implies that, when dealing with the latter ones, the Conjecture, a vital element of the 1981 paper, can merely be considered a hypothesis or, more appropriately, just a definition of what it is decided to call “local causality”. And, apparently, it is indeed in this sense that it was understood by physicists who, not as convinced as Einstein and Bell were of the necessity of coming back to a realism of some sort, aimed at interpreting the latter’s work on the subject as proving non-locality quite independently from realism. But, as it seems, the proof thus constructed, interesting and valuable as it is, still is one of a violation of a notion whose relationship with ‘local causality in the first sense’ is not crystal clear.

2.3 The 1990 premisses

While, apparently, Bell did not point out in writings the hardly binding character of the Conjecture, he surely was aware of it and this is probably the reason why, in his 1990 paper, he preferred strict inference to maximal generality and substituted to his 1981 approach a procedure very much akin to the 1976 one. Indeed, both explicitly invoke realism and make use of the word 'beable'; which confirms the continuity of Bell's strong belief in realism, considered necessary, not because of obscure metaphysical reasons but simply because he held it to be one of the main conditions a physical theory must satisfy in order to achieve sufficient preciseness.

Another strong analogy between these two modes of proof is that both make use of the standard probability rule (2.1). In sub-Section 2.1 we saw how from this rule and the local causality hypothesis Bell could derive the Bell-CHSH inequalities, by preventing that an information about B might affect the probability $P(A)$ of A without B being in any way a direct cause of A in the causality-action sense. To that end *it is necessary that every beable a variation of which might cause events to happen jointly in both R and R' should be kept fixed*, a requisite that, for future reference, will be called here "*Condition C*". It is easily seen that assuming all the parameters (beables) in region N are specified, hence fixed, indeed fulfills Condition C. But it so happens that this region is not the only one in this respect and in [(Bell, 1990)] Bell made the same assumption concerning another one he called "Region *3*" and which is suitable as well. It is a spacelike one that cuts both backward lightcones in the future of region N.

Let it be recalled that in any theory fulfilling Condition C the experimentally observed violation of the CHSH inequality cannot be explained otherwise than by assuming some direct causal action of B on A (or of A on B). Since these events are spacelike separated such a causal action constitutes a violation of 'local causality in the first sense', which is indeed what Bell actually meant to prove. On the other hand, here as in (Bell, 1976), Condition C restricts the types of theories for which nonlocality is thus proven; and, admittedly, it might be feared that it restricts them to an unsatisfactory degree. In this respect however, it should be noted, as T. Norsen appropriately pointed out (Norsen, 2011), that Condition C may well be taken to merely concern the particular hypothetical theory that is being considered as a possible candidate. It then does not mean we should "know everything" concerning a specified space-time region but merely that, given a candidate theory, the space-time region in question should include all of the beables (called λ here) the existence of which is assumed by this theory.

Bell's premisses

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Is realism a necessary premiss for proving nonlocality ?

For most powerful reasons he often explained John Bell, as we already stressed, was a strong supporter of Einsteinian realism. His papers were written in this spirit and it was noted above that indeed realism has a significant role in most of his proofs of non-locality and in particular in the one to which, finally, he seems to have attached the greatest weight. To a large extent this justifies the often appearing statement that Bell disproved 'local realism'. On the other hand, such a statement suggests the possibility of a choice. It seems it implies not only that realism and locality cannot both be true but also that if realism is not assumed locality might conceivably be preserved. Now it has been recently claimed that this, in fact, is not the case. That what is really proved goes beyond this, in that it refutes locality independently of whether or not any additional premiss such as realism is assumed. This section is an attempt at clarifying this question. Which of course forces us to, provisionally at least, set aside the afore-mentioned Bell's reasons for advocating realism.

One of the arguments (see e.g. (Gisin, 2012)) by means of which the just mentioned thesis was defended went as follows. It started from the observation that the correlation between Alice's and Bob's measurement outcomes "may actually arise out of a statistical mixture of different situations" traditionally labelled λ. It then proceeded in four steps, namely:

1. writing down the standard probability rule (2.1) as:

$$P(A, B|a, b, \lambda) = P(A|a, b, B, \lambda)P(B|a, b, \lambda) \tag{3.1}$$

2. defining the locality assumption to be that for any λ, "what happens on Alice's side does not depend on what happens on Bob's side, and vice versa";

3. inferring from this that:

$$P(A|a,b,B,\lambda) = P(A|a,\lambda) \tag{3.2}$$

   and

$$P(B|a,b,\lambda) = P(B|b,\lambda) \tag{3.3}$$

4. inserting (3.2) and (3.3) into (3.1), which yields, for each λ, the factorization

$$P(A,B|a,b,\lambda) = P(A|a,\lambda)P(B|b,\lambda) \tag{3.4}$$

from which the Bell-CHSH inequalities are derived in the standard way recalled above; and local causality is said to be thereby refuted. It is then pointed out (as Bell himself had done in ((Bell, 1981)) that the λ's may specify any state of affairs in the outside world, including even quantum states, the notion of which is generally not considered to particularly rely on realism; and that this also holds true concerning the instrument orientations a and b, which may be considered mere observables. In view of this all it is finally claimed that proving nonlocality necessitates no additional premiss and in particular no reference to realism.

Now, this is a conclusion that might conceivably have been drawn from Bell's 1981 approach, but it was shown above that the latter was not really convincing. The point here is that the reasoning above is at variance with the stand Bell finally took in his 1990 article, in which, as we saw, he stressed the necessity, for the argument to go through, of fulfilling Condition C above. And indeed, bearing this in mind it must be observed that, in Step 2 above, the notion 'dependence' is ambiguous. In one acceptation of the word it means that B is one of the direct causes of A in the traditional causality-action sense, whereas in probability calculus it has the wider meaning that A depends on B whenever the probability $P(A)$ of A depends on B, the dependence being due either to B being the direct cause of A or to the fact that A and B had a common cause in their past. If what is required is that the investigated notion should somehow be connected with local causality in the causality-action sense (i.e. in what we called the "first sense", the one Bell clearly adopted) the acceptation to be retained is clearly the first one. But the fact that A does not depend on B in this sense or, in other terms, that B is not a direct cause of A, does not imply that the probabilities $P(A)$ and $P(B)$ are independent –in other words it does not imply factorization of $P(A,B)$– since, to repeat, A and B may have shared a common direct cause in their past. Hence the reasoning fails.

On the other hand a remark by T. Norsen (Norsen, 2006) may dissuade us from being categorical on this point for, while she agreed on the necessity of taking Condition C into account she pointed out that the latter “commits us, really, to nothing”, presumably meaning by this that theories are produced by us and that therefore *we* decide of what premisses we impart to them; that, for example, we may freely decide that in region *3* of (Bell, 1990) just one causal factor exists, which is the pair wave function, and that it is specified, which just means it is kept the same in all of the measurements on individual pairs that compose a given experiment. This is true, but still it does not bar out the possibility that the not yet discovered ‘truly correct’ theory, the one describing nature ‘as it really is’, is not the one we have in mind. So, as we see, the conclusion of the just reported reasoning may still be retained, but only provided the view (deemed obsolete by many) that nature obeys fixed eternal laws that are not yet known (and may never be) be definitely rejected.

Is realism a necessary premiss for proving nonlocality ?

4

Back to the question raised by the article's title and conclusion

As we just saw, the validity of the statement that non-locality is true quite independently of whether or not Einsteinian realism is true cannot be completely salvaged in the mind of any thinker, even by taking Norsen's remark into account. But anyhow, in order to answer the question this section is meant to deal with the safest procedure could well be another one, namely the one of searching for a conception compatible both with local causality and with all the verifiable predictions from quantum mechanics. If, by any chance, one can be found, the question will automatically be answered in the negative. Now, we know already that no realist theory (in the sense of Einsteinian realism) can satisfy both requisites. But conceivably some non-realist one might.

Admittedly the conception we think of lies rather far from beaten tracks. It is suggested by significant aspects of Rovelli's *Relational Quantum Mechanics* (Rovelli, 1996), a theory quite a number of physicists deem acceptable. From the latter, it borrows the idea that the notion 'relativity' should be extended to the perceptions individuals have of the properties (attributes) of systems. Same as the speed of a vehicle is not the same relative to someone standing on the sidewalk and relative to a driver overtaking it, this conception assumes that the properties of any object must not be considered attached to that object but rather to its observers, and are therefore relative to each observer observing it. As we see, it is clearly centered on observation and information. It is incompatible with Einsteinian realism, and since realism has here been identified with the latter we should indeed consider it to be a variety of anti-realism.

Let one of the most salient feature of this theory be briefly reminded. As we know, in the eighty years old EPR reasoning the, there defined, notion 'elements of reality' enjoys an absolute meaning in the sense that once the existence of one such element has been duly derived it is valid for any observer whatsoever (in its name the word 'reality', akin to Bell's 'beable', is meant to stress precisely that). In an EPRB type experiment once, via a reasoning making use of local causality, the value of some quantity has been shown to be an element of reality it is the said absoluteness that entails that the value in question remains the same also when, instead of the measurement that induced it, the measurement of some other quantity is performed; and that also entails it must be the same for any observer. This is what made it possible for EPR to infer that quantum mechanics is not a complete theory. But within the here considered non-realist conception such a derivation is impossible for if we tried to define in it, in EPR's way, some kind of 'element of reality' notion, the latter could not enjoy absoluteness since it would be necessary to distinguish within it the 'elements of reality' relative to the various observers; those, for example, relative to Alice and to Bob respectively in an EPRB experiment. From this it follows that the well-known reasoning by means of which EPR proved the incompatibility between standard quantum mechanics (without hidden variables) and local causality does not go through. In other words the here considered conception appears compatible with local causality.

Of course the question then arises whether or not its compatibility with the quantum mechanical observational predictions can be refuted in Bell's manner, by showing it entails an inequality of the type of those of Bell and CHSH. But it is easily seen it can't. For indeed according to the conception in question the set of notions that cannot be defined so that they should be observer-independent includes not only elements of reality but also the probabilities that some definite observation should be made. In it, for example, in the second member of equation (2.4) the probability $P(A|a, \lambda_i)$ has a meaning only for Alice. For Bob it does not have any, which implies of course that, for Bob, its product with any given number has no sense either. For Alice the reverse is true: neither $P(B|b, \lambda_i)$ nor its product with any number has a meaning. Which implies that the product $P(A|a, \lambda_i)P(B|b, \lambda_i)$ has a meaning neither for Bob nor for Alice (nor for any other observer of course). Consequently a joint probability such as $P(A, B|a, b)$, as yielded by (2.4), simply has no meaning at all. And since the significant terms in both the Bell and the CHSH inequalities are ultimately composed of such expressions it follows that the said inequalities are meaningless as well. Their possible incompatibility with the quantum mechanical predictions and with experiment is therefore void of signification.

A non-realist conception borrowed from an interpretation of the quantum mechanical formalism considered acceptable and interesting by a number of physicists has been described, whose incompatibility with local causality cannot be established by any presently known method. Admittedly this result is *not* sufficient for proving that, to derive non-locality from the validity of the quantum observational predictions, realism must be assumed. The reason is that, in this respect, what we saw is merely a counter-example to the view that such a derivation is possible in complete generality, without anything else being postulated, which only means that some other premiss must be assumed, not that the latter *must* be realism. On the other hand, presently, for efficiently playing that role no other premiss is available, so that the result in question reinforce an impression the reading of Bell's articles already imparts, the one that assuming realism is by far the safest way to establish non locality on truly firm grounds. And anyhow, to the question that forms the title of this article it gives with certainty (except for people bluntly rejecting anything approaching the Rovelli approach !) a definite answer: the answer 'no'.

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Note 1. All the Bell articles here referred to are reprinted in Bell's *Speakable and unspeakble in quantum mechanics*, Cambridge University Press, Second ed. 2004.

Back to the question raised by the article's title and conclusion